PHYS 3090: Homework 3 (due Friday Oct. 10)

1. Let $z_1 = 3 + i$ and $z_2 = -2 - 2i$. Compute the following:

   - $z_1 z_2$
   - z_1/z_2 and z_2/z_1
   - $|z_1|$ and $|z_2|$
   - $\arg(z_2)$
   - $\ln(z_2)$

2. Show that for any complex numbers z_1 and z_2, the following are true:

   - $|z_1 z_2| = |z_1| \, |z_2|$
   - $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
   - $\arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$

3. Find the solutions of the equation $z^4 = -16i$.

4. Find the solutions of the equation $z^4 + 6z^2 = -8$.

5. Any complex function can be expressed as $f(z) = u(x, y) + iv(x, y)$, where u, v are real functions and $z = x + iy$. Find $u(x, y)$ and $v(x, y)$ for the following complex functions:

   - $f(z) = e^{-2iz}/z^2$
   - $f(z) = z^2 \ln(z^2)$
   - $f(z) = \cos(5z)/z$