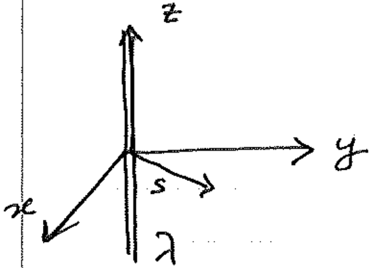
Tutorial #8

Problem 1:



$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\,\hat{s}$$

(a) $$\phi_{21} = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{ds}\cdot\vec{E} = -\int_{s_1}^{s_2} ds\,\frac{\lambda}{2\pi\varepsilon_0 s} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{s_2}{s_1}\right)$$

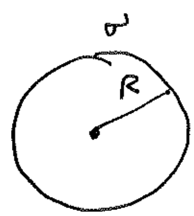
$$s_1 = d\,,\quad s_2 = d \;\rightarrow\; \phi_{21} = 0$$

(b) $$s_1 = d\,,\quad s_2 = d\sqrt{3^2+4^2} = 5d$$

Problem 2

$$W = q\,\phi_{21}$$

Problem 3

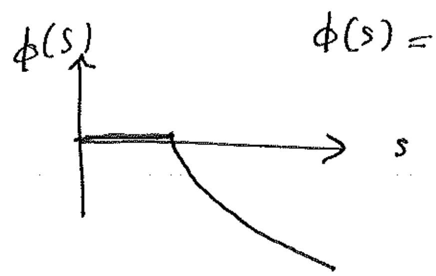


Gauss's law: $$\vec{E} = \begin{cases} 0 & s<R \\ \dfrac{\lambda}{2\pi\varepsilon_0 s}\,\hat{s} & s>R \end{cases}$$

$$\phi(s) = -\int_R^s \vec{ds}\cdot\vec{E}(s')$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{s}{R}\right) \qquad s>R$$

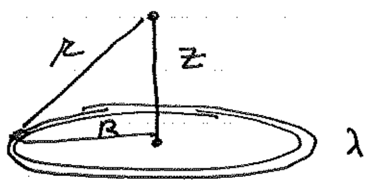
$$\phi(s) = 0 \qquad s<R$$



Problem 4

$$\vec{E} = -\vec{\nabla}\phi = -\frac{\partial \phi}{\partial s}\hat{s} = \begin{cases} \dfrac{\lambda}{2\pi\varepsilon_0 s}\hat{s} & s > R \\ 0 & s < R \end{cases}$$

Problem 5



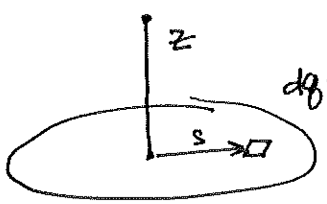
$$r = \sqrt{R^2+z^2}$$

$$dq = R\,d\theta\,\lambda$$

$$\phi(z) = \int \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda R}{4\pi\varepsilon_0}\int_0^{2\pi} d\theta \frac{1}{\sqrt{R^2+z^2}}$$

$$= \frac{\lambda R}{2\varepsilon_0\sqrt{R^2+z^2}}$$

Problem 6



$$dq = \sigma\, s\,ds\,d\theta \qquad r = \sqrt{s^2+z^2}$$

$$\phi(z) = \int \frac{dq}{4\pi\varepsilon_0 r} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi} \frac{s\,ds\,d\theta}{\sqrt{s^2+z^2}}$$

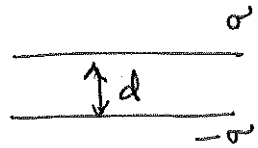
$$= \frac{\sigma}{4\pi\varepsilon_0} 2\pi \int_0^R ds \frac{s}{\sqrt{s^2+z^2}}$$

let $u = s^2+z^2 \longrightarrow du = 2s\,ds$

$$\phi(z) = \frac{\sigma}{4\varepsilon_0}\int_{z^2}^{R^2+z^2} du \frac{1}{u^{1/2}} \cancel{= \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - \sqrt{z^2}\right) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - |z|\right)$$

Problem 7



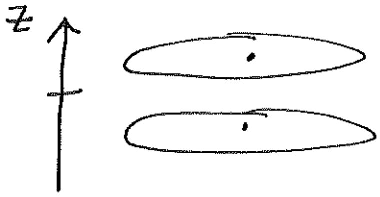
$$\Delta\phi = \frac{\sigma d}{\varepsilon_0}$$

Problem 8

$$\Delta\phi = \frac{Q}{A}\frac{d}{\varepsilon_0} = \frac{Q}{C} \rightarrow C = \frac{A\varepsilon_0}{d}$$

$$A = \pi R^2$$

Problem 9



$Q \rightarrow \sigma = \frac{Q}{\pi R^2}$

$-Q \rightarrow \sigma = -\frac{Q}{\pi R^2}$

plate 1: $\Delta\phi_1 = \phi(-L) - \phi(0)$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+L^2} - L\right) - \frac{\sigma}{2\varepsilon_0} R$$

plate 2: $\Delta\phi_2 = -\phi(0) + \phi(L)$

$$= -\frac{\sigma}{2\varepsilon_0} R + \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+L^2} - L\right)$$

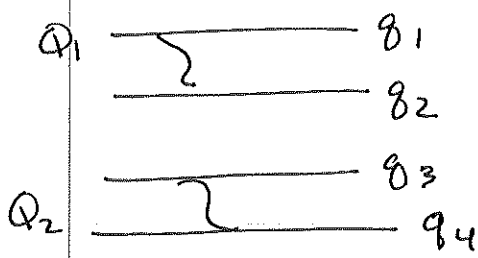
total: $\Delta\phi = \frac{\sigma}{\varepsilon_0}\left(\sqrt{R^2+L^2} - R - L\right)$

$$\approx -\frac{\sigma}{\varepsilon_0} L$$

Problem 10

$$C = \frac{4\pi R^2 \varepsilon_0}{R+L - \sqrt{R^2+L^2}}$$

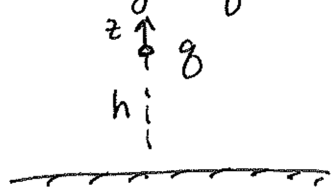
P&M 3.55



Same as 3.20

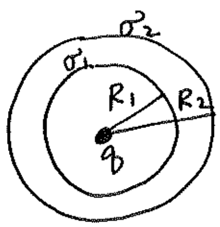
Image charges

Point charge q above infinite conducting plane.



What is the surface charge σ?
How much work does it take to move q from h to $z = +\infty$?

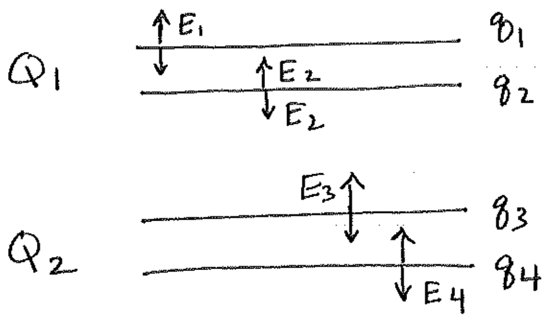
Conducting spherical shell.



Shell has net charge zero. Suppose we have a point charge q inside at the center.
what is $\phi(r)$?
what is σ_1, σ_2?

P&M, 3.20

Parallel plate capacitor with charges Q_1 and Q_2.



both sides at same potential.

$$q_1 + q_2 = Q_1$$
$$q_3 + q_4 = Q_2$$

Inside plate 1: $E_1 - E_2 - E_3 - E_4 = 0$

$$q_1 - q_2 - q_3 - q_4 = 0$$

Inside plate 2: $q_1 + q_2 + q_3 - q_4 = 0$

$$q_1 - q_2 - Q_2 = 0$$

$$2q_1 - Q_1 - Q_2 = 0 \quad \rightarrow \quad q_1 = \frac{Q_1 + Q_2}{2}$$

by symmetry $\qquad q_4 = \dfrac{Q_1 + Q_2}{2}$

$$q_2 = q_1 - Q_2 = \frac{Q_1 - Q_2}{2}$$

$q_3 = \dfrac{Q_2 - Q_1}{2}$ by symmetry.