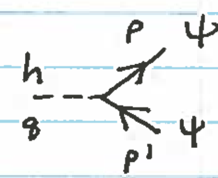
Higgs decays & Higgs production

Higgs boson has largest coupling to particles with largest mass, $W^\pm, Z, t$. However, $h \to W^+W^-, ZZ, t\bar{t}$ are all forbidden kinematically since $m_h = 125$ GeV. This is fortuitous for studying the Higgs since we can explore not only its largest couplings to $t\bar{t}, WW, ZZ$ (via higher order processes) but also couplings that are quite a bit smaller $(b\bar{b}, \tau\bar{\tau})$.

Tree-level decays:

- $h \to$ fermions. $\mathcal{L}_{int} = -\frac{m_\psi}{v} h \bar{\psi}\psi$



$= -i\frac{m_\psi}{v}$

$$i\mathcal{M} = -i\frac{m_\psi}{v}\bar{u}(p)v(p') \qquad \text{neglect } m_\psi \ll m_h.$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{m_\psi^2}{v^2} 4p\cdot p' = \frac{2m_\psi^2}{v^2} m_h^2 \qquad q^2 = m_h^2 = (p+p')^2 = 2p\cdot p'$$

$$\Gamma(h \to \psi\bar{\psi}) = \frac{1}{16\pi m_h}\sum|\mathcal{M}|^2 \qquad \text{(do 2-body phase space integral, } m_\psi = 0\text{)}$$

$$= \frac{m_\psi^2 m_h}{8\pi v^2}$$

$$\Gamma(h \to \tau\bar{\tau}) = \frac{m_\tau^2 m_h}{8\pi v^2} \approx 0.26 \text{ MeV} \quad (6.3\%)$$

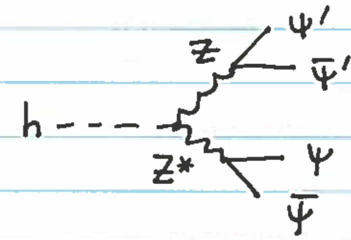
$$\Gamma(h \to b\bar{b}) = \frac{3m_b^2 m_h}{8\pi v^2} \approx 4.3 \text{ MeV}^* \quad (58\%)$$

$$\Gamma(h \to c\bar{c}) \approx \frac{3m_c^2 m_h}{8\pi v^2} \approx 0.4 \text{ MeV}^* \quad (2.9\%)$$

* Renormalization group running of $m_{c,b}$ with energy from $m_{c,b}$ to m_h reduces these Γ for $b\bar{b}$ by ~ 2 and $c\bar{c}$ by ~ 4.

- $h \to W^+W^-, ZZ$. forbidden unless one (or both) gauge bosons are off-shell.

e.g.



"golden mode" $\psi, \psi' = e, \mu$
$h \to ZZ^* \to 4$ leptons
where leptons $= e^+e^-$ or $\mu^+\mu^-$

$$\Gamma(h \to ZZ^*) \simeq 0.11 \text{ MeV} \quad (2.6\%)$$
$$\Gamma(h \to WW^*) \simeq 0.88 \text{ MeV} \quad (22\%)$$

Loop decays: two other important channels

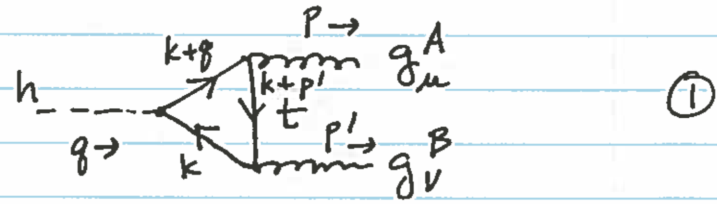
$$\Gamma(h \to gg) \simeq 0.35 \text{ MeV} \quad (8.6\%)$$
$$\Gamma(h \to \gamma\gamma) \simeq 6 \times 10^{-3} \text{ MeV} \quad (0.15\%)$$

Although gluons & photons are massless, h can couple to them at 1-loop order.

$h \to \gamma\gamma$ is rare, but experimentally clean (the other "golden mode")
It was how h was first discovered.

First do (easier) decay $h \to gg$.

$i\mathcal{M} = i\mathcal{M}_1 + i\mathcal{M}_2 =$



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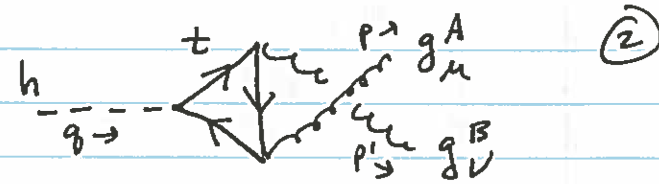


Note: $h \to gg$ is a way to indirectly probe $h\bar{t}t$ coupling.

$$i\mathcal{M}_1 = \int \frac{d^dk}{(2\pi)^d} (-1)\,\mathrm{Tr}\Big[\frac{i(\not{k}+m_t)}{k^2-m_t^2}\; i g_s T^B \gamma^\nu \;\frac{i(\not{k}+\not{p}'+m_t)}{(k+p')^2-m_t^2}$$

$$\cdot\, i g_s T^A \gamma^\mu \;\frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2-m_t^2}\cdot\left(-i\frac{m_t}{v}\right)\Big]\,\varepsilon_\mu\,\varepsilon_\nu$$

$$= -\,g_s^2\,\mathrm{Tr}(T^AT^B)\left(\frac{m_t}{v}\right)\varepsilon_\mu(p)\,\varepsilon_\nu(p')$$

$$\times \int \frac{d^dk}{(2\pi)^d}\,\frac{\mathrm{Tr}\big[(\not{k}+m_t)\gamma^\nu(\not{k}+\not{p}'+m_t)\gamma^\mu(\not{k}+\not{q}+m_t)\big]}{(k^2-m_t^2)((k+p')^2-m_t^2)((k+q)^2-m_t^2)}$$

Use Feynman parameters:

$$i\mathcal{M}_1 = -g_s^2\,\mathrm{Tr}(T^AT^B)\,\frac{m_t}{v}\,\varepsilon_\mu(p)\,\varepsilon_\nu(p')$$

$$\times\int dx\,dy\,dz\; 2\,\delta(1-x-y-z)\cdot\frac{\mathrm{Tr}[\cdots]}{\big[(k^2-m_t^2)x+((k+p')^2-m_t^2)y+((k+q)^2-m_t^2)z\big]^3}$$

Factor in brackets is:

$$(k^2-m_t^2)x+((k+p')^2-m_t^2)y+((k+q)^2-m_t^2)z$$

$$= k^2(x+y+z) + 2k\cdot(yp'+qz)$$

$$+\, \overset{\nearrow 0}{p'^2}\,y + \overset{\nearrow m_h^2}{q^2}\,z - m_t^2(x+y+z)$$

$$= (k+yp'+qz)^2 - (yp'+qz)^2 + m_h^2 z - m_t^2$$

$$= \ell^2 - \underbrace{\left(m_t^2 + z(z-1)m_h^2 + 2yz\,p'\cdot q\right)}_{=M^2}$$

Note: $p'\cdot q = p'\cdot p + p'^2 = \frac{1}{2}q^2 = \frac{1}{2}m_h^2$

$M^2 = m_t^2 - zx\,m_h^2\,,\quad \ell = k+yp'+zq = k+(1-x)p'+zp$

In terms of ℓ: the trace in the numerator is

$$\mathrm{Tr}[\cdots] = 16 m_t \ell^\mu \ell^\nu - 4\ell^2 m_t \eta^{\mu\nu}$$
$$+ 4m_t^3 \eta^{\mu\nu} - 2m_t m_h^2 (1-2xz)\eta^{\mu\nu} + 4(1-4xz) p_1^\nu p_2^\mu m_t$$

We have thrown away terms $\sim p^\mu, p'^\nu$ (vanish when contracted with $\epsilon_\mu(p), \epsilon_\nu(p')$)
and terms proportional to one power of ℓ (vanishes by antisym)

In d-dimensions, $\ell^\mu \ell^\nu \to \frac{1}{d}\eta^{\mu\nu}\ell^2$ under the integral.
Check: $\eta_{\mu\nu}\ell^\mu\ell^\nu = \ell^2 = \frac{1}{d}\eta_{\mu\nu}\eta^{\mu\nu}\ell^2 = \frac{d}{d}\ell^2$. ✓

Putting pieses together:

$$i\mathcal{M}_1 = -g_s^2 \mathrm{Tr}(T^A T^B)\frac{m_t^2}{V}\epsilon_\mu \epsilon_\nu \int \frac{d^d\ell}{(2\pi)^d}$$
$$\times \int dx\,dy\,dz\; 2\,\delta(1-x-y-z)\cdot \frac{1}{(\ell^2-M^2)^3}$$
$$\times \Big(\big(\tfrac{16}{d}-4\big)\ell^2\eta^{\mu\nu} + 2\big(2m_t^2 - m_h^2(1-2xz)\big)\eta^{\mu\nu}$$
$$+ 4(1-4xz)\,p_1^\nu p_2^\mu \Big)$$

Momentum integrals:

$$(1) = \int \frac{d^d\ell}{(2\pi)^d}\frac{\ell^2}{(\ell^2-M^2)^3} = \frac{-i}{(4\pi)^{d/2}}\frac{d}{2}\frac{\Gamma(3-\frac{d}{2}-1)}{\Gamma(3)}\left(\frac{1}{M^2}\right)^{3-\frac{d}{2}-1}$$

$$(2) = \int \frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2-M^2)^3} = \frac{i}{(4\pi)^{d/2}}\frac{\Gamma(3-d/2)}{\Gamma(3)}\left(\frac{1}{M^2}\right)^{3-d/2}$$

Integral (1) is divergent. But we only need to keep the divergent part $\sim \frac{1}{4-d}$ since it is proportional to $\left(\frac{16}{d}-4\right) = 4\left(\frac{4-d}{d}\right) \to 0$ as $d \to 4$

$$(1) = \frac{-i}{16\pi^2}\,\frac{d}{2}\,\frac{\Gamma\left(\frac{4-d}{2}\right)}{2} + \dots \text{ finite terms}$$

$$= -\frac{i}{16\pi^2}\,\frac{2}{4-d} + \dots \quad \text{using } \Gamma(x) = \frac{1}{x} + \dots \text{ finite. for } x \to 0.$$

$$(2) = \frac{i}{16\pi^2}\,\frac{1}{2}\,\frac{1}{M^2}$$

So we have:

$$i\mathcal{M}_1 = -\frac{i}{16\pi^2} g_s^2 \,\mathrm{Tr}(T^A T^B)\, \frac{m_t^2}{v}\, \varepsilon_\mu(p)\, \varepsilon_\nu(p') \int_0^1 dx \int_0^{1-x} dz$$

$$\times \Big\{ -4\left(\frac{4-d}{d}\right)\cdot\frac{2}{4-d}\cdot 2\,(m_t^2 - xz\, m_h^2)\,\eta^{\mu\nu}$$

$$+ (4m_t^2 - 2m_h^2 + 4xz\, m_h^2)\,\eta^{\mu\nu}$$

$$+ 4(1-4xz)\, p_1^\nu p_2^\mu \Big\} \frac{1}{M^2}$$

$$= -\frac{i}{16\pi^2} g_s^2 \,\mathrm{Tr}(T^A T^B)\, \frac{m_t^2}{v}\, \varepsilon_\mu(p)\, \varepsilon_\nu(p')$$

$$\times \int dx\, dz \cdot \left(4 p_1^\nu p_2^\mu - 2 m_h^2 \eta^{\mu\nu}\right)$$

$$\times \frac{1-4xz}{m_t^2 - xz\, m_h^2}$$

$$i\mathcal{M}_1 = \frac{-i}{16\pi^2}\frac{g_s^2}{v}\,\mathrm{Tr}(T^A T^B)\,\varepsilon_\mu(p)\varepsilon_\nu(p')\left(4p_1^\nu p_2^\mu - 2m_h^2\eta^{\mu\nu}\right)$$

$$\times \int_0^1 dx \int_0^{1-x} dz\, \frac{1-4xz}{1-xz(m_h^2/m_t^2)}$$

Let's consider limit $m_t \gg m_h$. Even though it doesn't hold, it works reasonably well numerically even if $m_t \sim m_h$.

$$\int_0^1 dx \int_0^{1-x} dz\,(1-4xz) \doteq \frac{1}{3}$$

Also we have $\mathrm{Tr}(T^A T^B) = \frac{1}{2}\delta^{AB}$

And $i\mathcal{M}_2 = i\mathcal{M}_1 \rightarrow$ factor of 2.

$$i\mathcal{M} = -i\,\frac{\alpha_s}{3\pi v}\,\varepsilon_\mu(p)\varepsilon_\nu(p')\left(p_1^\nu p_2^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right)\delta^{AB}$$

$$\sum|\mathcal{M}|^2 = \left(\frac{\alpha_s}{3\pi v}\right)^2 \underbrace{\delta_{AB}\delta^{AB}}_{8}\left(p_1^\nu p_2^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right)\left(p_{1\nu}p_{2\mu} - \frac{m_h^2}{2}\eta_{\mu\nu}\right)$$

$$= 8\,\frac{\alpha_s^2}{9\pi^2 v^2}\left(m_h^4 - m_h^2\,\underbrace{p_1\cdot p_2}_{m_h^2/2}\right)$$

$$= \frac{4\alpha_s^2 m_h^4}{9\pi^2 v^2}$$

also need to divide by 2 for identical gluons in final state.

$$\Gamma(h\to gg) = \frac{\alpha_s^2 m_h^3}{64\pi^3 v^2}$$

$$\simeq 0.23 \text{ MeV} \quad (5.7\%)^*$$

QCD corrections enhance it by $\sim 60\%$

What about light quark contribution to $h \to gg$?
Note: m_t dependence cancels, but this is only true for heavy states in the loop ($2m \gtrsim m_h$).

The loop integral is:

$$\int_0^1 dx \int_0^{1-x} dz \, \frac{1-4xz}{1-xz(m_h^2/m_q^2)} \approx \begin{cases} 1/3 & \text{for } m_q^2 \gg m_h^2 \\ \frac{1}{2}\frac{m_q^2}{m_h^2}\log^2(m_h^2/m_q^2) & m_q^2 \ll m_h^2 \end{cases}$$

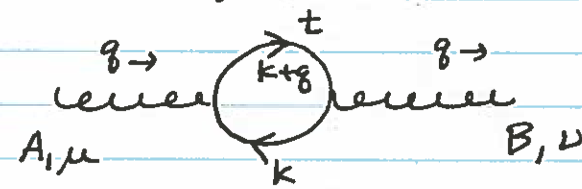
$\to 0$ for m_q.

Light quarks (here, u,d,s,c,b) give a small contribution.

Higgs low-energy theorem

There is a nice trick to calculate the $h \to gg$ amplitude from the QCD β-function. (in the $m_t \to \infty$ limit)

Let's compute the contribution from t to the gluon propagator: (vacuum polarization)

$i\mathcal{M} =$ 

This will contribute to the gluon field strength renormalization, which contributes to the QCD β-function. Let's compute it.

Use dim-reg in $d = 4-2\varepsilon$ dimensions. Amplitude will be divergent.

$$i\mathcal{M} = \int \frac{d^dk}{(2\pi)^d} (-1)\,\mathrm{Tr}\left[\frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2-m_t^2}(-ig_s\gamma^\mu T^A \mu^\varepsilon) \times \frac{i(\not{k}+m_t)}{k^2-m_t^2}(-ig_s\gamma^\nu T^B \mu^\varepsilon)\right]$$

Recall: factors of μ^{ε} are due to the fact that in d dim, gauge coupling has mass dim $(2-\frac{d}{2})=\varepsilon$.

Dim reg: $\quad g_s$ (4-dim) $\longrightarrow$ $g_s \mu^{\varepsilon}$ (d-dim)

$$i\mathcal{M} = -g_s^2 \mu^{2\varepsilon} \mathrm{Tr}[T^A T^B] \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[(\not{k}+\not{q}+m_t)\gamma^\mu(\not{k}+m_t)\gamma^\nu]}{(k^2-m_t^2)((k+q)^2-m_t^2)}$$

Feynman parameters:

$$\int \frac{d^4k}{(2\pi)^d} \frac{\mathrm{Tr}[\cdots]}{(k^2-m_t^2)((k+q)^2-m_t^2)} = \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[\cdots]}{(((k+q)^2-m_t^2)x+(k^2-m_t^2)(1-x))^2}$$

denominator:

$$((k+q)^2-m_t^2)x + (k^2-m_t^2)(1-x)$$
$$= k^2 + 2k\cdot q\, x + xq^2 - m_t^2 = (k+xq)^2 - m_t^2 - x(x-1)q^2$$
$$= \ell^2 - M^2$$

where $\ell = k + xq$, $M^2 = m_t^2 - x(1-x)q^2$

Now shift integral, setting $k = \ell - xq$.

$$i\mathcal{M} = -g_s^2 \mu^{2\varepsilon} \mathrm{Tr}[T^A T^B] \int_0^1 dx \int \frac{d^d\ell}{(2\pi)^d} \frac{1}{(\ell^2-M^2)^2}$$
$$\times \mathrm{Tr}[(\not{\ell}+\not{q}(1-x)+m_t)\gamma^\mu(\not{\ell}-x\not{q}+m_t)\gamma^\nu]$$

$$= -g_s^2 \mu^{2\varepsilon} \mathrm{Tr}[T^A T^B] \int_0^1 dx \int \frac{d^d\ell}{(2\pi)^d} \frac{1}{(\ell^2-M^2)^2}$$
$$\times \Big\{ \left(\frac{8}{d}-4\right) \ell^2 \eta^{\mu\nu} - 8(1-x)x\, q^\mu q^\nu + 4x(1-x) q^2 \eta^{\mu\nu}$$
$$+ 4m_t^2 \eta^{\mu\nu} \Big\}$$

Using $\ell^\mu \ell^\nu = \frac{1}{d} \ell^2 \eta^{\mu\nu}$ and terms linear in ℓ vanish.

Momentum integrals:

$$\int \frac{d^d l}{(2\pi)^d} \mu^{2\epsilon} \frac{1}{(l^2-M^2)^2} = \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \frac{\mu^{2\epsilon}}{(M^2)^{2-d/2}}$$

$$= \frac{i}{16\pi^2}(4\pi)^\epsilon \Big(\frac{1}{\epsilon} - \gamma_E + \underset{\mathcal{O}(\epsilon)}{\hat{\ldots}}\Big) \Big(\frac{\mu^2}{M^2}\Big)^\epsilon$$

$$= \frac{i}{16\pi^2}\Big(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\Big(\frac{\mu^2}{M^2}\Big) + \ldots \mathcal{O}(\epsilon) \ldots\Big)$$

We've used identity $z^\epsilon = e^{\epsilon \log z} \simeq 1 + \epsilon \log z$ and expanded everything to $\mathcal{O}(\epsilon^0)$.

$$\int \frac{d^d l}{(2\pi)^d} \mu^{2\epsilon} \frac{l^2}{(l^2-M^2)^2} = \frac{-i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(2-\frac{d}{2}-1)}{\Gamma(2)} \frac{\mu^{2\epsilon}}{(M^2)^{2-d/2-1}}$$

$$= \frac{+i}{16\pi^2}(4\pi)^\epsilon \frac{d}{2}\Big(\frac{1}{\epsilon} - \gamma_E + 1 + \ldots\Big) M^2 \Big(\frac{\mu^2}{M^2}\Big)^\epsilon$$

this term is multiplied by $(\frac{8}{d} - 4)$.

$$\Big(\frac{8}{d}-4\Big)\int \frac{d^d l}{(2\pi)^d} \mu^{2\epsilon} \frac{l^2}{(l^2-M^2)^2} = \frac{i}{16\pi^2}(4\pi)^\epsilon \overbrace{(4-2d)}^{-4+4\epsilon}\Big(\frac{1}{\epsilon} - \gamma_E + 1\Big) \times M^2 \Big(\frac{\mu^2}{M^2}\Big)^\epsilon$$

$$= \frac{-i}{16\pi^2} 4M^2 \Big(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log \frac{\mu^2}{M^2} + \ldots\Big)$$

$$i\mathcal{M} = \frac{i}{16\pi^2} g_s^2 \, \mathrm{Tr}[T^A T^B] \int_0^1 dx \Big(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log \frac{\mu^2}{M^2} + \ldots\Big)$$
$$\times \Big\{ (\cancel{4 m_t^2} - 4x(1-x)q^2)\eta^{\mu\nu} + 8x(1-x) q^\mu q^\nu - 4x(1-x) q^2 \eta^{\mu\nu} - \cancel{4 m_t^2 \eta^{\mu\nu}} \Big\}$$

$$i\mathcal{M} = i\frac{\alpha_s}{4\pi}\mathrm{Tr}[T^A T^B](g^{\mu}g^{\nu} - g^2\eta^{\mu\nu})\int_0^1 dx\; 8x(1-x)\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_t^2 - g^2 x(1-x)}\right)$$

Divergent part is removed by introducing a counter term:

$$\mathcal{L}_{CT} = -\delta_3 \frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

where $\delta_3 = -\frac{1}{2}\frac{\alpha_s}{4\pi}\int_0^1 dx\; 8x(1-x)\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi\right)$

$$= -\frac{\alpha_s}{6\pi}\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi\right) \qquad (\overline{MS} \text{ prescription})$$

"Bare Lagrangian": $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[G^{(0)}_{\mu\nu}G^{(0)\mu\nu}] + \ldots$

$$= -\frac{1}{2}(1+\delta_3)\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

Shift gluon fields: $G^A_\mu \to (1 - \frac{1}{2}\delta_3)G^A_\mu$

Gluon interaction: $\mathcal{L}_{int} = g_s^{(0)}\bar{q}^{(0)}\not{G}^{(0)A}T^A q^{(0)}$

$$= g_s^{(0)}\bar{q}\not{G}^A T^A q\,(1 - \frac{1}{2}\delta_3)$$

Identify $g_s\mu^{\varepsilon} = g_s^{(0)}(1 - \frac{1}{2}\delta_3)$

Bare coupling

$\Rightarrow \quad g_s^{(0)} = g_s\mu^{\varepsilon}(1 + \frac{1}{2}\delta_3)$

$$\alpha_s^{(0)} = \alpha_s\mu^{2\varepsilon}(1+\delta_3)$$

$$0 = \frac{\partial\alpha_s^{(0)}}{\partial\ln\mu} = \frac{\partial\alpha_s}{\partial\ln\mu}\mu^{2\varepsilon}(1+\delta_3) + \alpha_s\cdot 2\varepsilon(1+\delta_3)$$

$$\Rightarrow \beta_s^{(t)} = -\lim_{\varepsilon\to 0} 2\alpha_s\varepsilon\delta_3 = \frac{\alpha_s^2}{3\pi} \qquad (\text{only } t \text{ contribution})$$

Now consider the hgg interaction: in the $m_t \gg q^2$ limit:

$$i\mathcal{M} = i\frac{\alpha_s}{3\pi}(q^\mu q^\nu - q^2\eta^{\mu\nu})\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_t^2}\right)\mathrm{Tr}[T^A T^B]$$

The Higgs boson field couples to $\bar{t}t$ through interaction of form $m_t(1+\frac{h}{v})$. So if we shift $m_t \to m_t(1+h/v)$ this will include an interaction with the Higgs field.

$$i\mathcal{M} = \dots - i\frac{\alpha_s}{3\pi}(q^\mu q^\nu - q^2\eta^{\mu\nu})\log\left[\left(1+\frac{h}{v}\right)^2\right]\mathrm{Tr}[T^A T^B]$$

$$= \dots - i\frac{\alpha_s}{3\pi}(q^\mu q^\nu - q^2\eta^{\mu\nu})\mathrm{Tr}[T^A T^B]\, 2\frac{h}{v} + \mathcal{O}(h^2/v^2)$$

This can be represented by an effective hgg vertex:

$$\mathcal{L}_{eff} = \frac{\alpha_s}{6\pi v} h\, \mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

Note: the coefficient is automatically related to the β-fn.

i.e. $\mathcal{L}_{eff} = \left(\frac{\varepsilon\delta_3}{v}\right) h\,\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] = \frac{1}{2}\frac{\beta_s^{(t)}}{v} h\,\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$

Similar argument can be used to compute $h \to \gamma\gamma$:

$$\mathcal{L}_{eff} = \frac{1}{4}\frac{\beta_e^{(t+W)}}{v} h F_{\mu\nu}F^{\mu\nu}$$

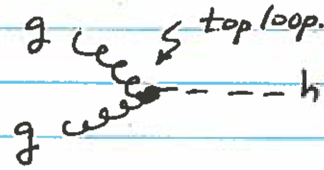
$\beta_e^{(t)} = \frac{8}{9}$, $\beta_e^{(W)} = -\frac{7}{2}$ (for $m_W \gtrsim m_h$, which isn't a great approx.)

Higgs production at hadron colliders

LHC: pp collider at $\sqrt{s} = 7, 8, 13$ TeV.
What is the cross section for producing h? How many h are produced?

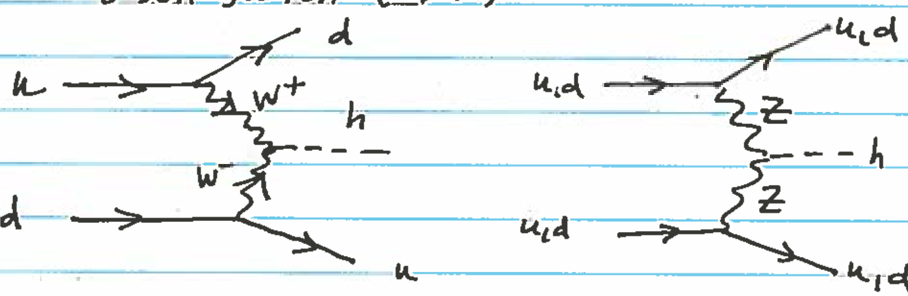
h produced several possible processes: (parton-level)
In order of importance:

1. gluon-gluon fusion (ggF)
   $gg \to h$ inverse decay

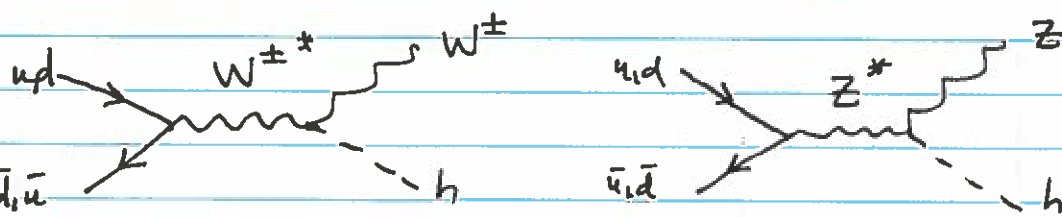


probes h coupling to $t\bar{t}$ and all possible massive colored states.

2. vector-boson fusion (VBF)

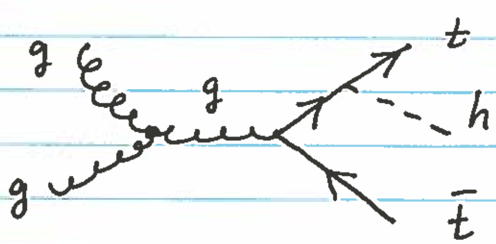


3. vector boson (W,Z) associated production: (WH, ZH)



Higgs boson produced in association with W, Z

4. top associated production ($t\bar{t}H$)



h produced with $t\bar{t}$ pair.
probe $ht\bar{t}$ coupling directly

$$\hat{\sigma}(gg\to h) = \frac{1}{2E_1 2E_2 |\underline{v}_{rel}|} \int \frac{d^3q}{(2\pi)^3 2E_q} (2\pi)^4 \delta^4(q-p_1-p_2)$$

$$\times \left(\frac{1}{8\cdot 2}\right)^2 \sum_{\substack{\text{spins,}\\ \text{colors}}} |\mathcal{M}(gg\to h)|^2$$

Work in CM frame: $|\underline{v}_{rel}| = 2$, $4E_1E_2 = \hat{s}$, $E_q = m_h$.

$$\hat{\sigma}(gg\to h) = \frac{1}{2\hat{s}} \frac{1}{2m_h} (2\pi)\, \delta(m_h - E_1 - E_2)$$

$$\times \frac{1}{(16)^2}\, 2\sum |\mathcal{M}(h\to gg)|^2$$

extra 2 for nonidentical initial gluons. using T-reversal invariance.

Note: $\Gamma(h\to gg) = \frac{1}{16\pi m_h} \sum |\mathcal{M}(h\to gg)|^2$

$$\Rightarrow \quad \sum |\mathcal{M}(h\to gg)|^2 = 16\pi\, m_h\, \Gamma(h\to gg)$$

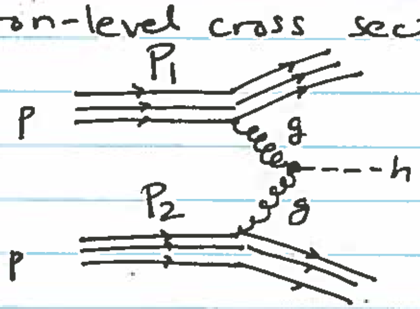
$$\hat{\sigma}(gg\to h) = \frac{2\cdot 8\pi^2\, \Gamma(h\to gg)}{16^2\, \hat{s}}\, \delta(m_h \overbrace{- E_1 - E_2}^{\sqrt{\hat{s}}})$$

$$= \frac{\pi^2\, \Gamma(h\to gg)}{16\, \hat{s}}\, \delta(\sqrt{\hat{s}} - m_h)$$

Now use $\delta(\sqrt{\hat{s}} - m_h) = \delta(\hat{s} - m_h^2)\, 2\sqrt{\hat{s}} = 2m_h\, \delta(\hat{s} - m_h^2)$

$$\hat{\sigma}(gg\to h) = \frac{\pi^2\, \Gamma(h\to gg)}{8\, m_h}\, \delta(\hat{s} - m_h^2)$$

Hadron-level cross section:



$\Big\}$ sum over all possible hadronic states $= X$

$$\sigma(pp\to hX) = \int_0^1 dx_1\, f_g(x_1) \int_0^1 dx_2\, f_g(x_2)\, \hat{\sigma}(gg\to h)$$

$P_{1,2}$ = initial ~~pros~~ proton momenta

$p_{1,2}$ = initial parton (g) momenta.

$x_{1,2}$ = momentum fractions.

$p_1 = x_1 P_1$, $p_2 = x_2 P_2$. Can relate $\hat{s}$ to s.

$$\hat{s} = (p_1 + p_2)^2 = 2 p_1 \cdot p_2 = x_1 x_2 \, 2 P_1 \cdot P_2 = s x_1 x_2 .$$

$$\sigma(pp \to hX) = \int_0^1 dx_1 \, f_g(x_1) \int_0^1 dx_2 \, f_g(x_2)$$

$$\times \frac{\pi^2 \Gamma(h \to gg)}{8 m_h} \delta(x_1 x_2 s - m_h^2)$$

We can use δ-fn to evaluate $\int dx_2$ integral.
Define $\tau = m_h^2/s$.

$\Rightarrow$

$$\delta(x_1 x_2 s - m_h^2) = \frac{1}{s} \delta(x_1 x_2 - \tau)$$

$$= \frac{1}{s x_1} \delta\left(x_2 - \frac{\tau}{x_1}\right)$$

So we have $x_2 = \dfrac{\tau}{x_1}$
Still must have $0 < x_2 < 1$. So

$$0 < \frac{\tau}{x_1} < 1 \quad \Rightarrow \quad 0 < \tau < x_1$$

Only $x_1 > \tau$ part of $\int_0^1 dx_1$ contributes.

$$\sigma(pp \to hX) = \int_\tau^1 \frac{dx_1}{x_1} f_g(x_1) f_g(\tau/x_1) \frac{\pi^2 \Gamma(h \to gg)}{8 m_h s}$$

$f_g(x)$ is obtained by fitting to experimental data.
Reasonably accurate analytic expression (Peskin & Schroeder)

$$f_g(x) = \frac{8}{x}(1-x)^7.$$

At LHC (at LO in α_s):

	P&S ansatz for $f_g(x)$	Actual $f_g(x)$
$\sqrt{s} = 7$ TeV	5.3 pb	10.5 pb
$\sqrt{s} = 8$ TeV	6.8 pb	11.4 pb
$\sqrt{s} = 13$ TeV	15.7	14.8 pb

State-of-the-art SM calculations: (NNLO) i.e. $\mathcal{O}(\alpha_s^2)$ corrections.

$\sqrt{s} = 7$ TeV	15.1 pb	$\mathcal{O}(\alpha_s)$ corrections are very important — many more diagrams to consider.
$\sqrt{s} = 8$ TeV	19.3 pb	
$\sqrt{s} = 13$ TeV	43.9 pb	

How many h are produced? Introduce concept of luminosity. Consider simplified version of collider expt., colliding A & B.

$$\frac{\text{Rate}}{\text{volume}} = n_A \, n_B \, |v_{rel}| \, \sigma$$

number density of A & B

$$\text{Rate} = \underbrace{(n_A \, n_B \, v_{rel} \, \text{Volume})}_{\mathcal{L} = \text{luminosity}} \cdot \sigma = \frac{\text{number of collisions}}{\text{time}}$$

volume of beam overlap region

Depends purely on expt. setup. Has units of $\frac{1}{\text{area} \cdot \text{time}}$

Integrated Luminosity: $\int dt \, \mathcal{L}$ = Number of collisions$/\sigma$.

Higgs discovery based on $\int dt\,\mathcal{L} \sim 5\ fb^{-1}$ at 7 TeV & same at 8 TeV. $1\ fb = 10^{-3}\ pb$.

Number of 7 TeV h events $= 5\ fb^{-1} \times 15.1\ pb \simeq 75{,}000$
— " — 8 TeV — " — $= 5\ fb^{-1} \times 19.3\ pb \simeq 100{,}000$.

However, the dominant decays $(b\bar{b}, WW^*, gg)$ are more challenging experimentally. Cleanest signals: ZZ^*, $\gamma\gamma$.

$BR(h \to \gamma\gamma) \approx 0.15\%$

Number of $h \to \gamma\gamma$ events ≈ 250 events. (Reduced further by experimenal detection efficiency & cuts.)

<u>Hadron spectroscopy & group theory</u>

Quarks are confined in color-neutral mesons $\sim (q_1 \bar{q}_2)$ or baryons $\sim (q_1 q_2 q_3)$. The typical mass scale for QCD is set by $\sim \Lambda_{QCD} \sim 200$ MeV.

Light quark hadrons: only containing (u, d, s) [or just (u, d)]
Heavy quark hadrons: one or more heavy quarks (c, b)
t-quark is irrelevant for hadron states since t decays very rapidly, before hadronization occurs (although virtual t can be very important in certain contexts).

Mesons: (masses in MeV, lifetimes in sec.)

	State	quark content	mass	lifetime	J^P	leading decay(s)
pions	π^0	$u\bar{u} - d\bar{d}$	135	8.5×10^{-17}	0^-	$\pi^0 \to \gamma\gamma$ (EM)
	$\pi^\pm$	$u\bar{d}, d\bar{u}$	140	2.6×10^{-8}	0^-	$\pi^+ \to \nu_\mu \mu^+$ (Weak)
kaons	$K^\pm$	$u\bar{s}, s\bar{u}$	494	1.2×10^{-8}	0^-	$\pi\pi, \mu^\pm \nu_\mu$ (W)
	$K^0, \bar{K}^0$	$d\bar{s}, s\bar{d}$	498	$\sim 10^{-8}$ or 10^{-10}	0^-	$2\pi, 3\pi$, (W) $\pi e \nu_e, \pi \mu \nu_\mu$
	f_0	$\pi\pi$ state?	~ 550	$\sim 10^{-27}$ s	0^+	$\pi\pi$
	η	$\sim u\bar{u} + d\bar{d} - 2s\bar{s}$	548	6×10^{-22} s	0^-	$\eta \to \gamma\gamma$ (EM), 3π
rho	$\rho^0, \rho^\pm$	same as π	~ 775	4×10^{-27} s	1^-	$\rho \to \pi\pi$
	ω	$u\bar{u} + d\bar{d}$	782	10^{-24} s	1^-	$\omega \to \pi^+ \pi^- \pi^0$
	$K^{*\pm}, K^{*0}, \bar{K}^{*0}$	same as K	~ 892	$\sim 10^{-26}$ s	1^-	$K\pi$ (strong)
	η'	$\sim u\bar{u} + d\bar{d} + s\bar{s}$	958	3×10^{-24} s	0^-	$\rho^0 \gamma, 3\pi$
	ϕ	$s\bar{s}$	1020	1.5×10^{-25} s	1^-	$\phi \to K\bar{K}, \rho\pi$

Baryons:

p	uud	938	$\gtrsim 10^{33}$ yrs	$\frac{1}{2}^+$	—
n	udd	939	880 s	$\frac{1}{2}^+$	$n \to p e^- \bar{\nu}_e$
Λ	uds	1116	2.6×10^{-10}	$\frac{1}{2}^+$	$p\pi^-, n\pi^0$ (w)
Σ^+	uus	1189	0.8×10^{-10}	$\frac{1}{2}^+$	$p\pi^0, n\pi^+$ (weak)
Σ^0	uds	1193	7×10^{-20}	$\frac{1}{2}^+$	$\Lambda\gamma$ (EM)
Σ^-	dds	1197	1.5×10^{-10}	$\frac{1}{2}^+$	$n\pi^-$ (Weak)
Δ^{++}	uuu				
Δ^+	uud	~ 1232	$\sim 10^{-27}$ s	$\frac{3}{2}^+$	$n\pi, p\pi$ (strong)
Δ^0	udd				
Δ^-	ddd				
"cascade" Ξ^0	uss	1314	3×10^{-10} s	$\frac{1}{2}^+$	$\Lambda\pi^0$ (weak)
Ξ^-	dss	1321	1.6×10^{-10}	$\frac{1}{2}^+$	$\Lambda\pi^-$ (weak)
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	same as Σ	~ 1385	$\sim 10^{-26}$ s	$\frac{3}{2}^+$	$\Lambda\pi, \Sigma\pi$ (strong)
Ξ^{*0}, Ξ^{*-}	same as Ξ	~ 1530	$\sim 7 \times 10^{-26}$ s	$\frac{3}{2}^+$	$\Xi\pi$ (Strong)
Ω^-	sss	1672	8×10^{-11} s	$\frac{3}{2}^+$	$\Lambda K^-, \Xi^0\pi^-$ (weak)

+ many other states...

Each particle is an energy level of the Hamiltonian.

Hints toward an organizing principle:

(1) Some hadrons (e.g. kaons, Λ, $\Sigma^\pm$, Ξ, Ω^-), although they could be produced in collisions very easily, had decays much longer than other states.

(Nature of s quarks & weak interaction unknown.)
$\Rightarrow$ New additive quantum number "strangeness" S that is conserved by strong interaction, but must be violated in the decay (weak interaction).
The convention was that each s quark gives strangeness $S=-1$,
e.g. $S(K^+) = 1$, $S(\Lambda) = -1$, $S(\Omega^-) = -3$.

(2) Hadron states come in nearly degenerate multiplets:

e.g. $\begin{pmatrix} p \\ n \end{pmatrix}$, $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$, Λ, $\begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix}$, etc.

doublet ($I=\frac{1}{2}$) triplet ($I=1$) singlet ($I=0$) quadruplet ($I=\frac{3}{2}$)

Points to a symmetry called "isospin", rotation between components.
Different components have different electric charges & (possibly) decays $\Rightarrow$ EM and weak interaction violate this symmetry.
However, the strong interaction Hamiltonian should preserve this symmetry, since it is generating the masses.

That is, multiplets are representations of $SU(2)$ symmetry.

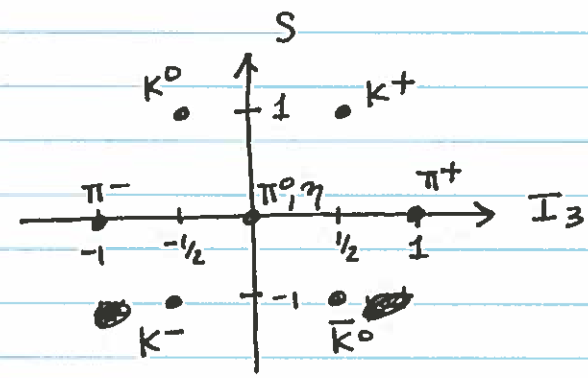
e.g. $I_3|p\rangle = +\frac{1}{2}|p\rangle$, $I_3|n\rangle = -\frac{1}{2}|n\rangle$
$|I|^2|p\rangle = (I(I+1))|p\rangle = \frac{3}{4}|p\rangle$, same for $|n\rangle$

Works just like spin.

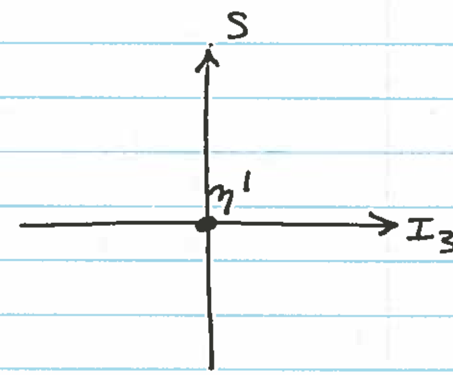
Eightfold Way: Gell-mann & Ne'eman showed how to embed these two approximate symmetries within a larger symmetry group ($SU(3)$).

First, consider plotting states of a given J^P in the $I_3 - S$ plane.

Pseudoscalar mesons:

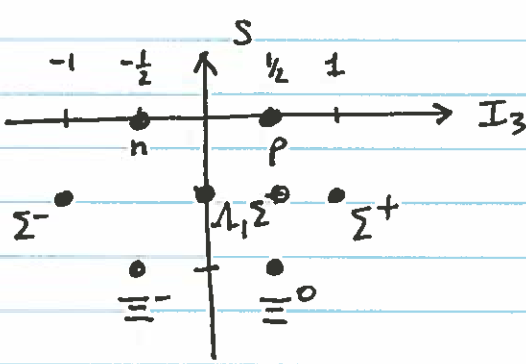


pseudoscalar octet



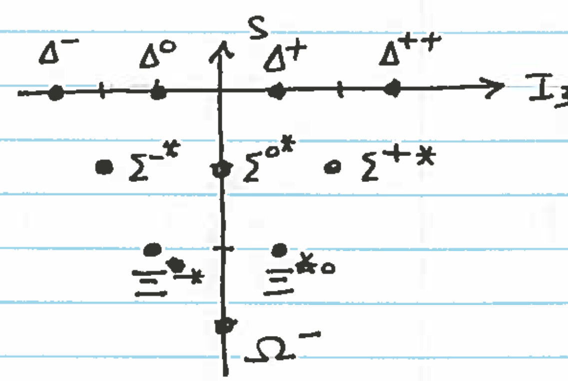
singlet

spin-½ baryons



baryon octet

spin-3/2 baryons



baryon decuplet.

These patterns follow representations of $SU(3)$ symmetry.

Symmetry of the strong interaction:

Starting from the SM Lagrangian it is easy to see where the SU(3) symmetry comes from. Neglecting electroweak interactions, the quark Lagrangian is:

$$\mathcal{L} = \sum_q \bar{q}\left(i(\not{\partial} - i g_s T^A \not{G}^A) - m_q\right) q - \frac{1}{2} \mathrm{Tr}[G_{\mu\nu} G^{\mu\nu}]$$

sum over all quark fields u, d, s, c, b, t.
G^A_μ is gluon field.
m_q = quark mass for quark q.
$T^A = SU(3)_c$ generator

~~Since hadrons~~

For light quark hadrons, the hadron mass comes predominantly from strong interactions. Those quarks with masses $m_q \ll \Lambda_{QCD}$ can be taken to be approximately degenerate (or massless).

Considering only u, d, s quarks; and setting $m_{u,d,s} \to m_0$:

$$\mathcal{L} = \bar{q}(i(\not{\partial} - i g_s T^A) - m_0) q - \frac{1}{2} \mathrm{Tr}[G_{\mu\nu} G^{\mu\nu}]$$

where $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$, triplet of quark flavors.

$\mathcal{L}$ has SU(3) flavor symmetry: $q \to Uq$, where U is SU(3) matrix.

$$\bar{q}(i(\not{\partial} - i g_s T^A \not{G}^A) - m_0) q \to \bar{q} U^\dagger (i(\not{\partial} - i g T^A \not{G}^A) - m_0) U q$$
$$= \bar{q}(i(\not{\partial} - i g T^A \not{G}^A) - m_0) q$$

Can also write e.g. EM interactions as

$$\mathcal{L}_{int} = -e \bar{q} Q \gamma^\mu q A_\mu \qquad \text{where } Q = \begin{pmatrix} 2/3 & & 0 \\ & -1/3 & \\ 0 & & -1/3 \end{pmatrix}$$

Not invariant under SU(3) flavor $\Rightarrow$ EM interactions violate symmetry.