PHYS 3090: Homework 8 (due Friday Nov. 28)

Problem 1: A function $f(t)$ has a Laplace transform

$$F(s) = \frac{2s(s^2 - 3a^2)}{(s^2 + a^2)^3}\,,$$

where a is a real, positive number. Compute $f(t)$ using the Bromwich integral.

Problem 2:

- Let $f(t)$ be a function with Laplace transform $F(s)$. Show that

$$\int_s^\infty ds'\, F(s') = \mathcal{L}\big[f(t)/t\big]\,. \tag{1}$$

- Using the above result, compute $\mathcal{L}[\frac{\sin t}{t}]$.
- Using the above result, evaluate the sine function

$$\mathrm{Si}(\infty) = \int_0^\infty dt\, \frac{\sin t}{t}\,. \tag{2}$$

Hint: See Sections 5.4–5.5 in the text.

Problem 3: Suppose a radioactive isotope, with decay constant λ, starts leaking from a nuclear reactor at $t = 0$ with rate $R(t)$. The number of radioactive atoms satisfies the rate equation

$$\dot{n}(t) + \lambda\, n(t) = R(t)\,, \tag{3}$$

with the initial condition $n(0) = 0$.

- Supposing that the leak is periodic, with rate $R(t) = R_0\big(1 - \cos(t/T)\big)\theta(t)$, where R_0 and T are constants, determine $n(t)$ by taking the Laplace transform of Eq. (3).
- Compute the Green's function $g(t, \tau)$ for the above rate equation and use this to write the solution $n(t)$ for arbitrary $R(t)$.

Problem 4: Compute $F(s) = \mathcal{L}[\sinh(\alpha t)]$ where α is a real, positive number. What is the range of s for which $F(s)$ is defined?