PHYS 3090: Homework 5 (due Friday Oct. 24)

Problem 1: Identify the singular points in the following functions and demonstrate whether they are poles, essential singularities, or removable singularities.

- $f(z) = \cos(z + 1/z)$
- $f(z) = \frac{z^3+6z^2+5z-12}{3z^2-6z+3}$
- $f(z) = \frac{z^2-4}{z+2}$
- $f(z) = \cot(z) - 1$

If any of the above functions have poles, determine their order.

Problem 2: Compute the following contour integrals $\oint_C dz\, f(z)$, where

- $f(z) = \frac{1}{z^2+1}$, where C is the circle $|z| = 2$
- $f(z) = \frac{1}{z^4+1}$, where C is the rectangle with corners at $z = \pm 2i$ and $z = 2 \pm 2i$.
- $f(z) = \tan(z)$, where C is the circle $|z| = 5$.
- $f(z) = \frac{e^z}{z^2-2iz}$, where C is the circle $|z - 2i| = 1$.

Problem 3: Consider the function $f(z) = g(z)/h(z)$, where $g(z)$ is analytic for all z. Suppose $f(z)$ has a simple pole at $z = a$.

- Show that the residue at $z = a$ is given by $\text{Res}\, f(a) = g(a)/h'(a)$. (*Hint:* Taylor expand something!)
- For the function $f(z) = \frac{e^z-1}{e^z+1}$, determine the locations and orders for all poles and compute their residues.