PHYS 3090: Homework 4 (due Friday Oct. 17)

Problem 1: Show explicitly that the following functions satisfy the Cauchy-Riemann relations and identify points z, if any, where $f(z)$ is not analytic.

- $f(z) = \tan(z)$
- $f(z) = e^{1/z}$

Problem 2: Compute the integral $\int_{z_1}^{z_2} dz\, z^2$, where $z_1 = 0$ and $z_2 = 1 + 2i$, along the following contours:

- First vertically from $z = 0$ to $z = 2i$, and then horizontally from $z = 2i$ to $z = 1 + 2i$.
- Along the straight line defined by $y = 2x$.
- Along a parabolic curve defined by $y = 2x^2$.

Verify that you obtain the same result for each path. (*Hint:* For an arbitrary path defined by $y = y(x)$, you can write $dz = dx + idy = dx + i\frac{dy}{dx}dx$.)

Problem 3: Compute the integral $\oint_C dz\, \frac{1}{z}$, where C is a circle of unit radius, defined by $|z| = 1$. (*Hint:* Express the integral in polar coordinates, $z = re^{i\theta}$, and perform the integration over θ. If r is fixed, then $dz = \frac{dz}{d\theta}d\theta$.)

How does your result change if you consider C to be a circle of arbitrary radius R, defined by $|z| = R$?