PHYS 5180: Homework 6 (due Friday 4pm Mar. 20)

1. PS problem 5.1

2. PS problem 5.5

3. The part of the weak interaction governing muon decay, $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, can be written as

$$\mathscr{L}_{\text{int}} = \frac{G_F}{\sqrt{2}}\, \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \mu \, \bar{e} \gamma_\mu (1 - \gamma^5) \nu_e \,,$$

known as Fermi theory, where $G_F \approx 1.166 \times 10^{-5}$ GeV^{-2}. Here, e, μ, ν_e, ν_μ are Dirac spinors for the electron, muon, electron neutrino, and muon neutrino.

- Verify that G_F has the correct mass dimension considering the fields appearing in $\mathscr{L}_{\text{int}}$.
- Compute the muon decay rate and lifetime, and compare your result numerically to the experimentally observed value, $\tau_\mu \approx 2.2 \times 10^{-6}$ s.

(You may neglect the electron and neutrino masses in your calculation.)

4. Consider the theory of *scalar* QED with complex scalar field ϕ, with Lagrangian

$$\mathscr{L} = |D_\mu \phi|^2 - m^2 \phi^* \phi - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where $D_\mu = \partial_\mu + ieA_\mu$.

- Obtain the Feynman rules for the theory.
- Compute $d\sigma/d\Omega$ for Compton scattering, $\gamma\phi \to \gamma\phi$.