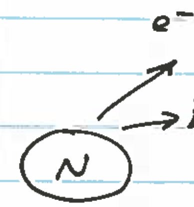
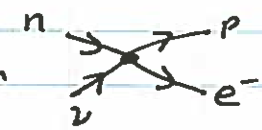
Brief history of the weak interaction



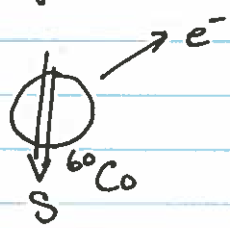
1899: Radioactive β-decay discovered by Becquerel

1930: neutrino proposed by Pauli to conserve energy & angular momentum in β-decay

1933: Fermi's theory of the weak interaction
Unknown what the Dirac structure is.



1956: Weak interaction proposed to violate parity by Lee & Yang (following suggestion by M. Block). Observed to violate P by C.S. Wu (1957)



$\underline{S} \cdot p_e < 0.$

This necessitates chiral couplings to fermions.
Correct V-A structure discovered by Marshak, Sudarshan, Feynman, Gell-Mann.

1960's: SM proposed by Glashow, Salam, Weinberg, w/ "Higgs mechanism" invented by Higgs, Brout, Englert, Guralnik, Hagen, Kibble (1963-1964)

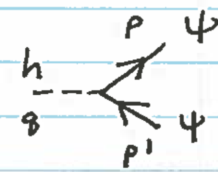
W, Z bosons discovered at CERN (1983)
h — " — (2012)

Higgs decays & Higgs production

Higgs boson has largest coupling to particles with largest mass, $W^\pm, Z, t$. However, $h \to W^+W^-, ZZ, t\bar{t}$ are all forbidden kinematically since $m_h = 125$ GeV. This is fortuitous for studying the Higgs since we can explore not only its largest couplings to $t\bar{t}, WW, ZZ$ (via higher order processes) but also couplings that are quite a bit smaller $(b\bar{b}, \tau\bar{\tau})$.

Tree-level decays:

- $h \to$ fermions. $\mathcal{L}_{int} = -\frac{m_\psi}{v} h \bar{\psi}\psi$



$= -i\frac{m_\psi}{v}$

$i\mathcal{M} = -i\frac{m_\psi}{v}\bar{u}(p)v(p')$ neglect $m_\psi \ll m_h$.

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{m_\psi^2}{v^2} 4p\cdot p' = \frac{2m_\psi^2}{v^2} m_h^2 \qquad q^2 = m_h^2 = (p+p')^2 = 2p\cdot p'$$

$$\Gamma(h\to\psi\bar{\psi}) = \frac{1}{16\pi m_h}\sum|\mathcal{M}|^2$$ (do 2-body phase space integral, $m_\psi = 0$)

$$= \frac{m_\psi^2 m_h}{8\pi v^2}$$

$$\Gamma(h\to\tau\bar{\tau}) = \frac{m_\tau^2 m_h}{8\pi v^2} \simeq 0.26 \text{ MeV} \quad (6.3\%)$$

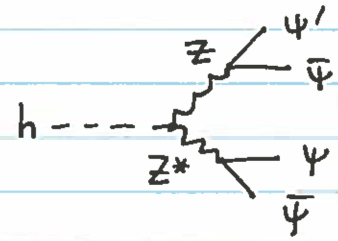
$$\Gamma(h\to b\bar{b}) = \frac{3 m_b^2 m_h}{8\pi v^2} \simeq 4.3 \text{ MeV}^* \quad (58\%)$$

$$\Gamma(h\to c\bar{c}) \approx \frac{3m_c^2 m_h}{8\pi v^2} \approx 0.4 \text{ MeV}^* \quad (2.9\%)$$

* Renormalization group running of $m_{c,b}$ with energy from $m_{c,b}$ to m_h reduces these Γ for $b\bar{b}$ by ~ 2 and $c\bar{c}$ by ~ 4.

- $h \to W^+W^-, ZZ$. forbidden unless one (or both) gauge bosons are off-shell.

e.g.



"golden mode" $\psi, \psi' = e, \mu$

$h \to ZZ^* \to 4$ leptons

where leptons $= e^+e^-$ or $\mu^+\mu^-$

$$\Gamma(h \to ZZ^*) \simeq 0.11 \text{ MeV} \quad (2.6\%)$$
$$\Gamma(h \to WW^*) \simeq 0.88 \text{ MeV} \quad (22\%)$$

Loop decays: two other important channels

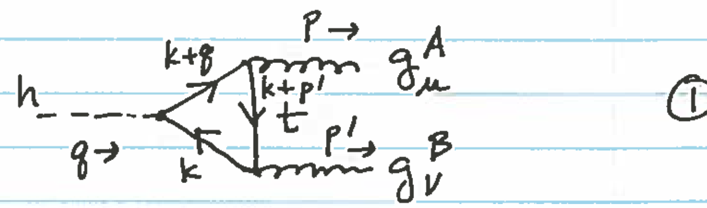
$$\Gamma(h \to gg) \simeq 0.35 \text{ MeV} \quad (8.6\%)$$
$$\Gamma(h \to \gamma\gamma) \simeq 6 \times 10^{-3} \text{ MeV} \quad (0.15\%)$$

Although gluons & photons are massless, h can couple to them at 1-loop order.

$h \to \gamma\gamma$ is rare, but experimentally clean (the other "golden mode")
It was how h was first discovered.

First do (easier) decay $h \to gg$.

$i\mathcal{M} = i\mathcal{M}_1 + i\mathcal{M}_2 =$



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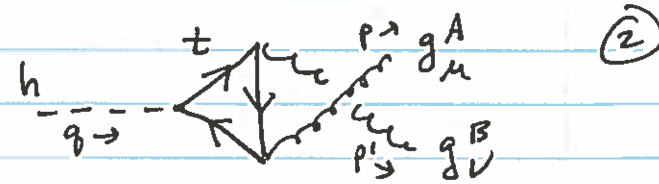


Note: $h \to gg$ is a way to indirectly probe $ht\bar{t}$ coupling.

$$i\mathcal{M}_1 = \int \frac{d^dk}{(2\pi)^d} (-1) \mathrm{Tr}\left[\frac{i(\not{k}+m_t)}{k^2-m_t^2}\, ig_s T^B \gamma^\nu \frac{i(\not{k}+\not{p}'+m_t)}{(k+p')^2-m_t^2} \right.$$

$$\left. \cdot\, ig_s T^A \gamma^\mu \frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2-m_t^2} \cdot \left(-i\frac{m_t}{v}\right) \right] \varepsilon_\mu \varepsilon_\nu$$

$$= -g_s^2 \mathrm{Tr}(T^A T^B) \left(\frac{m_t}{v}\right) \varepsilon_\mu(p)\, \varepsilon_\nu(p')$$

$$\times \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}\left[(\not{k}+m_t)\gamma^\nu(\not{k}+\not{p}'+m_t)\gamma^\mu(\not{k}+\not{q}+m_t)\right]}{(k^2-m_t^2)((k+p')^2-m_t^2)((k+q)^2-m_t^2)}$$

use Feynman parameters:

$$i\mathcal{M}_1 = -g_s^2 \mathrm{Tr}(T^A T^B) \frac{m_t}{v} \varepsilon_\mu(p)\, \varepsilon_\nu(p')$$

$$\times \int dx\,dy\,dz\; 2\delta(1-x-y-z) \cdot \frac{\mathrm{Tr}[\cdots]}{\left[(k^2-m_t^2)x + ((k+p')^2-m_t^2)y + ((k+q)^2-m_t^2)z\right]^3}$$

Factor in brackets is:

$$(k^2-m_t^2)x + ((k+p')^2-m_t^2)y + ((k+q)^2-m_t^2)z$$

$$= k^2(x+y+z) + 2k\cdot(yp' + qz)$$

$$+ \underbrace{p'^2}_{\to 0} y + \underbrace{q^2}_{\to m_h^2} z - m_t^2(x+y+z)$$

$$= (k + yp' + qz)^2 - (yp'+qz)^2 + m_h^2 z - m_t^2$$

$$= \ell^2 - \underbrace{\left(m_t^2 + z(z-1)m_h^2 + 2yz\, p'\cdot q\right)}_{= M^2}$$

Note: $p'\cdot q = p'\cdot p + p'^2 = \frac{1}{2}q^2 = \frac{1}{2}m_h^2$

$M^2 = m_t^2 - zx\, m_h^2$, $\quad \ell = k + yp' + zq = k + (1-x)p' + zp$

In terms of l: the trace in the numerator is

$$\mathrm{Tr}[\cdots] = 16 m_t l^\mu l^\nu - 4 l^2 m_t \eta^{\mu\nu}$$
$$+ 4 m_t^3 \eta^{\mu\nu} - 2 m_t m_h^2 (1-2xz)\eta^{\mu\nu} + 4(1-4xz) p_1^\nu p_2^\mu m_t$$

We have thrown away terms $\sim p^\mu, p'^\nu$ (vanish when contracted with $\epsilon_\mu(p), \epsilon_\nu(p')$)
and terms proportional to one power of l (vanishes by antisym)

In d-dimensions, $l^\mu l^\nu \to \frac{1}{d}\eta^{\mu\nu} l^2$ under the integral.
Check: $\eta_{\mu\nu} l^\mu l^\nu = l^2 = \frac{1}{d}\eta_{\mu\nu}\eta^{\mu\nu} l^2 = \frac{d}{d} l^2$. ✓

Putting pieses together:

$$i\mathcal{M}_1 = -g_s^2 \mathrm{Tr}(T^A T^B) \frac{m_t^2}{v} \epsilon_\mu \epsilon_\nu \int \frac{d^d l}{(2\pi)^d}$$
$$\times \int dx\, dy\, dz\; 2\,\delta(1-x-y-z) \cdot \frac{1}{(l^2-M^2)^3}$$
$$\times \left(\left(\frac{16}{d} - 4\right) l^2 \eta^{\mu\nu} + 2\left(2m_t^2 - m_h^2(1-2xz)\right)\eta^{\mu\nu} + 4(1-4xz)\, p_1^\nu p_2^\mu \right)$$

Momentum integrals:

$$(1) = \int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2-M^2)^3} = \frac{-i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(3-\frac{d}{2}-1)}{\Gamma(3)} \left(\frac{1}{M^2}\right)^{3-\frac{d}{2}-1}$$

$$(2) = \int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2-M^2)^3} = \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(3-d/2)}{\Gamma(3)} \left(\frac{1}{M^2}\right)^{3-d/2}$$

Integral (1) is divergent. But we only need to keep the divergent part $\sim \frac{1}{4-d}$ since it is proportional to $\left(\frac{16}{d}-4\right) = 4\left(\frac{4-d}{d}\right) \to 0$ as $d \to 4$

$$(1) = \frac{-i}{16\pi^2}\,\frac{d}{2}\,\frac{\Gamma\left(\frac{4-d}{2}\right)}{2} + \dots \text{ finite terms}$$

$$= -\frac{i}{16\pi^2}\,\frac{2}{4-d} + \dots \quad \text{using } \Gamma(x) = \frac{1}{x} + \dots \text{ finite. for } x \to 0.$$

$$(2) = \frac{i}{16\pi^2}\,\frac{1}{2}\,\frac{1}{M^2}$$

So we have:

$$i\mathcal{M}_1 = -\frac{i}{16\pi^2} g_s^2 \,\mathrm{Tr}(T^A T^B)\,\frac{m_t^2}{v}\,\varepsilon_\mu(p)\,\varepsilon_\nu(p') \int_0^1 dx \int_0^{1-x} dz \; \cancel{\delta(1-x-y-z)}$$

$$\times \Big\{ -4\left(\frac{4-d}{d}\right)\cdot\frac{2}{4-d}\cdot 2\,(m_t^2 - xz\,m_h^2)\,\eta^{\mu\nu}$$

$$+ (4m_t^2 - 2m_h^2 + 4xz\,m_h^2)\,\eta^{\mu\nu}$$

$$+ 4(1-4xz)\,p_1^\nu p_2^\mu \Big\} \quad \frac{1}{M^2}$$

$$= -\frac{i}{16\pi^2} g_s^2 \,\mathrm{Tr}(T^A T^B)\,\frac{m_t^2}{v}\,\varepsilon_\mu(p)\,\varepsilon_\nu(p')$$

$$\times \int dx\, dz \;\cdot \left(4 p_1^\nu p_2^\mu - 2m_h^2 \eta^{\mu\nu}\right)$$

$$\times \quad \frac{1-4xz}{m_t^2 - xz\,m_h^2}$$

$$i\mathcal{M}_1 = \frac{-i}{16\pi^2}\frac{g_s^2}{v}\,\mathrm{Tr}(T^A T^B)\,\varepsilon_\mu(p)\varepsilon_\nu(p')\,(4 p_1^\nu p_2^\mu - 2 m_h^2 \eta^{\mu\nu})$$
$$\times \int_0^1 dx \int_0^{1-x} dz \, \frac{1-4xz}{1-xz(m_h^2/m_t^2)}$$

Let's consider limit $m_t \gg m_h$. Even though it doesn't hold, it works reasonably well numerically even if $m_t \sim m_h$.

$$\int_0^1 dx \int_0^{1-x} dz\, (1-4xz) \doteq \frac{1}{3}$$

Also we have $\mathrm{Tr}(T^A T^B) = \frac{1}{2}\delta^{AB}$

And $i\mathcal{M}_2 = i\mathcal{M}_1 \rightarrow$ factor of 2.

$$i\mathcal{M} = -i\,\frac{\alpha_s}{3\pi v}\,\varepsilon_\mu(p)\varepsilon_\nu(p')\left(p_1^\nu p_2^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right)\delta^{AB}$$

$$\Sigma|\mathcal{M}|^2 = \left(\frac{\alpha_s}{3\pi v}\right)^2 \underbrace{\delta_{AB}\,\delta^{AB}}_{8}\left(p_1^\nu p_2^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right)\left(p_{1\nu} p_{2\mu} - \frac{m_h^2}{2}\eta_{\mu\nu}\right)$$

$$= 8\,\frac{\alpha_s^2}{9\pi^2 v^2}\left(m_h^4 - m_h^2\,\underbrace{p_1\cdot p_2}_{m_h^2/2}\right)$$

$$= \frac{4\alpha_s^2 m_h^4}{9\pi^2 v^2}$$

also need to divide by 2 for identical gluons in final state.

$$\Gamma(h\to gg) = \frac{\alpha_s^2 m_h^3}{64\pi^3 v^2}$$

$$\simeq 0.23 \text{ MeV} \quad (5.7\%)^*$$

QCD corrections enhance it by ~ 60%

What about light quark contribution to $h \to gg$?
Note: m_t dependence cancels, but this is only true for heavy states in the loop ($2m \gtrsim m_h$).

The loop integral is:

$$\int_0^1 dx \int_0^{1-x} dz \, \frac{1-4xz}{1-xz(m_h^2/m_q^2)} \approx \begin{cases} 1/3 & \text{for } m_q^2 \gg m_h^2 \\ \frac{1}{2}\frac{m_q^2}{m_h^2}\log^2(m_h^2/m_q^2) & m_q^2 \ll m_h^2 \end{cases}$$

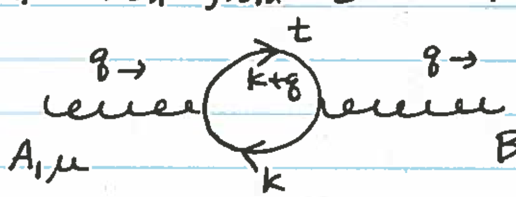
$\to 0$ for m_q.

Light quarks (here, u, d, s, c, b) give a small contribution.

Higgs low-energy theorem

There is a nice trick to calculate the $h \to gg$ amplitude from the QCD β-function. (in the $m_t \to \infty$ limit)

Let's compute the contribution from t to the gluon propagator:

$i\mathcal{M} =$ 

This will contribute to the gluon field strength renormalization, which contributes to the QCD β-function. Let's compute it

Use dim-reg in $d = 4-2\varepsilon$ dimensions. Amplitude will be divergent.

$$i\mathcal{M} = \int \frac{d^dk}{(2\pi)^d} (-1) \,\mathrm{Tr}\left[\frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2 - m_t^2} (-ig_s \gamma^\mu T^A \mu^\varepsilon) \right.$$

$$\left. \times \frac{i(\not{k}+m_t)}{k^2 - m_t^2} (-ig_s \gamma^\nu T^B \mu^\varepsilon) \right]$$

Recall: factors of μ^{ε} are due to the fact that in d dim, gauge coupling has mass dim $(2-\frac{d}{2})=\varepsilon$.

Dim reg: g_s (4-dim) $\longrightarrow g_s \mu^{\varepsilon}$ (d-dim)

$$i\mathcal{M} = -g_s^2 \mu^{2\varepsilon} \mathrm{Tr}[T^A T^B] \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[(\not{k}+\not{q}+m_t)\gamma^\mu(\not{k}+m_t)\gamma^\nu]}{(k^2-m_t^2)((k+q)^2-m_t^2)}$$

Feynman parameters:

$$\int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[\cdots]}{(k^2-m_t^2)((k+q)^2-m_t^2)} = \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[\cdots]}{(((k+q)^2-m_t^2)x+(k^2-m_t^2)(1-x))^2}$$

denominator:

$$((k+q)^2-m_t^2)x + (k^2-m_t^2)(1-x)$$
$$= k^2 + 2k\cdot q\, x + x q^2 - m_t^2 = (k+xq)^2 - m_t^2 - x(x-1)q^2$$
$$= l^2 - M^2$$

where $l = k + xq$, $M^2 = m_t^2 - x(1-x)q^2$

Now shift integral, setting $k = l - xq$.

$$i\mathcal{M} = -g_s^2 \mu^{2\varepsilon} \mathrm{Tr}[T^A T^B] \int_0^1 dx \int \frac{d^dl}{(2\pi)^d} \frac{1}{(l^2-M^2)^2}$$
$$\times \mathrm{Tr}[(\not{l}+\not{q}(1-x)+m_t)\gamma^\mu(\not{l}-x\not{q}+m_t)\gamma^\nu]$$

$$= -g_s^2 \mu^{2\varepsilon} \mathrm{Tr}[T^A T^B] \int_0^1 dx \int \frac{d^dl}{(2\pi)^d} \frac{1}{(l^2-M^2)^2}$$
$$\times \left\{ \left(\frac{8}{d}-4\right) l^2 \eta^{\mu\nu} - 8(1-x)x\, q^\mu q^\nu + 4x(1-x) q^2 \eta^{\mu\nu} + 4m_t^2 \eta^{\mu\nu} \right\}$$

Using $l^\mu l^\nu = \frac{1}{d} l^2 \eta^{\mu\nu}$ and terms linear in l vanish.

Momentum integrals:

$$\int \frac{d^d\ell}{(2\pi)^d} \mu^{2\epsilon} \frac{1}{(\ell^2-M^2)^2} = \frac{i}{(4\pi)^{d/2}} \quad \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \quad \frac{\mu^{2\epsilon}}{(M^2)^{2-d/2}}$$

$$= \frac{i}{16\pi^2} (4\pi)^{\epsilon} \Big(\frac{1}{\epsilon} - \gamma_E \underset{\overset{\wedge}{\mathcal{O}(\epsilon)}}{+ \ldots} \Big) \left(\frac{\mu^2}{M^2} \right)^{\epsilon}$$

$$= \frac{i}{16\pi^2} \left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\left(\frac{\mu^2}{M^2}\right) + \ldots \mathcal{O}(\epsilon) \ldots \right)$$

We've used identity $z^{\epsilon} = e^{\epsilon \log z} \simeq 1 + \epsilon \log z$ and expanded everything to $\mathcal{O}(\epsilon^0)$.

$$\int \frac{d^d\ell}{(2\pi)^d} \mu^{2\epsilon} \frac{\ell^2}{(\ell^2-M^2)^2} = \frac{-i}{(4\pi)^{d/2}} \; \frac{d}{2} \; \frac{\Gamma(2-\frac{d}{2}-1)}{\Gamma(2)} \; \frac{\mu^{2\epsilon}}{(M^2)^{2-d/2-1}}$$

$$= \frac{+i}{16\pi^2} (4\pi)^{\epsilon} \frac{d}{2} \left(\frac{1}{\epsilon} - \gamma_E + 1 + \ldots \right) M^2 \left(\frac{\mu^2}{M^2} \right)^{\epsilon}$$

This term is multiplied by $\left(\frac{8}{d} - 4\right)$.

$$\left(\frac{8}{d}-4\right) \int \frac{d^d\ell}{(2\pi)^d} \mu^{2\epsilon} \frac{\ell^2}{(\ell^2-M^2)^2} = \frac{i}{16\pi^2} (4\pi)^{\epsilon} \overbrace{(4-2d)}^{-4+4\epsilon} \left(\frac{1}{\epsilon} - \gamma_E + 1\right)$$
$$\times M^2 \left(\frac{\mu^2}{M^2}\right)^{\epsilon}$$

$$= \frac{-i}{16\pi^2} 4M^2 \left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log \frac{\mu^2}{M^2} + \ldots \right)$$

$$i\mathcal{M} = \frac{i}{16\pi^2} g_s^2 \, \mathrm{Tr}[T^A T^B] \int_0^1 dx \left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log \frac{\mu^2}{M^2} + \ldots \right)$$
$$\times \Big\{ \big(\cancel{4 m_t^2} - 4x(1-x) q^2\big) \eta^{\mu\nu} + 8x(1-x) q^\mu q^\nu$$
$$- 4x(1-x) q^2 \eta^{\mu\nu} - \cancel{4 m_t^2} \eta^{\mu\nu} \Big\}$$

$$i\mathcal{M} = i\frac{\alpha_s}{4\pi}\mathrm{Tr}[T^A T^B](q^\mu q^\nu - q^2\eta^{\mu\nu})$$
$$\times \int_0^1 dx \; 8x(1-x)\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi + \log\left(\frac{\mu^2}{m_t^2 - x(1-x)q^2}\right)\right)$$

Note: the divergent part is removed by a counter term by renormalizing the gluon field strength. The coefficient of the $1/\varepsilon$ term corresponds to the resulting contribution to the QCD β-function.

$$i\mathcal{M} = i\frac{\alpha_s}{4\pi}\mathrm{Tr}(T^A T^B)(q^\mu q^\nu - q^2\eta^{\mu\nu})\, b_t$$

$$\Rightarrow \beta_s^{(t)} = \frac{d\alpha_s}{d\log\mu} = \frac{\alpha_s^2}{4\pi} b_t \qquad t\text{-only contribution}$$

Recall: $\beta_s = -\left(11 - \frac{2}{3}N_f\right)\frac{\alpha_s^2}{2\pi}$

Now consider the $h\to gg$ amplitude, in the $m_t\to\infty$ limit.

$$i\mathcal{M} = i\frac{\alpha_s}{3\pi}\mathrm{Tr}[T^A T^B](q^\mu q^\nu - q^2\eta^{\mu\nu})\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_t^2}\right)$$

The Higgs field h couples via $m_t(1+\frac{h}{v})$. So if we shift $m_t \to m_t(1+h/v)$, we can include interactions with ~~[illegible]~~ the Higgs. Note:

$$\log m_t^2\left(1+\frac{h}{v}\right)^2 = \log m_t^2 + 2\log\left(1+\frac{h}{v}\right)$$
$$\approx \log m_t^2 + 2\, h/v + \mathcal{O}\left(\frac{h^2}{v^2}\right)$$

Only need term linear in h.

Contracting h with an external higgs state, we get:

$$i\mathcal{M}(hgg) = -i\frac{\alpha_s}{3\pi v}\delta^{AB}(q^\mu q^\nu - q^2\eta^{\mu\nu})$$

Same as full loop calculation, (with zero momentum h)

We can match this onto an effective theory for hgg interaction:

$$\mathcal{L}_{eff} = +\frac{\alpha_s}{6\pi v} h \,\mathrm{Tr}\left[G_{\mu\nu} G^{\mu\nu}\right]$$

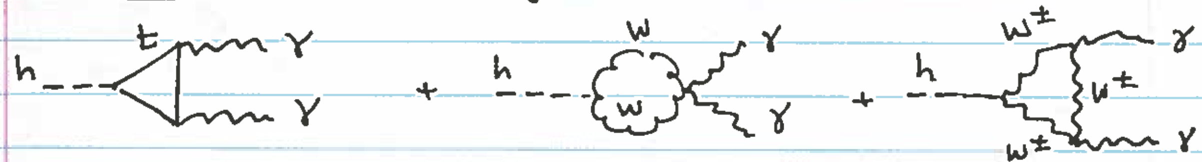
or expressed in terms of the β-fn:

$$\mathcal{L}_{eff} = +\frac{1}{2} \frac{\beta_s^{(t)}}{\alpha_s v} h \,\mathrm{Tr}\left[G_{\mu\nu} G^{\mu\nu}\right]$$

Useful way to compute hgg vertex: easier to compute divergent part of gluon propagator vs. full hgg loop diagram.

Note: only include massive ($\gtrsim m_h$) states contributing to β-fn.

<u>$h \to \gamma\gamma$ decay</u> : leading contributions are t & $W^\pm$ loops



Difficult to compute using unitary gauge, but can be computed more easily from the β-function for QED:

$$\mathcal{L}_{eff} = \frac{1}{4} \frac{\beta_e^{(t+W)}}{\alpha} \frac{h}{v} F^{\mu\nu} F_{\mu\nu}$$

$$\beta_e = \frac{d\alpha}{d\log\mu}$$

only t & W contributions