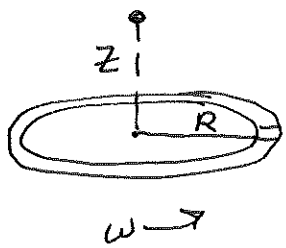
Tutorial

Rotating loop of charge with angular frequency ω.



period to go around loop

$$\omega = \frac{2\pi}{\tau} \rightarrow \tau = \frac{2\pi}{\omega}$$

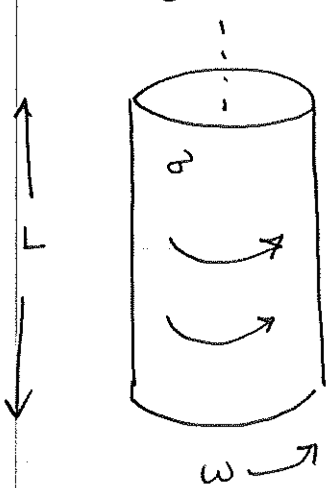
$$\text{Current} = \frac{\text{charge}}{\text{time}} = \frac{\text{charge}}{\text{length}} \cdot \frac{\text{length}}{\text{time}}$$

$$= \lambda \frac{2\pi R}{2\pi/\omega} = \lambda R \omega = I$$

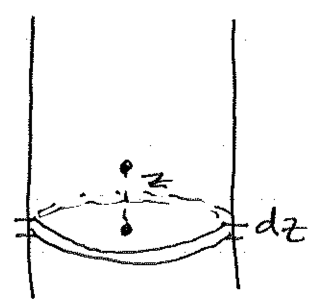
$$\vec{B} = \frac{\mu_0 I R^2 \hat{z}}{2(R^2 + z^2)^{3/2}} = \frac{\mu_0 \lambda \omega R^3}{2(R^2+z^2)^{3/2}} \hat{z}$$

at the center $\vec{B} = \dfrac{\mu_0 \lambda \omega}{2} \hat{z}$

Rotating ~~sphere~~ cylinder of fixed surface charge σ, rotating with angular frequency ω, radius R.



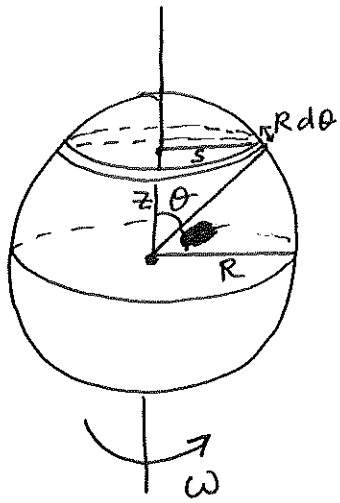
Divide into circular loops.
Same as above:



$\lambda \rightarrow \sigma dz$

$$\vec{B} = \int_{-\infty}^{\infty} dz \, \frac{\mu_0 \sigma \omega R^3}{2(R^2+z^2)^{3/2}} \hat{z} = \mu_0 \sigma \omega R$$

Rotating sphere : radius R & surface charge σ
Find $\vec{B}$ at the center.



Divide into slices of width $R\,d\theta$

Each loop has linear charge density $\lambda = \sigma R\,d\theta$ and radius $s = R\sin\theta$

Each loop generates $\vec{B}$ of

$$\vec{B} = \frac{\mu_0 \lambda \omega s^3}{2(s^2 + \underset{\uparrow\, z = R\cos\theta}{z^2})^{3/2}}\,\hat{z}$$

Adding up all the loops from $\theta = 0$ to $\theta = \pi$.

$$\vec{B} = \int_0^{\pi} \frac{\mu_0\, \sigma R\,d\theta\ \ \omega\, s^3}{2(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}\,\hat{z}$$

$$= \int_0^{\pi} \frac{\mu_0\, \sigma\, R\,d\theta\ \omega\, R^3 \sin^3\theta}{2R^3}\,\hat{z}$$

$$= \frac{\mu_0\, \sigma R \omega}{2}\hat{z} \underbrace{\int_0^{\pi} d\theta\, \sin^3\theta}_{=4/3} = \underline{\frac{2\mu_0 \sigma R\omega}{3}\hat{z}}$$