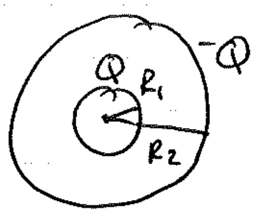
Tutorial 7

P&M, 3.28

Maximizing the energy of a capacitor.
Spherical capacitor of radius R_1, R_2.



$$\Delta\phi = Q/C$$

$$= -\int_{R_2}^{R_1} dr \, \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{C} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = 4\pi\varepsilon_0 \Big/ \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Take R_2 fixed (limiting total volume) and
max $\vec{E}$ at inner surface is E_0
(avoid electrical breakdown - spark)

$$E \text{ at } R_2 \text{ is } E = \frac{Q}{4\pi\varepsilon_0 R_1^2} \le E_0$$

So max charge is $Q = 4\pi\varepsilon_0 R_1^2 E_0$

Energy:
$$U = \frac{Q^2}{2C} = \frac{(4\pi\varepsilon_0 R_1^2 E_0)^2}{2 \cdot 4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

maximize wr.t $R_1 \rightarrow R_1 = 3/4 \cdot R_2$

E_0 at R_1, then $E(R_2) = \dfrac{E_0 R_1^2}{R_2^2}$

$$E(r) = \frac{E_0 R_1^2}{r^2}$$

$$U = \frac{\varepsilon_0}{2} \int_{R_1}^{R_2} dV \left(\frac{E_0 R_1^2}{r^2}\right)^2 = \frac{\varepsilon_0}{2} \cdot 4\pi \int_{R_1}^{R_2} r^2 dr \, \frac{E_0^2 R_1^4}{r^4}$$

$$= \frac{\varepsilon_0}{2} \cdot 4\pi \, E_0^2 \, R_1^4 \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

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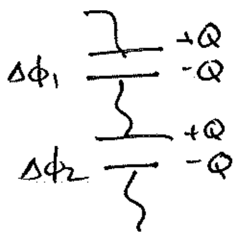
<u>problem 3.18</u> : Adding capacitors.

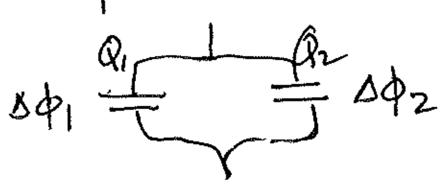
Suppoce we have two capacitors C_1 & C_2 in series. what is net capacitance C?



$$\Delta\phi = \Delta\phi_1 + \Delta\phi_2 = \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

In parallel:



same potential
$\Delta\phi_1 = \Delta\phi_2 = \Delta\phi$

$$Q = Q_1 + Q_2 = C\Delta\phi$$

$$= C_1\Delta\phi + C_2\Delta\phi = C\Delta\phi$$

$$\Rightarrow C_1 + C_2 = C.$$