Standard Model

"Gauge theory of Elementary particle physics"
Cheng & Li

"Periodic table" of elementary particles & forces.

Fermions (spin 1/2)

name		mass (approx)	electric charge (proton charge e)
quarks	up (u)	2 MeV	$+\frac{2}{3}e$
	charm (c)	~~1.5~~ 1.3 GeV	
	top (t)	173 GeV	
	down (d)	5 MeV	$-\frac{1}{3}e$
	strange (s)	~~100 MeV~~ 95 MeV	
	bottom (b)	4.2 GeV	
leptons (charged leptons)	e	0.511 MeV	$-e$
	μ	106 MeV	
	τ	1.8 GeV	
(neutrinos)	ν_e		0
	ν_μ	≈ 0 *	
	ν_τ	($\lesssim$ eV)	

* Neutrinos do have a small, non-zero mass. The Standard model is defined with massless neutrinos. Evidence for neutrino masses is technically physics beyond the SM, but mass can be included in SM in two possible ways (but we don't know which way is correct)

Bosons (spin 0 or spin 1)

gauge bosons ($S=1$)	photon (γ)	(EM)	0	0
	gluon (g)	(strong)	0	0
	$W^{\pm}$	(weak)	80.4 GeV	$\pm e$
	Z		91.2 GeV	0
Higgs boson ($s=0$)	h		125 GeV	0

Forces in SM are mediated by gauge bosons

(1) EM force → photon. γ couples to particles with electric charge.
QED : QFT of electron & photon.

Feynman rule

$V(r) = \pm \frac{\alpha_{em}}{r}$ Coulomb potential

(2) Strong force → gluon. g couples to particles with "color" (color is like electric charge for the strong force.)

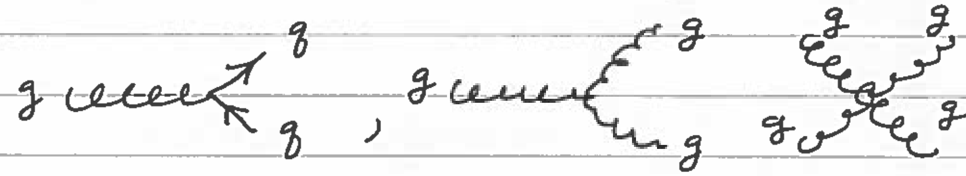
3 types of color : red (r), green (g), blue (b)
3 types of anticolor: $\bar{r}$, $\bar{g}$, $\bar{b}$

quarks carry color $q = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$

antiquarks carry anticolor $\bar{q} = \begin{pmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{pmatrix}$

gluons carry both color & anticolor, e.g. $r\bar{b}$, $g\bar{r}$, etc.
only <u>eight</u> possible combinations → eight types of gluons.

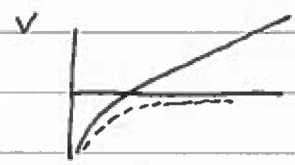
Feynman rules:



where $q = u, d, c, s, t, b$.

no analogous diagrams for QED since photons do not carry electric charge.

Potential between quark & antiquark $V(r) \simeq -\frac{\alpha_s}{r} + br$



Takes infinite amount of energy to separate quark & antiquark.
⇒ no free quarks (confinement)

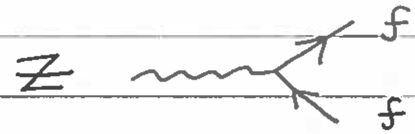
All quarks & antiquarks must be confined into color-neutral <u>hadrons</u> (bound states with no net color, i.e. "white")

meson: quark-antiquark state ($q\bar{q}$) $r\bar{r}$ = white
baryon: quark-quark-quark state (qqq) rgb = white.

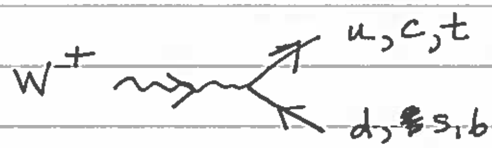
e.g. proton (uud), neutron (udd)

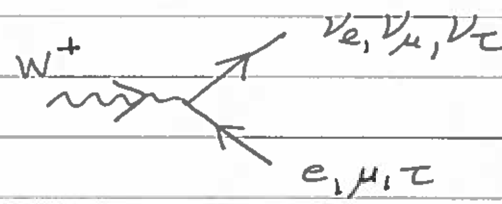
$Q_p = +\frac{2}{3}e \times 2 - \frac{1}{3}e = +e$ proton charge
$Q_n = +\frac{2}{3}e - 2 \times \frac{1}{3}e = 0$ neutron charge

Weak interaction/force: all fermions couple to weak force.



$f =$ any SM fermion.

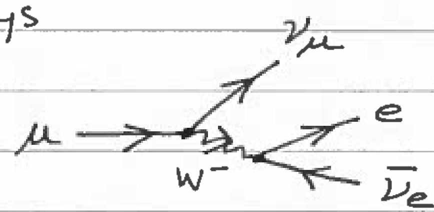




Only the $W^{\pm}$ interaction can change SM fermions from one type to another. If not for W, every SM fermion would be stable.

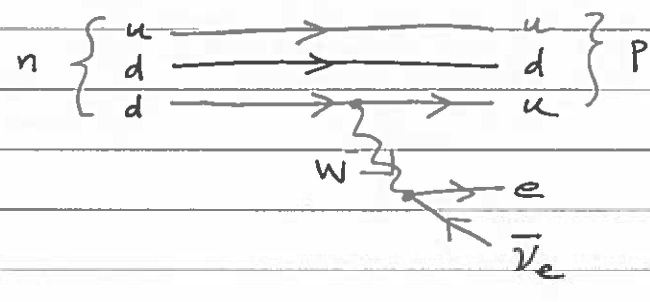
examples: Weak decays

(1) muon decay



$$\mu^- \to e^- \nu_\mu \bar{\nu}_e$$

(2) neutron β-decay $n \to p\, e^- \bar{\nu}_e$



Goal: write down Lagrangian to describe all known particles & interactions (except gravity, darkmatter, etc.)

Key ingredients: renormalizability (want a predictive theory)
gauge symmetry (abelian & nonabelian)

Higgs mechanism & Spontaneous Symmetry breaking

$W^{\pm}, Z$ are massive gauge bosons. Also they have chiral couplings to fermions (different couplings to LH & RH spinors)

$\Rightarrow$ inconsistent with both gauge symmetry & renormalizability

QED-like theory with massive photon A_μ:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu + \bar{\Psi}(i\not{D} - m_\Psi)\Psi$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$D_\mu = \partial_\mu + i(g_L P_L + g_R P_R) A_\mu$

$g_{L,R}$ = gauge couplings for $\Psi_{L,R}$

$P_{L,R} = \frac{1 \mp \gamma^5}{2}$, $\quad \Psi_{L,R} = P_{L,R}\Psi$

Gauge transformations: $A_\mu \to A_\mu + \partial_\mu \alpha(x)$

$$\Psi_{L,R} \to e^{-i g_{L,R}\alpha(x)} \Psi_{L,R}$$

$F_{\mu\nu}$ & $\bar{\Psi}\not{D}\Psi$ are gauge invariant, but mass terms aren't

$$A_\mu A^\mu \to A_\mu A^\mu + 2A_\mu \partial^\mu \alpha + (\partial^\mu \alpha)^2$$

$$\bar{\Psi}\Psi = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L \to e^{i(g_L - g_R)\alpha}\bar{\Psi}_L\Psi_R + e^{-i(g_L-g_R)}\bar{\Psi}_R\Psi_L$$

$$\neq \bar{\Psi}\Psi \text{ unless } g_L = g_R$$

What else goes wrong? Theory is also nonrenormalizable.

Consider the Feynman propagator for the photon:

$$D_{\mu\nu}(x,y) = \langle T\left(A_\mu(x)\, A_\nu(y)\right)\rangle$$

for massless photon Fourier transform is $D_{\mu\nu}(k) = \dfrac{-i\eta_{\mu\nu}}{k^2+i\varepsilon}$

Compute $D_{\mu\nu}(x,y)$ using mode expansion:

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_k}\sum_{\text{pol } i}\left(\varepsilon_\mu^{(i)}(k)\, a_k^{(i)} e^{-ik\cdot x} + \varepsilon_\mu^{(i)*}(k)\, a_k^{(i)\dagger} e^{ik\cdot x}\right)$$

$$\langle A_\mu(x) A_\nu(y)\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{1}{2E_k}\int\frac{d^3k'}{(2\pi)^3}\frac{1}{2E_{k'}}\sum_{i,j}\varepsilon_\mu^{(i)}(k)\,\varepsilon_\nu^{(j)}(k')^* \times \underbrace{\langle a_k^{(i)} a_{k'}^{(j)\dagger}\rangle}_{2E_k(2\pi)^3\delta^3(\underline{k}-\underline{k}')\,\delta^{ij}} e^{-i(k\cdot x - k'\cdot y)}$$

$$= \int\frac{d^3k}{(2\pi)^3}\frac{1}{2E_k}\underbrace{\sum_i \varepsilon_\mu^{(i)}(k)\,\varepsilon_\nu^{(i)}(k)^*}_{=-\eta_{\mu\nu} \text{ for massless photon}}$$

Massive photon: 3 polarizations. For photon at rest, we can take $\varepsilon_\mu^{(1)}(0) = (0,1,0,0)$, $\varepsilon_\mu^{(2)}(0) = (0,0,1,0)$, $\varepsilon_\mu^{(3)}(0) = (0,0,0,1)$

$$\sum_i \varepsilon_\mu^{(i)}(0)\,\varepsilon_\nu^{(i)}(0) = \begin{pmatrix}0&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}_{\mu\nu}$$

Boost to momentum $k^\mu = (E_k, 0, 0, k)$

Boost matrix: $\Lambda = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix}$, $\gamma = \frac{E_k}{m_A}$, $\beta = \frac{k}{E_k}$

$$\sum_i \varepsilon_\mu^{(i)}(0)\, \varepsilon_\nu^{(i)}(0) \xrightarrow{\text{boost}} \sum_i \varepsilon_\mu^{(i)}(k)\, \varepsilon_\nu^{(i)}(k) = \Lambda \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \Lambda$$

$$= \begin{pmatrix} k^2/m_A^2 & 0 & 0 & kE_k/m_A^2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ kE_k/m_A^2 & 0 & 0 & E_k^2/m_A^2 \end{pmatrix}_{\mu\nu}$$

$$= -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}$$

General rule: massless vector boson $\sum_i \varepsilon_\mu^{(i)} \varepsilon_\nu^{(i)*} \to -\eta_{\mu\nu}$

massive vector boson $\sum_i \varepsilon_\mu^{(i)} \varepsilon_\nu^{(i)*} \to -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}$

Massive vector propagator (unitary gauge)

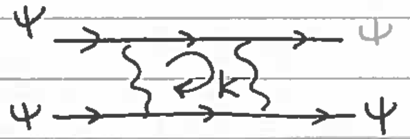
$$D_{\mu\nu}(k) = \frac{-i}{k^2 - m_A^2 + i\varepsilon}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}\right)$$

Extra $k_\mu k_\nu$ term can spoil renormalizability of theory. Recall in QED, divergences arise in 2-point & 3-point functions.

e.g. 

Can be absorbed by renormalizing mass, electric charge, wavefunction.

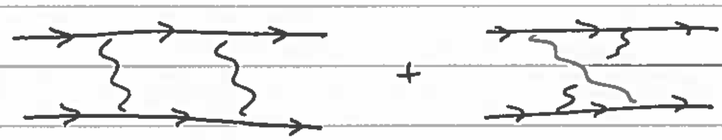
In massive QED, this is not the case. Consider 4-point function at 1-loop (e.g. $\psi\psi \to \psi\psi$ scattering)



$$\sim g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k}\right)^2 \left(\frac{k_\mu k_\nu}{k^2}\right)^2 \sim g^4 \Lambda^2$$

keeping only leading divergent terms
Λ = momentum cut-off.

Actually, this divergence cancels out from summing over diagrams, but still log divergent

 $\sim g^4 m_\psi^2 \log \Lambda^2$

Divergence absorbed in a dimension-6 counterterm:

$$\mathcal{L} = c\, \bar{\psi}\psi\bar{\psi}\psi$$, where c has mass dim -2.

$\Rightarrow$ not possible to compute $\psi\psi \to \psi\psi$ scattering since depends on new coupling c.

$i\mathcal{M}(\psi\psi \to \psi\psi)$ = 

Similar problem with higher point functions as well.

Spontaneous symmetry breaking

Violating a gauge symmetry explicitly (at the Lagrangian level) can introduce problems. However a symmetry can be broken spontaneously even if $\mathcal{L}$ is symmetric.

Ground state with broken symmetry can have lower energy than state with unbroken symmetry.

Let U be a symmetry that leaves Hamiltonian H invariant: $H \to U H U^\dagger = H$.

Let $|A\rangle$ and $|B\rangle$ be two states that are related by U: i.e. $U|A\rangle = |B\rangle$.

In terms of creation operators:

$$|A\rangle = a_A^\dagger |0\rangle \quad \text{and} \quad |B\rangle = a_B^\dagger |0\rangle$$

and we must have $U a_A^\dagger U^\dagger = a_B^\dagger$.

Then $U|A\rangle = U a_A^\dagger |0\rangle = \underbrace{U a_A^\dagger U^\dagger}_{a_B^\dagger} \underbrace{U|0\rangle}_{|0\rangle} = |B\rangle$

provided ground state is invariant: $U|0\rangle = |0\rangle$.

If this holds then A & B have the same energy:

$$E_A = \langle A|H|A\rangle = \langle B|U H U^\dagger|B\rangle = \langle B|H|B\rangle = E_B$$

If the ground state preserves the symmetry, then ~~the spectrum of states~~ the symmetry will be manifest in the spectrum of states

If the ground state violates the symmetry, the symmetry will not be manifest in the spectrum. However, there will be certain symmetry relations that reflect the symmetry of the Lagrangian.

Discrete symmetry

Theory with a real scalar field ϕ with $\mathbb{Z}_2$ symmetry $\phi \to -\phi$.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4$$

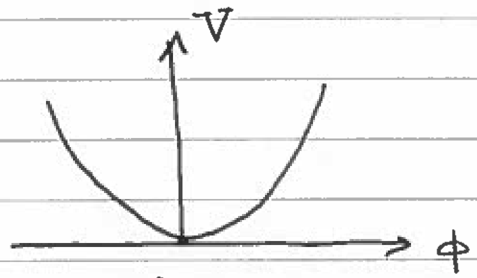
$\mathcal{L}$ invariant under $\phi \to -\phi$ (no linear or cubic terms)

Hamiltonian: $H = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\nabla\phi|^2 + V(\phi)$

where $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$

Since $\dot{\phi}^2$ and $|\nabla\phi|^2$ are both positive, energy minimized by $\phi = \text{const}$ ($\dot{\phi} = 0$, $\nabla\phi = 0$) determined by minimizing V.

Note: $\lambda > 0$, otherwise H is unbounded from below. But μ^2 can have either sign.

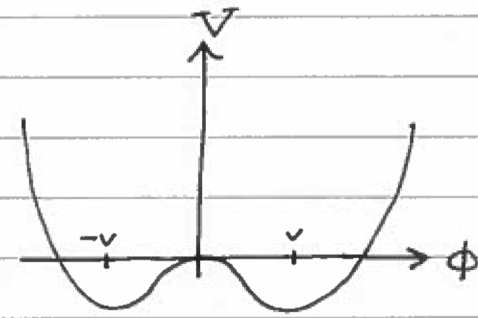


$\mu^2 > 0$

minimum at $\phi = 0$

$\langle 0|\phi|0\rangle = 0$

unbroken sym.



$\mu^2 < 0$.

minimum at $\phi = v \neq 0$

$\langle 0|\phi|0\rangle = v$

spon. broken. sym.

$$0 = \left.\frac{\partial V}{\partial \phi}\right|_{\phi = v} = \mu^2 v + \lambda v^3 \Rightarrow v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$$

v is called the <u>vacuum expectation value</u> (vev).

Physical states are small oscillations about the vacuum state. Define a shifted field $\phi = v + \phi'$ in broken case. (take $v > 0$)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi')^2 - \frac{1}{2}\mu^2 (v + \phi')^2 - \frac{1}{4}\lambda (v + \phi')^4$$

$$= \frac{1}{2}(\partial_\mu \phi')^2 - (-\mu^2)\phi'^2 - \lambda v \phi'^3 - \frac{\lambda}{4}\phi'^4$$

Now ϕ' doesn't have original symmetry ($\mathcal{L}$ not invar. under $\phi' \to -\phi'$)

Mass of ϕ' is $m_{\phi'}^2 = -2\mu^2 = 2\lambda v^2$.

Original symmetry reflected in relation between mass, cubic, and quartic terms.

Abelian (global) symmetry: complex scalar ϕ

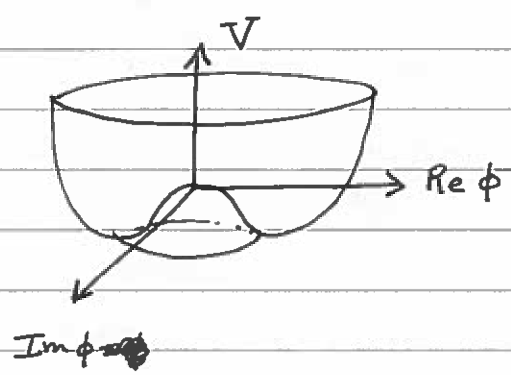
$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

$\mathcal{L}$ invariant under global $U(1)$ symmetry: $\phi \to e^{i\alpha}\phi$

Assume $\mu^2 > 0 \longrightarrow$ spontaneous symmetry breaking

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\frac{\partial V}{\partial \phi} = (-\mu^2 + \lambda \phi^\dagger \phi)\,\phi^\dagger = 0 \quad \text{for} \quad |\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$$



Infinite number of degenerate minima, all related by symmetry trans.

Let's write: $\phi = \dfrac{\phi_1 + i\phi_2}{\sqrt{2}}$ and assume only ϕ_1 gets a vev (free to do this w/o loss of generality)

$$\langle 0 | \phi_1 | 0 \rangle = v = \sqrt{\frac{\mu^2}{\lambda}}, \qquad \langle 0 | \phi_2 | 0 \rangle = 0.$$

Expand $\mathcal{L}$ in terms of shifted fields:

$$\phi_1 = v + \phi_1', \quad \phi_2 = \phi_2'$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1')^2 + \frac{1}{2}(\partial_\mu \phi_2')^2 - \mu^2 \phi_1'^2 - \lambda v \phi_1' (\phi_1'^2 + \phi_2'^2)$$
$$- \frac{\lambda}{4}(\phi_1'^2 + \phi_2'^2)^2$$

Note: ϕ_1' has mass $m_{\phi_1'}^2 = 2\mu^2$

ϕ_2' is massless. No energy cost to move in ϕ_2' direction.

This is a consequence of a general result known as Goldstone's theorem. ~~For every generator of~~

For every symmetry that broken, there is one ~~[illegible]~~ massless Goldstone boson per generator. that is no longer a symmetry.

Here $U(1)$ has one generator.

We'll come back to Goldstone's theorem later, but for now we move on to local symmetries.

Abelian Higgs model: complex scalar ϕ with a local $U(1)$ gauge symmetry (scalar QED)

$$\mathcal{L} = (D_\mu \phi^\dagger)(D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu = \partial_\mu + i g A_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

As before, $V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$ has minimum at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$

Expand $\mathcal{L}$ as above: write $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$, and shift the fields $\phi_1 = v + \phi_1'$, $\phi_2 = \phi_2'$.

New feature is covariant derivative term:

$$|D_\mu \phi|^2$$
$$\parallel$$
$$\mathcal{L} \supset \left| (\partial_\mu + i g A_\mu) \left(\frac{v + \phi_1' + i\phi_2'}{\sqrt{2}} \right) \right|^2$$

$$= \frac{1}{2} \left| \partial_\mu \phi_1' + i \partial_\mu \phi_2' + i g v A_\mu + i g A_\mu \phi_1' + g A_\mu \phi_2' \right|^2$$

$$= \frac{1}{2} (\partial_\mu \phi_1')^2 + \frac{1}{2} (\partial_\mu \phi_2')^2 + g v A_\mu \partial^\mu \phi_2' + \frac{1}{2} g^2 v^2 A_\mu A^\mu$$
$$+ \ldots \text{ cubic \& quartic terms} \ldots$$

First note that the vev for ϕ has generated a photon mass $m_A^2 = g^2 v^2$.

However we also have a mixing term $\sim A_\mu \partial^\mu \phi_2'$ in the Lagrangian that we want to get rid off to put the kinetic term into canonical form.

Instead of expanding ϕ in terms of Re/Im components, expand in polar components:

$$\phi(x) = \frac{1}{\sqrt{2}} (v + h(x))\, e^{i \xi(x)/v} = \frac{1}{\sqrt{2}} (v + \underbrace{h(x)}_{\approx \phi_1'} + i \underbrace{\xi(x)}_{\approx \phi_2'} + \ldots)$$

for small fluc.

where $h(x)$, $\xi(x)$ are real fields.

We can make ϕ purely real by gauge transformation:

$$\phi \to e^{-i\xi/v} \phi = \frac{1}{\sqrt{2}} (v + h)$$

This gauge choice is called unitary gauge.

Note also: $A_\mu \to A_\mu' = A_\mu + \frac{1}{gv} \partial_\mu \xi(x)$

So $D_\mu \phi = (\partial_\mu + i g A_\mu) \phi \to \frac{1}{\sqrt{2}} (\partial_\mu + i g A_\mu')(v + h)$

We have eliminated ξ from the Lagrangian by absorbing it into gauge field A'_μ. The Lagrangian is now:

$$\mathcal{L} = \frac{1}{2}\left|(\partial_\mu + igA_\mu)(v+h)\right|^2 + \frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

(we can drop the primes. Note: $F'_{\mu\nu} = F_{\mu\nu}$)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}g^2v^2 A_\mu A^\mu + g^2 v A_\mu A^\mu h + \frac{1}{2}g^2h^2A_\mu A^\mu$$
$$+ \left(\frac{\mu^2}{2} - \frac{6}{4}\lambda v^2\right)h^2 - \lambda v h^3 - \frac{\lambda}{4}h^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

use $v^2 = \mu^2/\lambda$

$$= \frac{1}{2}(\partial_\mu h)^2 - \mu^2h^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2v^2A_\mu A^\mu + \mathcal{L}_{int}$$

$m_h^2 = 2\mu^2$, $m_A^2 = g^2v^2$

$m_h = \sqrt{2\mu^2}$, $m_A = gv$

$$\mathcal{L}_{int} = g^2 v A_\mu A^\mu h + \frac{1}{2}g^2h^2A_\mu A^\mu - \lambda v h^3 - \frac{\lambda}{4}h^4$$

} interactions fixed in terms of few parameters — λ, m_h, m_A

The would-be-Goldstone boson ξ has been "eaten" by A_μ.

massless A_μ (2 d.o.f.) + massless real scalar ξ (1 d.o.f.) $\longrightarrow$ massive A_μ (3 d.o.f.)

Fermion masses: add fermion ψ to model

Assume chiral couplings ($\psi_{L,R}$ transform under gauge symmetry differently)

mass term $\bar{\psi}\psi$ is forbidden. But we can generate a mass term for ψ spontaneously using ϕ.

Some notation: let g be gauge coupling, Q is the charge of a field in units of g.

Quantum numbers for fields:

ϕ :	$Q_\phi = +1$	(to be consistent with above)	
ψ_L :	Q_L	$g_L = g Q_L$	$Q_L \neq Q_R$ (chiral)
ψ_R :	Q_R	$g_R = g Q_R$	

Gauge transforms:

$$\phi \to e^{-igQ_\phi \alpha} \phi$$

$$\psi_{L,R} \to e^{-igQ_{L,R}\alpha} \psi_{L,R}$$

If we assume $Q_L = Q_R + Q_\phi$, then we can write down another interaction that is gauge invariant:

$$\mathcal{L}_{Yukawa} = -y \bar{\psi}_L \psi_R \phi + h.c. \qquad y = \text{Yukawa coupling}$$

After SSB, we have (in unitary gauge) $\phi = \frac{1}{\sqrt{2}}(v+h)$

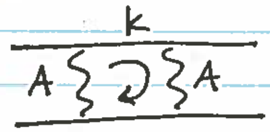
$$\mathcal{L}_{Yukawa} = -y \bar{\psi}_L \psi_R \frac{1}{\sqrt{2}}(v+h) + c.c.$$

$$= -\frac{yv}{\sqrt{2}} \bar{\psi}\psi - \frac{y}{\sqrt{2}} \bar{\psi}\psi h$$

SSB generates a fermion mass $m_\psi = \frac{yv}{\sqrt{2}}$.

We've shown how a gauge-invariant theory can produce a theory that appears not to be gauge invariant via Spontaneous symmetry breaking. "Higgs mechanism"

Also fixes issue of renormalizability. Example: $\psi\psi \to \psi\psi$ at 1-loop:



$+$

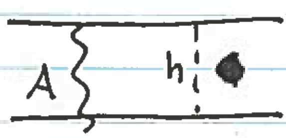
$$\sim g^4 \int \frac{d^4k}{(2\pi)^4} \left(\frac{\not{k} + m_\psi}{k^2}\right)^2 \left(\frac{k_\mu k_\nu}{m_A^2 k^2}\right)^2$$

($\not{k}$: cancels out)

$$\sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$$

$$\sim \frac{y^2}{v^2} \log \Lambda^2$$

Now we have additional contributions:

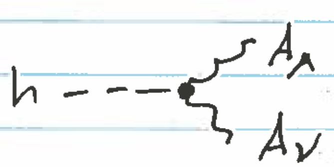


$$+ \text{ perm.} \sim g^2 y^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{\not{k}}{k^2}\right)^2 \left(\frac{k_\mu k_\nu}{m_A^2 k^2}\right)^2$$

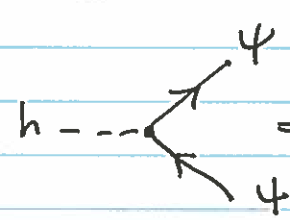
$$\sim \frac{g^2 y^2}{m_A^2} \log \Lambda^2 \sim \frac{y^2}{v^2} \log \Lambda^2$$

Divergences cancel out due to fact that masses m_A, m_ψ and couplings g, y are related.

Prediction from spontaneously broken gauge theory: "extra" scalar h whose couplings to gauge bosons & fermions is related to their masses:



$$= 2i \frac{m_A^2}{v} \eta_{\mu\nu}$$



$$= \frac{i m_\psi}{v}$$

Nonabelian Higgs model

example: SU(2) gauge theory. $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$, doublet of complex fields $\phi_{1,2}$

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$
$$- \frac{1}{2} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu})$$

where $D_\mu \Phi = (\partial_\mu + i g T^a A^a_\mu) \Phi$

$T^a = \frac{\sigma^a}{2}$, generators of SU(2)

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g \varepsilon^{abc} A^b_\mu A^c_\nu$

Potential is $V(\Phi) = -\mu \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$

Minimum of potential at $\Phi^\dagger \Phi = \frac{v^2}{2}$ where $v = \sqrt{\frac{\mu^2}{\lambda}}$

Free to work in unitary gauge where only real component of ϕ_2 is nonzero.

i.e. $\Phi(x) = \exp\left(i T^a \xi^a(x)/v\right) \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$

in terms of real fields $\xi^{1,2,3}$ & h.
Rotate away ξ^a using gauge transform.

$$\Phi(x) \to \exp(-i T^a \xi^a(x)/v)\, \Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

Covariant derivative term:

$$\mathcal{L} \supset (D_\mu \Phi^\dagger)(D^\mu \Phi) = \frac{1}{2} (0, v+h) (\overleftarrow{\partial}_\mu - i g T^a A^a_\mu)$$
$$\cdot (\overrightarrow{\partial}_\mu + i g T^b A^b_\mu) \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(0, v+h)\begin{pmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{pmatrix}^2 \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(v+h)^2\left((A^1_\mu)^2 + (A^2_\mu)^2 + (A^3_\mu)^2\right)$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left(\frac{gv}{2}\right)^2 A^a_\mu A^{\mu a}\left(1 + \frac{h}{v}\right)^2$$

We get three massive vector bosons with masses

$$m_A = \frac{gv}{2}. \qquad \text{(3 massless Goldstones } \xi^a \text{ eaten)}$$

This is almost like the SM: 3 massive vector bosons

but in SM $m_W \neq m_Z$ and also one massless boson γ
(and eight massless gluons)

SM Lagrangian

We are now ready to write down the SM Lagrangian.
Need to specify the gauge group, the d.o.f. (fields)
and their quantum numbers.

Gauge group: $\underbrace{SU(3)_C}_{\text{QCD (color)}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Electroweak (L = left, Y = hypercharge)}}$

Field	quantum numbers $(SU(3)_C, SU(2)_L, U(1)_Y)$
$Q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$	$(3, 2, \frac{1}{6})$
u^i_R	$(3, 1, +\frac{2}{3})$
d^i_R	$(3, 1, -\frac{1}{3})$

$L_L^i = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $\quad (1, 2, -\tfrac{1}{2})$

e_R^i $\quad (1, 1, -1)$

$[\nu_R^i$ $\quad (1, 1, 0)\ ]$

$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ $\quad (1, 2, \tfrac{1}{2})$ $\quad$ Scalar Higgs field (same as Φ)

i labels generation ($i = 1, 2, 3$). 3 copies of each field w/ same quantum numbers, different mass

Gauge bosons: $\quad$ gauge coupling

			gauge coupling
$SU(3)_C$	:	gluon field g_μ^A ($A = 1, \ldots, 8$),	g_s
$SU(2)_L$	:	W_μ^a ($a = 1, 2, 3$)	g
$U(1)_Y$	:	B_μ	g'

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{scalar} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{gauge} = -\frac{1}{2}\mathrm{Tr}(g_{\mu\nu} g^{\mu\nu}) - \frac{1}{2}\mathrm{Tr}(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{fermion} = \sum_{\text{fermions } \psi} \bar{\psi}\, i \not{D} \psi$$

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D^\mu H) - V(H)$$

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

First consider scalar Lagrangian:

Work in unitary gauge: $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$, $v = \sqrt{\frac{\mu^2}{\lambda}}$

Covariant derivative term:

$$D_\mu H = \left(\partial_\mu + i\frac{g}{2}\sigma^a W_\mu^a + i\frac{g'}{2}B_\mu \right) H$$

$$= \left(\partial_\mu + i\frac{1}{2}\begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g'B_\mu \end{pmatrix} \right) H$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + i\frac{1}{\sqrt{2}}\frac{1}{2}(v+h)\begin{pmatrix} g(W_\mu^1 - iW_\mu^2) \\ -gW_\mu^3 + g'B_\mu \end{pmatrix}$$

$$(D_\mu H)^\dagger (D^\mu H) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(v+h)^2\left((W_\mu^1)^2 + (W_\mu^2)^2\right)$$

$$+ \frac{1}{8}(v+h)^2\left(gW_\mu^3 - g'B_\mu\right)^2$$

Define $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$$

$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$$

where $\tan\theta_W = g'/g$

$$\Rightarrow g' = g\tan\theta_W = g\frac{\sin\theta_W}{\cos\theta_W}$$

$$gW_\mu^3 - g'B_\mu = g\left(W_\mu^3 - \frac{\sin\theta_W}{\cos\theta_W}B_\mu\right)$$

$$= \frac{g}{\cos\theta_W} Z_\mu$$

$$D_\mu H^\dagger D^\mu H = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2 v^2}{4} W^+_\mu W^- \left(1+\frac{h}{v}\right)^2$$

$$+ \frac{1}{8}\frac{g^2 v^2}{c_W^2} Z_\mu Z^\mu \left(1+\frac{h}{v}\right)^2$$

$$= \frac{1}{2}(\partial_\mu h)^2 + m_W^2 W^+_\mu W^- \left(1+\frac{h}{v}\right)^2$$

$$+ \frac{1}{2} m_Z^2 Z_\mu Z^\mu \left(1+\frac{h}{v}\right)^2$$

$$m_W = \frac{gv}{2}, \qquad m_Z = \frac{gv}{2c_W}$$

$$\Rightarrow \quad \frac{m_W^2}{c_W^2} = m_Z^2 \qquad \text{Define } \rho = \frac{m_W^2}{m_Z^2 c_W^2}$$

Then we have $\rho = 1$ in SM (at tree-level).

Note: A_μ remains massless $\rightarrow$ photon field.
$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

Next, consider fermion terms:

$$\mathcal{L}_{ferm} = \sum_\Psi \bar{\Psi}\, i \not{D} \Psi$$

$$= \bar{Q}^i_L\, i\left(\not{\partial} + i\frac{g}{2}\sigma^a W^a_\mu + i g' \frac{1}{6} B_\mu + i \frac{g_s}{2}\lambda^A g^A_\mu\right) Q^i_L$$

$$+ \bar{u}^i_R\, i\left(\not{\partial} + i g' \cdot \frac{2}{3} B_\mu + i\frac{g_s}{2}\lambda^A g^A_\mu\right) u^i_R$$

$$+ \bar{d}^i_R\, i\left(\not{\partial} + i g'\left(-\frac{1}{3}\right) B_\mu + i\frac{g_s}{2}\lambda^A g^A_\mu\right) d^i_R$$

$$+ \bar{L}^i_L\, i\left(\not{\partial} + i g'\left(-\frac{1}{2}\right) B_\mu + i\frac{g}{2}\sigma^a W^a_\mu\right) L^i_L$$

$$+ \bar{e}^i_R\, i\left(\not{\partial} + i g'(-1) B_\mu + \text{[illegible]}\right) e^i_R$$

$\not{\partial}$ terms are the usual kinetic terms: e.g. for quarks

$$\bar{Q}_L^i \not{\partial} Q_L^i + \bar{u}_R^i \not{\partial} u_R^i + \bar{d}_R^i \not{\partial} d_R^i$$

$$= \bar{u}_L^i \not{\partial} u_L^i + \bar{d}_L^i \not{\partial} d_L^i + \bar{u}_R^i \not{\partial} u_R^i + \bar{d}_R^i \not{\partial} d_R^i$$

$$= \bar{u}^i \not{\partial} u^i + \bar{d}^i \not{\partial} d^i$$

gluon terms: $-g_s\, g_\mu^A \left(\bar{u}^i \gamma^\mu \frac{\lambda^A}{2} u^i + \bar{d}^i \gamma^\mu \frac{\lambda^A}{2} d^i \right)$

Electroweak interactions: $W^\pm, Z, \gamma$ interactions

Charged current interactions ($W^\pm$ only)

$$\mathcal{L}_{CC} = i\, \bar{Q}_L^i \left(i\frac{g}{2}\right) \begin{pmatrix} 0 & \not{W}^1 - i\not{W}^2 \\ \not{W}^1 + i\not{W}^2 & 0 \end{pmatrix} Q_L^i$$

$$+ i\, \bar{L}_L^i \left(i\frac{g}{2}\right) \begin{pmatrix} 0 & \not{W}^1 - i\not{W}^2 \\ \not{W}^1 + i\not{W}^2 & 0 \end{pmatrix} L_L^i$$

$$= -\frac{g}{\sqrt{2}} (\bar{u}_L^i, \bar{d}_L^i) \begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix} \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$$

$$- \frac{g}{\sqrt{2}} (\bar{\nu}_L^i, \bar{e}_L^i) \begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix} \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$$

$$= -\frac{g}{\sqrt{2}} \bar{u}_L^i \not{W}^+ d_L^i - \frac{g}{\sqrt{2}} \bar{\nu}_L^i \not{W}^+ e_L^i + \text{c.c.}$$

Neutral current interactions (A, Z only)

$$\mathcal{L}_{NC} = -\bar{Q}_L^i \left(\frac{g}{2} \begin{pmatrix} \not{W}^3 & 0 \\ 0 & -\not{W}^3 \end{pmatrix} + \frac{g'}{6} \begin{pmatrix} \not{B} & 0 \\ 0 & \not{B} \end{pmatrix} \right) Q_L^i$$

$$- \bar{u}_R^i \left(\frac{2}{3} g' \not{B} \right) u_R^i - \bar{d}_R^i \left(-\frac{1}{3} g' \not{B} \right) d_R$$

$$- \bar{L}_L^i \left(\frac{g}{2} \begin{pmatrix} \not{W}^3 & 0 \\ 0 & -\not{W}^3 \end{pmatrix} - \frac{g'}{2} \begin{pmatrix} \not{B} & 0 \\ 0 & \not{B} \end{pmatrix} \right) L_L^i$$

$$- \bar{e}_R^i \left(-g' \not{B} \right) e_R^i$$

Use $W^3_\mu = c_W Z_\mu + s_W A_\mu$

$B_\mu = -s_W Z_\mu + c_W A_\mu$

General fermions ψ_L, ψ_R

$$\mathcal{L}_{NC} = -\bar{\psi}_L \left(g T_3 (c_W \not{Z} + s_W \not{A}) + g' Y_L(-s_W \not{Z} + c_W \not{A}) \right) \psi_L$$
$$- \bar{\psi}_R \left(g' Y_R (-s_W \not{Z} + c_W \not{A}) \right) \psi_R$$

$Y_{L,R}$ = hypercharge of LH, RH components (e.g. $Y_{Q_L} = \frac{1}{6}$, etc
$T_{3L} = \pm\frac{1}{2}$ for upper/lower component of $SU(2)_L$ doublet.

$$\mathcal{L}_{NC} = -\bar{\psi} \left\{ (g T_{3L} s_W + g' Y_L c_W) \not{A} P_L + g' Y_R c_W \not{A} P_R \right.$$
$$\left. + (g T_{3L} c_W - g' Y_L s_W) \not{Z} P_L - g' Y_R s_W \not{Z} P_R \right\} \psi$$
$$= -\bar{\psi} \left\{ g s_W \not{A} \left((T_{3L} + Y_L) P_L + Y_R P_R \right) \right.$$
$$\left. + \frac{g}{c_W} \Big(\underbrace{\left(T_{3L}(1 - s_W^2) - Y_L s_W^2 \right)}_{T_{3L} - (T_{3L} + Y_L) s_W^2} P_L + (-Y_R s_W^2) P_R \Big) \not{Z} \right\} \psi$$

Note: we have chosen hypercharges such that $T_{3L} + Y_L = Y_R$.

u_L: $T_{3L} + Y_L = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$
u_R: $Y_R = 2/3$
$\Big\} = Q_u$

ν_L: $T_{3L} + Y_L = \frac{1}{2} - \frac{1}{2} = 0$
$Q_\nu = 0$.

d_L: $T_{3L} + Y_L = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}$
d_R: $Y_R = -\frac{1}{3}$
$\Big\} = Q_d$

e_L: $T_{3L} + Y_L = -\frac{1}{2} - \frac{1}{2} = -1$
e_R: $Y_R = -1$
$\Big\} = Q_e$

$$\mathcal{L}_{NC} = -e Q_\psi \bar{\Psi} \not{A} \Psi - \frac{g}{c_W} \bar{\Psi} \not{Z} (T_{3L} P_L - Q_\psi s_W^2) \Psi$$

where $e = g s_W$, Q_ψ is EM charge in units of e.

Even though we started with a chiral theory, able to assign charges such that unbroken $U(1)_{em}$ is not chiral & couples in usual way to fermions.

e.g. for quarks we have:

$$\mathcal{L}_{NC} = -e Q_u \bar{u}^i \not{A} u^i - e Q_d \bar{d}^i \not{A} d^i$$
$$- \frac{g}{c_W} \bar{u}^i \not{Z} \left(\tfrac{1}{2} P_L - Q_u s_W^2 \right) u^i - \frac{g}{c_W} \bar{d}^i \not{Z} \left(-\tfrac{1}{2} P_L - Q_d s_W^2 \right) d^i$$

Fermion mass terms: $\bar{\Psi}\Psi$ terms are forbidden by gauge sym. but can be generated via Yukawa interaction.

First, need to recall some $SU(2)$ group theory. Suppose we have two 2-component vectors (fundamental reps) of $SU(2)$, denoted η, ξ. There are two ways to contract them in an $SU(2)$-invariant way:

$$\eta^\dagger \xi \quad \text{or} \quad \eta^T \varepsilon \xi \quad \text{where} \quad \varepsilon = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \text{ is}$$

antisym. tensor. acting on $SU(2)_L$ indices.

For general $SU(N)$, there are 2 invariant tensors:

δ_{ij} (Identity) : $\mathbb{1} \to U^\dagger \mathbb{1} U = \mathbb{1} U^\dagger U = \mathbb{1}$

$\underbrace{\varepsilon_{ijk\ldots}}_{N \text{ indices}}$ (antisym tensor) : $\varepsilon_{ijk\ldots} \to U_{ii'} U_{jj'} \ldots \varepsilon_{i'j'k'\ldots}$

$$= \underbrace{\det(U)}_{1} \varepsilon_{ijk\ldots}$$
$$= \varepsilon_{ijk\ldots}$$

Valid Yukawa interactions:

$$H^\dagger \bar{d}_R Q_L \;,\; \bar{u}_R Q_L^T \varepsilon H \;,\; H^\dagger \bar{e}_R L_L$$

Invalid Yukawa interactions (why?)

$$\bar{d}_R Q_L^T \varepsilon H \;,\; H^\dagger \bar{u}_R Q_L \;,\; \bar{e}_R L_L^T \varepsilon H$$

In general, Yukawa interaction can couple any two fermion generations i & j.

$$\mathcal{L}_{Yukawa} = -Y^{(u)}_{ij} \bar{u}^i_R Q^{j\,T}_L \varepsilon H - Y^{(d)}_{ij} \overset{H^\dagger}{\vee}\bar{d}^i_R Q^j_L - Y^{(e)}_{ij} H^\dagger \bar{e}^i_R L^j_L + h.c.$$

$$= -Y^{(u)}_{ij} \frac{v}{\sqrt{2}} \bar{u}^i_R u^j_L \left(1+\frac{h}{v}\right) - Y^{(d)}_{ij} \frac{v}{\sqrt{2}} \bar{d}^i_R d^j_L \left(1+\frac{h}{v}\right) - Y^{(e)}_{ij} \frac{v}{\sqrt{2}} \bar{e}^i_R e^j_L \left(1+\frac{h}{v}\right) + h.c.$$

We have mass terms, but in general they are not diagonal. Diagonalize them by performing a 3x3 rotation in generations

We can diagonalize any complex square matrix by using a biunitary transformation where resulting matrix is diagonal with real, positive entries.

proof: square matrix M. Note $MM^\dagger$ is Hermitian, diagonalized by unitary transform.

$$U^\dagger M M^\dagger U = M_d^2 = \text{diag. \& positive.}$$

$$U^\dagger M \underbrace{(M^\dagger U M_d^{-1})}_{V} = M_d$$

← take positive square root of each diag. entry

V is also unitary:

$$V^\dagger V = M_d^{-1} U^\dagger M M^\dagger U M_d^{-1} = M_d^{-1} M_d^2 M_d^{-1} = \mathbb{1}$$

Assumes M_d has nonzero entries (M_d^{-1} exists) → OK since all SM fermions have nonzero masses (except ν, neglected). Proof can be generalized to case of zero masses anyways.

Write Yukawa terms in matrix form:

$$\mathcal{L}_{Yukawa} = -\bar{u}_R M^{(u)} u_L \left(1+\frac{h}{v}\right) - \bar{d}_R M^{(d)} d_L \left(1+\frac{h}{v}\right)$$

$$- \bar{e}_R M^{(e)} e_L \left(1+\frac{h}{v}\right) + h.c.$$

$$M_{ij}^{(u,d,e)} = Y_{ij}^{(u,d,e)} \frac{v}{\sqrt{2}}$$ 3x3 matrices in generation space. ($i,j=1,2,3$)

$$u_L = \begin{pmatrix} u_L^1 \\ u_L^2 \\ u_L^3 \end{pmatrix} = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}$$, etc. 3-component vectors

Diagonalize using biunitary transformation:

$$U_{u_R}^\dagger M^{(u)} U_{u_L} = m^{(u)} = \mathrm{diag}\left(m_1^{(u)}, m_2^{(u)}, m_3^{(u)}\right) = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

U_{u_L}, U_{u_R} are unitary matrices.

Define new fermion fields: u_L', u_R' such that

$$u_{L,R} = U_{u_{L,R}} u_{L,R}'$$

$$\bar{u}_R M^{(u)} u_L = \bar{u}_R' m^{(u)} u_L'$$

mass terms are real & diagonal.

Similar for other fields

$$d_{L,R} = U_{d_{L,R}} d'_{L,R} \quad , \quad e_{L,R} = U_{e_{L,R}} e'_{L,R}$$

Since there is no neutrino mass, free to rotate ν_L as we like: define ν_L' such that

$$\nu_L^i = U_{e_L} \nu_L'$$

For all fermions $\psi = u^i, d^i, e^i$ we have:

$$\mathcal{L}_{Yukawa} = - \sum_{\psi} m_\psi \bar{\psi}\psi \left(1 + \frac{h}{v}\right)$$

Since we've redefined fermion fields, check what happens to charged & neutral currents.

$$\mathcal{L}_{NC} = -e Q_\psi \bar{\psi} \not{A} \psi - \frac{g}{c_W} \bar{\psi} \not{Z} (T_{3L} P_L - Q_\psi s_W^2) \psi$$

Consider $\psi = u^i$. All u^i have same Q_u & T_{3L}.

$$\mathcal{L}_{NC} = -e Q_u (\bar{u}_L^i \not{A} u_L^i + \bar{u}_R^i \not{A} u_R^i)$$
$$- \frac{g}{c_W} \left(\bar{u}_L^i \not{Z} (T_{3L} - Q_u s_W^2) u_L^i - \bar{u}_R^i \not{Z} Q_u s_W^2 u_R^i \right)$$

Now transform to primed fields (mass eigenstate fields)

$$\bar{u}_L^i \gamma^\mu u_L^i = \bar{u}_L'^j (U_{u_L}^\dagger)_{ji} \gamma^\mu (U_{u_L})_{ik} u_L'^k$$
$$= \bar{u}_L'^i \gamma^\mu u_L'^i \quad \text{since } U_{u_L}^\dagger U_{u_L} = \mathbb{1} \text{ (unitary)}$$

Same for $u_R, d_{L,R}, e_{L,R}, \nu_L$. NC unchanged: $\psi \to \psi'$.

Since NC was diagonal in original basis, it is diagonal in the mass basis. Z, γ interaction do not change the flavor of a fermion (generation).

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L^i \not{W}^+ d_L^i + \bar{\nu}_L^i \not{W}^+ e_L^i\right) + h.c.$$

$$= -\frac{g}{\sqrt{2}}\Big(\bar{u}_L'^{j} (U_{uL}^\dagger)_{ji} \not{W}^+ (U_{dL})_{ik}\, d_L'^{k}$$

$$+ \bar{\nu}_L'^{j} (U_{eL}^\dagger)_{ji} \not{W}^+ (U_{eL})_{ik}\, e_L'^{k}\Big) + h.c.$$

For quark CC interaction, unitary matrices don't cancel!

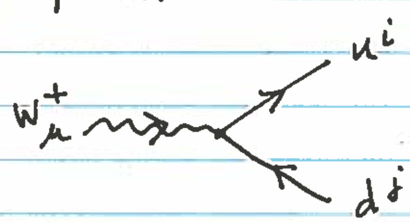
Define matrix $V = U_{uL}^\dagger U_{dL} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

"Cabibbo - Maskawa - Kobayashi (CKM) matrix"

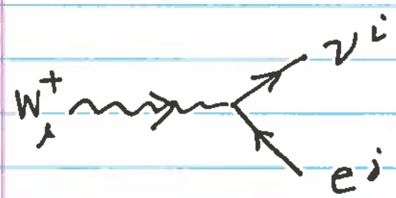
$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L'^{i} V_{ij} \not{W}^+ d_L'^{j} + \bar{\nu}_L'^{i} \not{W}^+ e_L'^{i}\right) + h.c.$$

CC interaction couples quarks of different generations i, j

Feynman rule:



$$= -\frac{ig}{\sqrt{2}} V_{ij} \gamma^\mu P_L$$



$$= -\frac{ig}{\sqrt{2}} \delta_{ij} \gamma^\mu P_L$$