# PHYS 3090: Homework 3 (due Wednesday Oct. 4)

**Problem 1**: Compute $\int_0^{2\pi} d\theta \, \frac{1}{(2-\cos\theta)^2}$ using contour integration. **(5 points)**

**Problem 2**: Compute p.v. $\int_{-\infty}^{\infty} dx \, \frac{\cos x - 1}{x^2}$ by contour integration. **(5 points)**

**Problem 3**: Compute p.v. $\int_0^{\infty} dx \, \frac{1}{1+x^{100}}$ by contour integration. **(5 points)**

**Hint 1:** You may use the result from problem 2a in HW 2 to compute the residues.

**Hint 2:** The equation for a finite geometric series $\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x}$ might be useful.

**Problem 4 (10 points):** Compute the integral $I = \text{p.v.} \int_{-\infty}^{\infty} dx \, \frac{e^x}{e^{3x}+1}$. We are going to evaluate $I$ using a rectangular contour $C$ shown in Fig. 1, with sides labeled by paths $P_1$ - $P_4$.

(a) Show that $f(z) = \frac{e^z}{e^{3z}+1}$ has poles along the imaginary axis at $z = \frac{\pi i}{3} + \frac{2\pi i n}{3}$, where $n = 0, \pm 1$, *etc.* **(2 points)**

(b) The original integral is $I = \int_{P_1} dz \, f(z)$. Show that $\int_{P_3} dz \, f(z) = -e^{2\pi i/3} \, I$. **(2 points)**

(c) Show that $\int_{P_2} dz \, f(z) = \int_{P_4} dz \, f(z) = 0$ in the limit $R \to \infty$. **(1 point)**

(d) Combining these results, you have

$$\lim_{R\to\infty} \int_{P_1} dz \, f(z) + \int_{P_2} dz \, f(z) + \int_{P_3} dz \, f(z) + \int_{P_4} dz \, f(z) = \left(1 - e^{2\pi i/3}\right) I = \oint_C dz \, f(z) \, . \qquad (1)$$

Now use the residue theorem to evaluate the right-hand side and solve for the original integral $I$. **(5 points)**

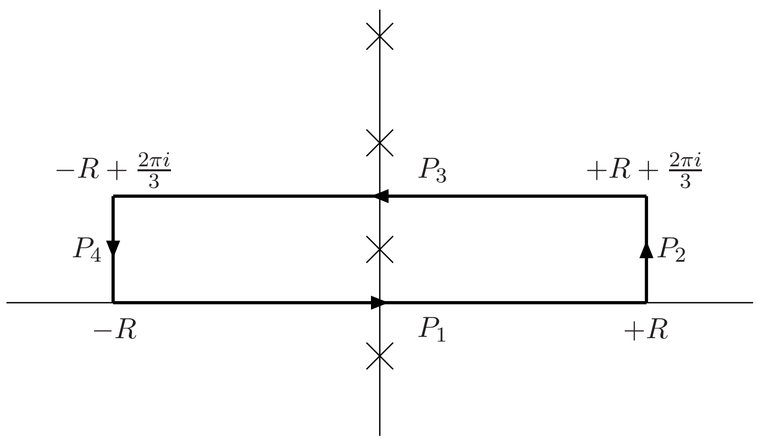


Figure 1: Rectangular contour $C$ defined by the corners $z = \pm R$ and $z = \pm R + \frac{2\pi i}{3}$, and sides $P_1$-$P_4$. The crosses show the poles.