Flavor physics

Hadrons can also include heavy quarks, c, b. Lightest pseudoscalar (0^-) states:

Charmed mesons (D):	quark content	mass $\sim m_c$
$D^0, \bar{D}^0$	$c\bar{u}, u\bar{c}$	1865
D^+, D^-	$c\bar{d}, d\bar{c}$	1870
D_s^+, D_s^-	$c\bar{s}, s\bar{c}$	1968

Bottom (beauty) mesons (B):

B^+, B^-	$u\bar{b}, b\bar{u}$	5279
$B^0, \bar{B}^0$	$d\bar{b}, b\bar{d}$	5280
$B_s^0, \bar{B}_s^0$	$s\bar{b}, b\bar{s}$	5367
B_c^+, B_c^-	$c\bar{b}, b\bar{c}$	6275

Also, vector states (1^-): same names, just add "*"

Quarkonium: $c\bar{c}$ or $b\bar{b}$

Charmonium:

η_c	0^-	$c\bar{c}$	2984
J/ψ	1^-	$c\bar{c}$	3097

Bottomonium:

η_b	0^-	$b\bar{b}$	9398
Υ	1^-	$b\bar{b}$	9460

+ many more excited states.

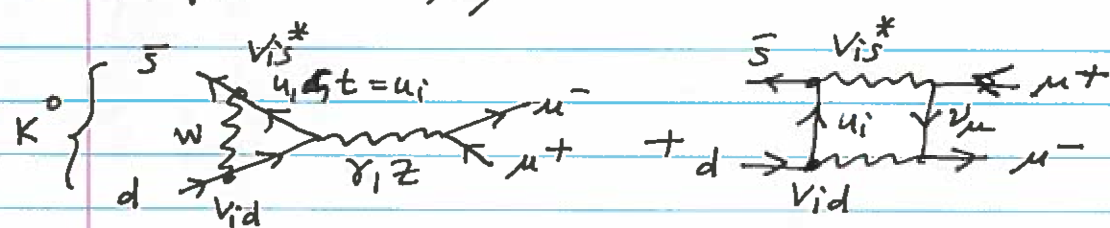
Similar to strangeness S, define other approx. conserved quantum numbers:

charm C: c has $C=+1$, $\bar{c}$ has $C=-1$
bottomness B: b has $B=-1$, $\bar{b}$ has $+1$.

Processes that violate S, C, B must occur via weak charge current interaction. Allows us to measure & test consistency of CKM matrix as sole source of quark flavor violation (and CP violation).

Important class of observables: Flavor-changing neutral currents (FCNCs). These are $s \leftrightarrow d$, $b \leftrightarrow d$, $b \leftrightarrow s$ transition, forbidden at tree-level since only CC violates flavor, while neutral current (Z, γ) preserves flavor. But FCNCs arise at one-loop order.

Example: $K^0 \to \mu^+ \mu^-$



Matrix element has the form: (sum over virtual quark flavors)

$$i\mathcal{M} \propto \sum_i V_{id} V_{is}^* \, f(m_i/m_W)$$

If m_{u_i} all the same (or equivalently if $m_{u_i}/m_W \to 0$), then

$$i\mathcal{M} \propto \underbrace{\sum_i V_{id} V_{is}^*}_{} \times (\text{independent of } i)$$

$$= (V^\dagger V)_{ds} = \mathbb{1}_{ds} = \delta_{12} = 0.$$

This is called the GIM mechanism (Glashow-Iliopoulis-Maiani).
In the limit of massless quarks (compared to m_W), FCNCs vanish.

t-quark is very massive, but contribution is suppressed since

$$|V_{td} V_{ts}| \approx 3 \times 10^{-4}$$

u,c-quark contributions suppressed by $m_{u,c}/m_W \ll 1$.

So FCNCs are both loop-suppressed & GIM-suppressed (or CKM-suppressed). Very sensitive probe of beyond the SM physics,

Brief history of kaons

Many discoveries have been made by studying kaons.

(1) Longevity of kaons ⇒ strangeness & eightfold way (1961) ⇒ quarks.

(2) Theta-tau puzzle & parity violation
~1950, two particles were discovered: "θ" & "τ", with same mass.
Their decays are:

$$\theta^+ \to \pi^+\pi^0$$
$$\tau^+ \to \pi^+\pi^0\pi^0,\ \pi^+\pi^+\pi^-$$

Since $P(\pi) = -1$,
$P(\theta) = (-1)^2 = +1$
$P(\tau) = (-1)^3 = -1$
(spatial wavefunction is $L=0$)

Radical idea: θ^+, τ^+ are the same particle (K^+) and weak interaction violates parity,

(3) Neutral Kaon mixing & the charm quark.

We have $K^0, \bar{K}^0$ which are eigenstates of strangeness, $S = -1, +1$.

Although P was violated by weak interaction, CP "seemed" to be good symmetry. Write $K^0, \bar{K}^0$ in terms of CP eigenstates:

First note: $P|K^0\rangle = -|K^0\rangle$, $C|K^0\rangle = |\bar{K}^0\rangle$, and similar for $|\bar{K}^0\rangle$.
$CP|K^0\rangle = -|\bar{K}^0\rangle$, $CP|\bar{K}^0\rangle = -|K^0\rangle$.

Then $|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$, $|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$
$CP|K_1\rangle = -|K_1\rangle$, $CP|K_2\rangle = +|K_2\rangle$

In the $|K^0\rangle, |\bar{K}^0\rangle$ basis Hamiltonian written as

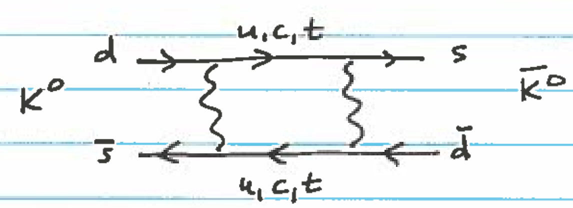
$$H = \begin{pmatrix} m_K & M_{12} \\ M_{12}^* & m_K \end{pmatrix} + \text{decay terms.}$$

By CPT, diagonal entries must be the same. If CP is a symmetry, then M_{12} = real.

$$M_{12} = \langle K^0|\hat{H}|\bar{K}^0\rangle \underset{(CP)(CP)}{\overset{(CP)^2}{=}} \langle \bar{K}^0|\hat{H}|K^0\rangle = M_{12}^*$$

Eigenstates are $|K_{1,2}\rangle$, $m_{1/2} = m_K \pm M_{12}$, $\Delta m = m_2 - m_1 = 2M_{12}$

M_{12} mixes ~~eigen~~ states of strangeness $S = \pm 1$. $\Rightarrow$ weak interaction



$K^0 \to \bar{K}^0$

$$M_{12} \propto \sum_{i,j} V_{id} V_{is}^* \, V_{jd} V_{js}^* \, f\left(m_i/m_W, m_j/m_W\right)$$

Charm provides largest contribution $\rightarrow$ allowed prediction of m_c.

Problems with the SM

(1) Neutrino masses. Neutrinos are massless in SM (only ν_L fields), but are observed to have a small ($<$ eV) mass that is non zero. Why are neutrinos so much lighter than other fermions?

We can include ν masses straightforwardly by adding a Yukawa term and adding RH ν fields, ν_R^i.

$$\mathcal{L}_{Yukawa} = -Y_{ij}^{(\nu)} \bar{\nu}_R^i L_L^{j\,T} \varepsilon H + h.c.$$

$$= -\bar{\nu}_R^i \frac{Y_{ij}^{(\nu)} v}{\sqrt{2}} \nu_L^j \left(1+\frac{h}{v}\right) + h.c.$$

Diagonalize using biunitary transform:

$$\nu_{L,R}^i = (U_{\nu_{L,R}})_{ij} \, \nu'^j_{L,R}$$

$$\mathcal{L}_{Yukawa} = -m_i^{(\nu)} \bar{\nu}'^i \nu'^i \left(1+\frac{h}{v}\right)$$

ν'_i fields are mass eigenstate fields.
ν_i fields are gauge eigenstate fields.

Now CC interaction is no longer diagonal for leptons:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} U_{ji}^{*} \bar{\nu}_L'^i \gamma^\mu e_L'^j + h.c.$$

where $U = U_{e_L}^\dagger U_{\nu_L} = $ PMNS matrix

Pontecorvo – Maki – Nakagawa – Sakata

But actually this is not the most general mass term. Since $\nu_R^i \sim (1,1,0)$, a singlet under $SU(3)_C \times SU(2)_L \times U(1)_Y$ we can include a Majorana mass term for ν_R.

$$\mathcal{L}_{majorana} = -\tfrac{1}{2} M_{ij} \left(\overline{\nu}_R^{\,i} \nu_R^{c\,j} + \overline{\nu}_R^{\,i\,c} \nu_R^{\,j} \right)$$

$C =$ charge conjugation. Notation: $\nu_R^c = (\nu_R)^c = (P_R \nu)^c$ not $P_R(\nu^c)$.

Let's just consider one flavor for simplicity.

$$\mathcal{L}_{\nu\,mass} = -m\left(\overline{\nu}_R \nu_L + \overline{\nu}_L \nu_R\right) - \tfrac{1}{2} M \left(\overline{\nu}_R \nu_R^c + \overline{\nu}_R^c \nu_R\right)$$

note: $\overline{\nu}_R \nu_L = \overline{\nu}_L^c \nu_R^c$, $\overline{\nu}_L \nu_R = \overline{\nu}_R^c \nu_L^c$; $m = \frac{yv}{\sqrt{2}}$ Dirac mass

$$\Rightarrow \mathcal{L}_{\nu\,mass} = -\tfrac{1}{2}\left(\overline{\nu}_L^c, \overline{\nu}_R\right) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$$

Diagonalize: we get two Majorana neutrinos; ν & N.

$$\mathcal{L}_{\nu\,mass} = -\tfrac{1}{2} m_\nu \overline{\nu}^c \nu - \tfrac{1}{2} M_N \overline{N}^c N$$

where $\nu \approx \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$, $N \approx \begin{pmatrix} \nu_R \\ \nu_R^c \end{pmatrix}$ In $M \gg m$ limit:

$m_N \approx M$, $m_\nu = \frac{m^2}{M}$ Eigenvalues of mass matrix.

Neutrinos are light because singlet neutrinos can acquire a very heavy mass M.

say $yv = 1$ GeV, $M = 10^9$ GeV $\Rightarrow m_\nu = 1$ eV.
$(y \sim 10^{-2})$

For Dirac neutrinos, $y \lesssim 10^{-12}$ — weirdly small.

Experimental consequences:

(1) ν oscillations (Majorana or Dirac). e.g. ν beam experiment

$$p \Rightarrow \boxed{\text{target}} \equiv \longrightarrow \pi^+ \xrightarrow[\sim 100\%]{} \mu^+ \nu_\mu$$

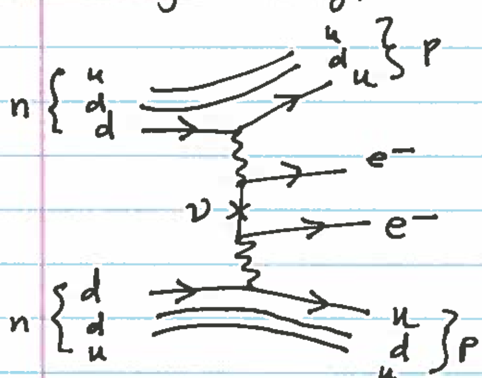
$$|\nu_\mu\rangle = U_{21}|\nu_1\rangle + U_{22}|\nu_2\rangle + U_{23}|\nu_3\rangle$$

flavor eigenstate = linear combination of mass eigenstates

Each mass eigenstate evolves with its own time evolution $e^{-iE_i t}$

$\Rightarrow$ linear combination changes as ν propagates
can have probability to be observed as ν_e or ν_τ.

(2) neutrinoless double β-decay. $(0\nu\beta\beta)$
Majorana only.



nucleus with N neutrons, Z protons
$\rightarrow$ N−2 neutrons, Z+2 protons
$+\ 2e^-$

relies on fact that ν violates lepton number.

(2) What is the origin of flavor? Why 3 generations?
Why huge hierarchy between $y_e \approx 3\times10^{-6}$ & $y_t \approx 1$?

Why are the CKM & PMNS matrices so different?

$$|V_{CKM}| = \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.999 \end{pmatrix}$$ closely aligned with $\mathbb{1}$.

$$|U_{PMNS}| \simeq \begin{pmatrix} 0.82 & 0.54 & 0.15 \\ 0.35 & 0.70 & 0.62 \\ 0.44 & 0.45 & 0.77 \end{pmatrix}$$ "anarchy"

(3) Why 3 different gauge groups? Why does $SU(2)_L$ violate parity? Can quarks & leptons be unified in a single representation? Why do fermions have seemingly adhoc hypercharges?

- Pati-Salam model: $SU(4)\times SU(2)_L \times SU(2)_R$
  Restores parity. Quarks & leptons unified together.

$$\Psi_L \sim (4,2,1) = \begin{pmatrix} u_L^r & d_L^r \\ u_L^g & u_L^g \\ u_L^b & u_L^b \\ \nu_L & e_L \end{pmatrix}$$ lepton number is the "fourth color"

$$\Psi_R \sim (4,1,2) = \text{same with } R \to L.$$

$U(1)_Y$ emerges as combination of $SU(4)$ & $SU(2)_R$ generators.

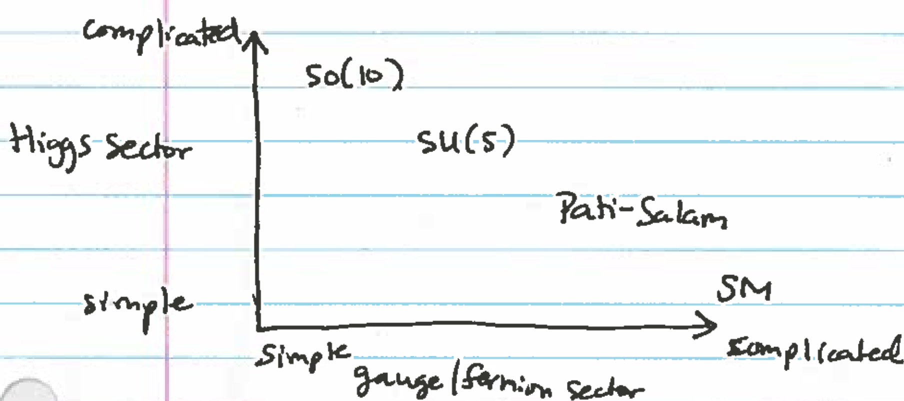
- Georgi-Glashow $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$
  Smallest simple gauge group unifying all three SM gauge groups
  (Grand unified theory, GUT)

SM fermions fit in 2 reps of $SU(5)$: $\bar{5} = \begin{pmatrix} (d_R^c) \\ --- \\ L_L \end{pmatrix}$ } 3 colors } $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

and $10 = \begin{pmatrix} 0 & (u_R^c)^b & -(u_R^c)^g & u_L^r & d_L^r \\ -(u_R^c)^b & 0 & (u_R^c)^r & u_L^g & d_L^g \\ & & 0 & u_L^b & d_L^b \\ & \ddots & & 0 & e_R^c \\ & & & & 0 \end{pmatrix}$

- Georgi's $SO(10) \supset SU(5) \rightarrow \supset SU(3)_C \times SU(2)_L \times U(1)_Y$
  $\supset SU(4) \times SU(2)_L \times SU(2)_R \nearrow$

All SM fermions ($+ \nu_R$) of a single generation fit into a single 16-dim rep. (2 quarks (u,d) $\times$ 2 chirality (L/R) $\times$ 3 colors
+ 2 leptons (e, ν) $\times$ 2 chirality (L/R) $= 16$)



conservation of complexity?

Experimental signature: proton decay: new gauge bosons transition between quark & leptons.

(4) Hierarchy problem:

SM has one fundamental dimensionful parameter $\mu \sim 100$ GeV.
The only other fundamental dimensionful parameter we know is from gravity $\rightarrow m_{pl} \sim 10^{19}$ GeV

Why is $\mu / m_{pl} \ll 1$?