PHYS 5180: Homework 5 (due Friday 4pm Mar. 6)

1. PS problem 3.2

2. PS problem 3.4 (a-c)

3. The neutrino ν is a spin-$\frac{1}{2}$ particle with a very tiny but nonzero mass ($m_\nu \lesssim 1$ eV).[1] Since only the left-handed neutrino field ν_L has been observed experimentally, it is unknown whether the neutrino is described by a Majorana field ν_L or a Dirac field $\nu = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$. If ν is a Dirac field, it is a mystery why the neutrino should have a Dirac mass that is so much smaller than the other Dirac fermions in the Standard Model.

One model to describe the neutrino mass is called the see-saw model. Let $N = i\sigma^2\nu_R^*$ be the left-handed spinor corresponding to ν_R. Consider the Lagrangian

$$\mathscr{L} = \bar{\nu}(i\gamma^\mu\partial_\mu - m)\nu + \frac{iM}{2}(N^T\sigma^2 N - N^\dagger\sigma^2 N^*)\,, \tag{1}$$

where m is a Dirac mass for $\nu_{L,R}$ and M is a Majorana mass for ν_R.

Working in the limit $M \gg m$, show that through a field redefinition

$$\nu_L \to i\nu_1 + \frac{m}{M}\nu_2\,, \qquad N \to \nu_2 - \frac{im}{M}\nu_1 \tag{2}$$

the Lagrangian (1) is diagonalized as two uncoupled Majorana fermions ν_1 and ν_2, at linear order in $1/M$. Compute the Majorana masses for ν_1 and ν_2 in terms of m, M.

Assuming ν_1 is the physical neutrino state that has been observed experimentally, compute the value of M needed to have $m_1 = 1$ eV if

- $m = m_e$ (the electron mass)
- $m = m_t$ (the top quark mass)

In this way, the see-saw "explains" the smallness of the neutrino mass by having a big Majorana mass for the unobserved right-handed state ν_R.

4. Consider the following interactions between a Dirac fermion ψ and a photon A^μ:

- $\bar{\psi}\gamma^\mu\psi A_\mu$ (coupling of a photon to fermion electric charge)

[1] In reality there are three flavors of neutrinos ν_L. For simplicity, we are only considing a single flavor, but this discussion generalize to multiple flavors.

- $\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$ (coupling of a photon to the fermion magnetic dipole)
- $\epsilon_{\mu\nu\alpha\beta}\,\bar{\psi}\sigma^{\mu\nu}\psi F^{\alpha\beta}$ (coupling of a photon to the fermion electric dipole)

Determine whether these interactions are even or odd under the following: C, P, T, CP, CPT. Assume that the photon field A^μ transforms as a Lorentz four-vector under P, T and is odd under C.

5. PS problem 3.7.