Z, μ, and τ Decays

7 January 2016

Due on Friday 15 January 2016.

1 Z Decays

Calculate (at tree level) the total Z decay width as well as the following branching ratios:

- $Br(Z \to e^+e^-)$,
- $Br(Z \to u\bar{u})$
- $Br(Z \to d\bar{d})$ and
- $Br(Z \to \nu_e\bar{\nu}_e)$.

(The branching ratio is the partial decay width divided by the total decay width.) Express the total width in GeV and find the Z lifetime in seconds. You can neglect the masses of the allowed final states. You can use $\sin^2\theta_W = 0.23$ (evaluated at the scale of the Z mass).

To determine which decays are kinematically allowed, you may want to consult the Particle Data Group (PDG) website (http://pdg.lbl.gov/index.html) for the particle masses. Do the quark colors play any role? When summing over the polarizations of massive gauge bosons, you should use the replacement $\sum \epsilon^\mu(p)\epsilon^{*\nu}(p) \to -\eta^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}$. The extra term (compared with what you learned in QFT I) is the contribution of the Higgs boson that was "eaten" by the massive gauge boson.

2 Muon and Tau Decays

a) In QFT I homework 2 you showed that the muon decay width $\Gamma(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu)$ is

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \tag{1}$$

by neglecting the masses of the electron and neutrinos and assuming that

$$\sum_{\text{final spins}} \langle|\mathcal{M}(\mu(p_1) \to \bar{\nu}_e(p_2) + \nu_\mu(p_3) + e(p_4))|^2\rangle_{\text{initial spins}} = 2\left(\frac{g}{M_W}\right)^4 (p_1\cdot p_2)(p_3\cdot p_4)\ . \tag{2}$$

where the Fermi constant G_F is

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{m_W^2} \tag{3}$$

Derive equation (2).

b) Calculate the following partial decay widths of the τ:

- $\Gamma(\tau \to e^- + \bar{\nu}_e + \nu_\tau)$,
- $\Gamma(\tau \to \mu + \bar{\nu}_\mu + \nu_\tau)$

Compare your results with the measured values using the PDG website.

The following result from QFT I homework 2 may be useful. The three-body phase space for massless particles is:

$$\int d\Phi_3 = \frac{Q^2}{128\pi^3}\int dx_1 dx_2 \tag{4}$$

where Q is the (constant) total 4-momentum: $Q = k_1 + k_2 + k_3$ and

$$x_i = \frac{2k_i \cdot Q}{Q^2} \tag{5}$$

The allowed region for $x_1\,, x_2$ is the triangle bounded by the lines $x_1 = 1$, $x_2 = 1$, $x_1 + x_2 = 1$.

c) Suppose we allow the electron, muon, and tau to have independent couplings to the W, i.e. $g \to g_e$, g_μ, g_τ. How are the partial widths modified?

d) Lepton universality (which is an automatic consequence of the gauge theory) requires $g_\mu/g_e = 1$ and $g_\tau/g_\mu = 1$ (which together imply $g_\tau/g_e = 1$).

By taking the appropriate ratios of partial widths and lepton masses (and using their experimentally determined values), show that lepton universality is satisfied.

Your result provides strong motivation for considering the W as a gauge boson, rather than an arbitrary vector field.