Quark Spin and Stops

15 January 2016

Due on Friday 22 January 2016.

1 Quarks Are Not Scalars

Hint:

Since we are doing a null result check in this problem, we can drop constant factors very liberally.

Pay attention to the consistency of your momentum and Mandelstam variable definition. Minus signs are important here.

In lecture we saw that the parton model (where the proton is made from spin-$\frac{1}{2}$ quarks) leads to the Callan-Gross relation $xF_1^e(x) = F_2^e(x)$. Show that $F_1^e = 0$ if you assume that the proton is made from spin-0 "quarks". To do that, calculate the subprocess differential cross section $\hat{\sigma}(e^- q \to e^- q')$ using the scalar QED Feynman rule for the scalar "quarks" and compare to the definition of F_1^e and F_2^e defined in the lecture notes.

2 Stops

Supersymmetry predicts that each Standard Model fermion has a scalar superpartner that lives in the same representation of $SU(3) \times SU(2) \times U(1)$. Superpartners of SM fermions refered to either by prepending an "s" to the name of the SM fermion or adding a tilde to the symbol for the SM fermion. In this problem you will look at (a simplified version of) how heavy "squarks" affect the higgs to two gluon and higgs to two photon decays.

a) Assume that the "stop" couples to the Higgs doublet H through the following term, and that there are no other terms that give the stop a mass:

$$\mathcal{L} \supset -y_{\tilde{t}} H^\dagger H \tilde{t}^\dagger \tilde{t} \tag{1}$$

Determine the stop mass $m_{\tilde{t}}^2$. Draw the higgs-stop vertices and determine the Feynman rules in unitary gauge.

b) The stop-gluon vertices come from the terms:

$$\mathcal{L} \supset \left[(\partial_\mu + ig_s G_\mu)\tilde{t}\right]^\dagger \left[(\partial^\mu + ig_s G^\mu)\tilde{t}\right] \tag{2}$$

Draw the gluon-stop vertices and determine the Feynman rules.

c) Draw the one-loop Feynman diagram(s) in which a higgs decays to two gluons via virtual stop(s). Write down the associated contribution to the matrix element $i\mathcal{M}_{\tilde{t}}^{gg}$.

d) Introduce Feynman parameters and use dimensional regularization to evaluate the momentum integral in $i\mathcal{M}_{\tilde{t}}^{gg}$. Assume the stop is heavy $m_{\tilde{t}}^2 \gg m_h^2$.

e) In lecture we found the contribution to the matrix element from top loops:

$$i\mathcal{M}_1 + i\mathcal{M}_2 = -i\frac{\alpha_s}{3\pi v}\epsilon_\mu^*(p)\epsilon_\nu^*(p')\left(p^\nu \left(p'\right)^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right)\delta^{AB} \tag{3}$$

Use this result to determine the partial decay width $\Gamma(h \to gg)$ including the contributions of both tops and stops. Assume that QCD corrections enhance the partial decay width by 60%.

f) The stop-photon interactions are the same scalar QED interactions you used in problem 1. Draw the one-loop Feynman diagram(s) in which a higgs decays to two photons via virtual stop(s). Write down the associated contribution to the matrix element $i\mathcal{M}_{\tilde{t}}^{\gamma\gamma}$. *Hint*: Apart from group theory factors this calculation is identical to part (c).

g) Introduce Feynman parameters and use dimensional regularization to evaluate the momentum integral in $i\mathcal{M}_{\tilde{t}}^{\gamma\gamma}$. Assume the stop is heavy $m_{\tilde{t}}^2 \gg m_h^2$. *Hint*: Apart from group theory factors this calculation is identical to part (d).

h) The one-loop matrix element for higgs to two photons is:

$$i\mathcal{M}(h \to \gamma\gamma) = i\mathcal{M}_{\tilde{t}}^{\gamma\gamma} + i\mathcal{M}_t^{\gamma\gamma} + i\mathcal{M}_W^{\gamma\gamma} \tag{4}$$

where the contributions of top loops and W loops are given by:

$$\begin{aligned} i\mathcal{M}_t^{\gamma\gamma} &= -i\frac{\alpha}{4\pi v}\epsilon_\mu^*(p)\epsilon_\nu^*(p')\left[p^\nu (p')^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right]\frac{16}{9} \\ i\mathcal{M}_W^{\gamma\gamma} &= -i\frac{\alpha}{4\pi v}\epsilon_\mu^*(p)\epsilon_\nu^*(p')\left[p^\nu (p')^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right](-8.3) \end{aligned} \tag{5}$$

Use this result to determine the partial decay width $\Gamma(h \to \gamma\gamma)$.

i) How would the existence of the heavy "stop" affect the branching ratios? Is this model consistent with current data? *Hint*: Table 11.2 of the Higgs boson physics review on PDG might be useful.