

PHYS 3090: Homework 5 (due Friday Oct. 23)

Problem 1: Compute the following contour integrals $\oint_C dz f(z)$, where

- $f(z) = \frac{1}{z^2+1}$, where C is the circle $|z| = 2$ **(5 points)**
- $f(z) = \frac{1}{z^4+1}$, where C is the rectangle with corners at $z = \pm 2i$ and $z = 2 \pm 2i$ **(5 points)**
- $f(z) = \tan(z)$, where C is the circle $|z| = 5$ **(5 points)**
- $f(z) = \frac{e^z}{z^2-2iz}$, where C is the circle $|z - 2i| = 1$ **(5 points)**

Problem 2: Evaluate the c_{-1} term in the Laurent expansion for the following functions

- $f(z) = \frac{\cot z}{z^2}$ about $z = 0$ **(5 points)**
- $f(z) = \frac{e^z}{z^2+1}$ about $z = i$ **(5 points)**
- $f(z) = \cos(z + \frac{1}{z})$ about $z = 0$ **(5 points)**

Problem 3: Compute $\oint_C dz e^{a/z}$ where C is the unit circle $|z| = 1$ and a is a complex number. **(5 points)**

Problem 4: Compute $\oint_C dz \frac{e^{1/z}}{1-z}$ where C is the circle $|z| = 0.1$. **(10 points)**

Hint: Recall the infinite geometric series formula $\sum_{n=1}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$.