PHYS 5180: Homework 4 (due Friday 4pm Feb. 20)

1. PS problem 4.2

2. PS problem 4.3

3. Consider two real scalars ϕ and Φ, with Lagrangian

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \Phi)^2 - \tfrac{1}{2}M^2\Phi^2 + \tfrac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{3!}\Phi\phi^3,$$

where Φ has mass M and ϕ is massless. The goal of this problem is to do the three-body phase space integral for the decay $\Phi(k) \to \phi(p_1)\phi(p_2)\phi(p_3)$, assuming Φ decays at rest:

$$I_3 = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^4(k - p_1 - p_2 - p_3)\,.$$

- First, do the $\int d^3p_3$ integral using $\delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)$. Note that after doing this integral E_3 is fixed to be $E_3 = |\mathbf{p}_1 + \mathbf{p}_2|$.
- Second, due to rotational invariance of I_3, you may perform three out of four angular integrals, such that
$$\int d^3p_1 \int d^3p_2 = 2(2\pi)^2 \int_0^\infty p_1^2\, dp_1 \int_0^\infty p_2^2\, dp_2 \int_{-1}^{+1} d\cos\theta$$
where $\cos\theta$ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$.
- Third, rewrite I_3 by performing a change of integration variables as follows:
$$\int dp_1 \int dp_2 \int d\cos\theta \;\longrightarrow\; \int dE_1 \int dE_2 \int dE_3.$$
What is the range of integration on E_3? Use the remaining δ-function to perform the integral over E_3. After performing this integral, show that the allowed range of integration for $\int dE_1 \int dE_2$ is
$$0 < E_1 < \frac{M}{2}\,, \quad \frac{M}{2} - E_1 < E_2 < \frac{M^2 - 2ME_1}{2M - 4E_1}\,. \tag{1}$$
(*Hint*: Look at where the δ-function has support.)
- Fourth, assuming the matrix element is constant, evaluate the remaining $\int dE_1 \int dE_2$ integrals to show
$$I_3 = \frac{M^2}{64\pi^3} \tag{2}$$
- Lastly, use your result to evaluate $\Gamma(\Phi \to \phi\phi\phi)$. (*Hint*: Don't forget a factor of 1/3! from identical final state particles.)