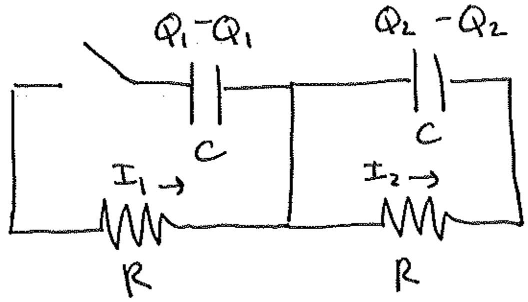
Tutorial

P&M, problem 4.18

(a)



$$\frac{Q_1}{C} - I_1 R = 0 \qquad \frac{Q_2}{C} - I_2 R = 0$$

$$I_1 = -\frac{dQ_1}{dt} \qquad I_2 = -\frac{dQ_2}{dt}$$

$\Rightarrow$ same as in lecture

$$Q_1(t) = A\, e^{-t/RC}, \quad Q_2(t) = B\, e^{-t/RC}$$

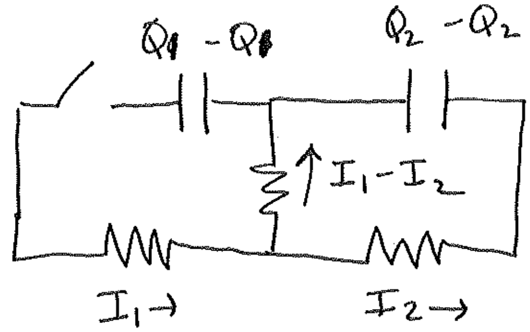
determine A, B by initial conditions.

$$Q_1(0) = A = Q_0, \quad Q_2(0) = B = 0.$$

$$\Rightarrow \quad Q_1(t) = Q_0\, e^{-t/RC}, \quad Q_2(t) = 0.$$

No charge goes onto capacitor 2, since current can take middle path with no resistance.

(b)



$$\frac{Q_1}{C} - I_1 R - (I_1 - I_2)R = 0 \qquad \frac{dQ_1}{dt} = -I_1$$

$$\frac{Q_2}{C} - I_2 R - (I_2 - I_1)R = 0 \qquad \frac{dQ_2}{dt} = -I_2$$

Take sum & difference:

$$Q = Q_1 + Q_2 \,, \quad \Delta Q = Q_1 - Q_2$$

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0 \qquad \frac{\Delta Q}{C} + 3R\frac{d\Delta Q}{dt} = 0 .$$

$$Q(t) = A e^{-t/RC} \,, \quad \Delta Q = B e^{-t/3RC}$$

$$Q_1(t) = \frac{Q + \Delta Q}{2} = \frac{1}{2}\left(A e^{-t/RC} + B e^{-t/3RC}\right)$$

$$Q_2(t) = \frac{Q - \Delta Q}{2} = \frac{1}{2}\left(A e^{-t/RC} - B e^{-t/3RC}\right)$$

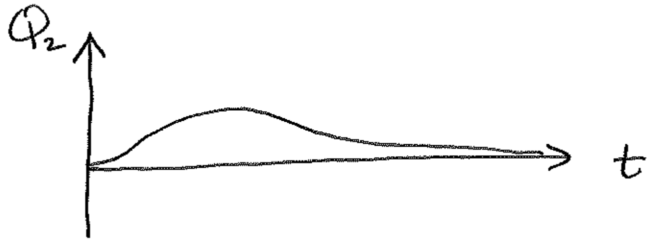
Initial conditions: $Q_1(0) = Q_0 = \frac{1}{2}(A + B)$
$Q_2(0) = 0 = \frac{1}{2}(A - B)$

$$\Rightarrow A = B = Q_0/2$$

So we have

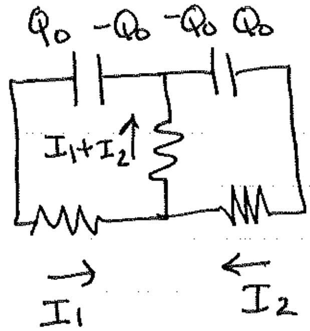
$$Q_1(t) = \frac{Q_0}{2}\left(e^{-t/RC} + e^{-t/3RC}\right)$$

$$Q_2(t) = \frac{Q_0}{2}\left(e^{-t/RC} - e^{-t/3RC}\right)$$

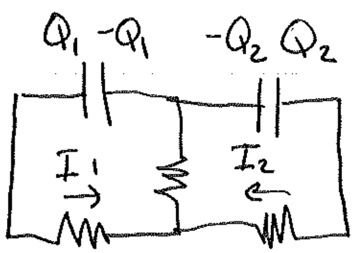


Some charge flows onto capacitor 2 temporarily.

(c) Consider an extra example: Suppose both capacitors start with charge Q_0 as follows.



What happens as a function of time?
Note circuit is symmetric, so $I_1 = I_2$, $Q_1 = Q_2$



$$\frac{Q_1}{C} - I_1 R - (I_1+I_2)R = 0$$

$$\frac{Q_1}{C} - 3RI_1 = 0$$

$$I_1 = -\frac{dQ_1}{dt}$$

Solution is:

$$Q_1(t) = Q_0\, e^{-t/3RC}$$

Initial potential energy in capacitors:

$$U = 2 \cdot \frac{Q_0^2}{2C} = \frac{Q_0^2}{C}$$

Work dissipated by lower resistors:

$$W = \int_0^\infty dt\, P = 2\int_0^\infty dt\, I_1^2 R$$

2 resistors with equal current $I_1 = I_2$

$$W = 2R\left(\frac{Q_0}{3RC}\right)^2 \int_0^\infty dt\, e^{-2t/3RC}$$

$$= 2R\,\frac{Q_0^2}{9R^2C^2}\left(\frac{3RC}{2}\right) = \frac{Q_0^2}{3C}$$

Work dissipated by central resistor

$$W = \int_0^\infty dt\, P = \int_0^\infty dt\, (I_1 + I_2)^2 R$$

$$= 4\int_0^\infty dt\, I_1^2 R = \frac{2Q_0^2}{3C}$$

Total work dissipated by all resistors is $\frac{Q_0^2}{C}$

same as initial potential energy of capacitors