PHYS 3090: Homework 2 (due Friday Sept. 26)

1. Consider the matrix $M = \begin{pmatrix} \cos\theta\, e^{i\alpha} & \sin\theta\, e^{i\beta} \\ \sin\theta\, e^{i\phi} & \cos\theta\, e^{i\eta} \end{pmatrix}$.

- What are the conditions on the phases $(\alpha, \beta, \phi, \eta)$ such that M is Hermitian for any value of θ?
- What are the conditions on the phases $(\alpha, \beta, \phi, \eta)$ such that M is unitary for any value of θ?

2. Consider a double-spring system, with equal masses m and spring constants k and $\frac{3}{2}k$, where x_1 and x_2 measure the displacements from the equilibrium (see Figure 1 left).

- What is the potential energy V?
- Determine the normal frequencies and the normal modes.
- Suppose at $t = 0$, the system had the initial condition

$$x_1(0) = 0\,, \quad x_2 = d\,, \quad \dot{x}_1(0) = \dot{x}_2(0) = 0\,. \tag{1}$$

  Using eigenvalue methods, determine $\vec{x}(t)$ for $t > 0$.

- Compute the linear momentum p_1 and p_2 of each of the two masses as a function of t.

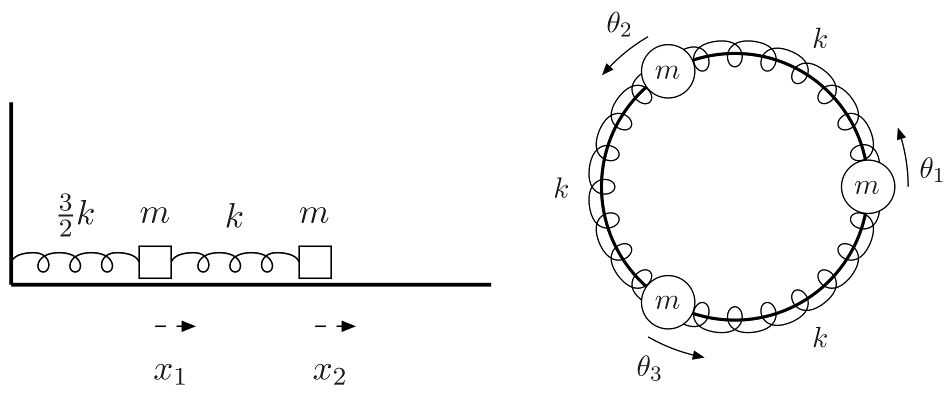

Figure 1: Left: Double spring system with spring constants k and $\frac{3}{2}k$ and masses m on a frictionless surface. Right: three masses m connected by springs in a ring configuration.

3. Consider circular configuration of three masses shown in Figure 1 (right). All three masses (with mass m) and all three springs (with spring constant k) are fixed to move along the circle of radius R. Let the variables $(\theta_1, \theta_2, \theta_2)$ be the angular displacements of each mass from its equilibrium position.

- What is the potential energy V in terms of k, R, and θ_i?
- Write the equation of motion for this system as $\ddot{\vec{\theta}} = -U\vec{\theta}$, where $\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$ and U is a 3×3 matrix.

  What is U in terms of k, m, and R?
- What are the normal frequencies and normal modes for this system?
- Suppose at time $t = 0$ all masses begin at their equilibrium positions and we give mass 1 a "kick" so that it has velocity v (and the other masses begin at rest). This corresponds to an initial condition:

$$\theta_1(0) = \theta_2(0) = \theta_3(0) = 0, \qquad R\,\dot{\theta}_1(0) = v, \quad \dot{\theta}_2(0) = \dot{\theta}_3(0) = 0\,. \tag{2}$$

  Using eigenvalue methods, determine $\vec{\theta}(t)$ for $t > 0$.
- Compute the angular momentum L_1, L_2, and L_3 for each of the three masses as a function of t. Show that the total angular momentum $L_1 + L_2 + L_3$ is constant in time.