

## PHYS 3090: Homework 3 (due Wednesday Oct. 5)

**Problem 1:** Compute  $\int_0^{2\pi} d\theta \frac{1}{(2-\cos\theta)^2}$  using contour integration. **(5 points)**

**Problem 2:** Compute p.v.  $\int_{-\infty}^{\infty} dx \frac{\cos x - 1}{x^2}$  by contour integration. **(5 points)**

**Problem 3:** Compute p.v.  $\int_0^{\infty} dx \frac{1}{1+x^{100}}$  by contour integration. **(5 points)**

**Hint 1:** You may use the result from problem 2a in HW 2 to compute the residues.

**Hint 2:** The equation for a finite geometric series  $\sum_{n=0}^N x^n = \frac{1-x^{N+1}}{1-x}$  might be useful.

**Problem 4 (10 points):** Compute the integral  $I = \text{p.v.} \int_{-\infty}^{\infty} dx \frac{e^x}{e^{3x}+1}$ . We are going to evaluate  $I$  using a rectangular contour  $C$  shown in Fig. 1, with sides labeled by paths  $P_1 - P_4$ .

- (a) Show that  $f(z) = \frac{e^z}{e^{3z}+1}$  has poles along the imaginary axis at  $z = \frac{\pi i}{3} + \frac{2\pi i n}{3}$ , where  $n = 0, \pm 1, \text{ etc.}$  **(2 points)**
- (b) The original integral is  $I = \int_{P_1} dz f(z)$ . Show that  $\int_{P_3} dz f(z) = -e^{2\pi i/3} I$ . **(2 points)**
- (c) Show that  $\int_{P_2} dz f(z) = \int_{P_4} dz f(z) = 0$  in the limit  $R \rightarrow \infty$ . **(1 point)**
- (d) Combining these results, you have

$$\lim_{R \rightarrow \infty} \int_{P_1} dz f(z) + \int_{P_2} dz f(z) + \int_{P_3} dz f(z) + \int_{P_4} dz f(z) = (1 - e^{2\pi i/3}) I = \oint_C dz f(z). \quad (1)$$

Now use the residue theorem to evaluate the right-hand side and solve for the original integral  $I$ . **(5 points)**

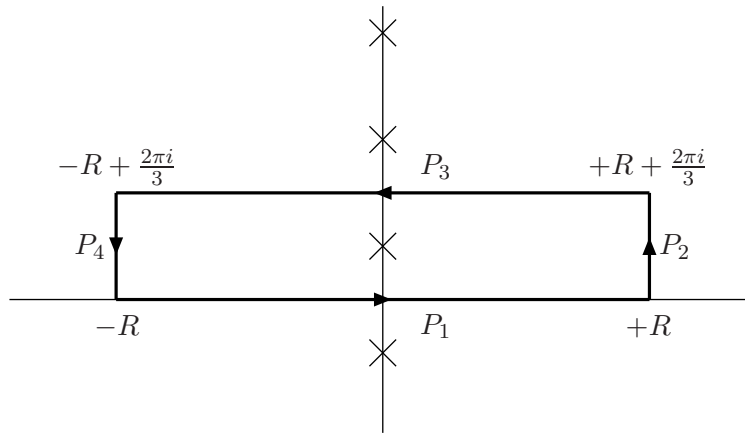


Figure 1: Rectangular contour  $C$  defined by the corners  $z = \pm R$  and  $z = \pm R + \frac{2\pi i}{3}$ , and sides  $P_1$ - $P_4$ . The crosses show the poles.