Deep Inelastic Scattering

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What other evidence supports a chiral theory for weak interactions?

How can we test the proposed charges of the quarks?

Does the miracle of factorization really work?

We will find the answer in this tutorial.

1 Charged Current Deep Inelastic Scattering

A collection of useful formula to compute traces of gamma matrices and the differential cross-section are at the end of the tutorial.

a) To understand charged current deep inelastic scattering, we first need to find the subprocess differential cross sections. By neglecting all fermion masses, show that

$$\frac{d\hat{\sigma}}{dy}(e^- q \to \nu q') = \frac{1}{2}\frac{d\hat{\sigma}}{dy}(\nu_e q' \to e^- q) = \frac{G_F^2 \hat{s}}{2\pi}|V_{qq'}|^2 \left(\frac{M_W^2}{M_W^2 + Q^2}\right)^2 \tag{1}$$

Quantities associated with the subprocess conventionally wear hats, e.g. the Mandelstam invariants for the subprocess are $\hat{s}, \hat{t}, \hat{u}$. The Mandelstam variable $\hat{t}$ has an alias: $Q^2 = -\hat{t}$. The dimensionless variable y is given by $y = -\hat{t}/\hat{s}$. The Fermi coupling is related to the $SU(2)_L$ coupling by $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$. Where does the factor of $\frac{1}{2}$ come from? If we assume the scales at which the scatterings happen are much lower than the mass the of the W boson, again we see that because of factorization, the fundamental scattering process is elastic.

b) Show that the parton differential cross sections with anti-neutrinos are given by

$$\frac{1}{2}\frac{d\hat{\sigma}}{dy}(\bar{\nu}_\mu q \to \mu^+ q') = \frac{d\hat{\sigma}}{dy}(\mu^+ q' \to \bar{\nu}_\mu q) = \frac{G_F^2 \hat{s}}{2\pi}|V_{qq'}|^2 \left(\frac{M_W^2}{M_W^2 + Q^2}\right)^2 (1-y)^2 \tag{2}$$

The difference between the neutrino and anti-neutrino cross sections shows that the weak interaction is chiral. For instance in class we derive the double differential cross section for the process of $e^- p \to e^- X$, the result would not change for $e^+ p \to e^+ X$. For more discussion about how helicity of the particles affect which Mandelstam variables show up in the cross section, we refer interested reader to section 5.2 of Peskin and Schroeder. Note that the result from the last part does not depend on y. The dependence on y for both neutrino and anti-neutrino scattering has been experimentally tested.

c) Most of the momentum in protons and neutrons is carried by the valence quarks (u and d) and by gluons. The remainder of the momentum is carried by the sea quarks (s, c, b, $\bar{u}$, $\bar{d}$,$\bar{s}$, $\bar{c}$, and $\bar{b}$). By neglecting the sea quarks (i.e. in the valence dominance approximation) and using the fact that the CKM matrix is close to the identity matrix show that

$$\begin{aligned} \frac{d\sigma(\nu_\mu p \to \mu^- X)}{dx} &= \frac{G_F^2 s}{\pi} x f_d(x) \\ \frac{d\sigma(\nu_\mu n \to \mu^- X)}{dx} &= \frac{G_F^2 s}{\pi} x f_u(x) \end{aligned} \tag{3}$$

where $f_u(x)$ and $f_d(x)$ are the up and down quark parton distribution functions in the proton, and $x = \frac{Q^2}{2P\cdot q}$, P is the nucleon momentum, and q is the W boson momentum. The PDF's of hadrons reflect the symmetries that relate different hadrons. Since the neutron and proton are related under the exchange of the u and d quarks, $f_u(x) \approx f_d^n(x)$ and $f_d(x) \approx f_u^n(x)$ where $f_u^n(x)$ and $f_d^n(x)$ are the parton distribution functions for the up and down quarks respectively in the neutron.

d) Use your results and the result from lecture

$$\frac{d^2\sigma(e^- p \to e^- X)}{dxdy} = \sum_q x f_q(x) \frac{2\pi\alpha^2 s Q_q^2}{Q^4} \left(y^2 + 2(1-y)\right) \tag{4}$$

and the following definition of the form factors

$$F_2^{\nu D}(x) = \left(\frac{2\pi}{G_F^2 s}\right) \left[\frac{d\sigma(\nu_\mu p \to \mu X)}{dx} + \frac{d\sigma(\nu_\mu n \to \mu X)}{dx}\right] \tag{5}$$

and

$$F_2^{eD}(x) = \left(\frac{3Q^4}{8\pi\alpha^2 s}\right) \left[\frac{d\sigma(e^- p \to e^- X)}{dx} + \frac{d\sigma(e^- n \to e^- X)}{dx}\right] \tag{6}$$

to show that in the valence dominance approximation

$$F_2^{\nu D}(x) = K F_2^{eD}(x) \tag{7}$$

where K is a constant, find K assuming the charge for up quark is Q_u and the charge for the down quark is Q_d.

e) Experimentally $K \approx \frac{18}{5}$. What does this tell you about the quark charges?

2 Useful Formulae

$$\gamma^\mu \gamma_\mu = 4 \tag{8}$$

$$\gamma^\mu \gamma^\nu \gamma_\mu = -2\gamma^\nu \tag{9}$$

$$\gamma^\mu \gamma^\nu \gamma^\lambda \gamma_\mu = 4g^{\nu\lambda} \tag{10}$$

$$\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma \gamma_\mu = -2\gamma^\sigma \gamma^\lambda \gamma^\nu \tag{11}$$

$$\mathrm{tr}\, \gamma^\mu \gamma^\nu \gamma^\lambda = 0 \tag{12}$$

$$\text{the same for arbitrary odd number of } \gamma^{\mu\prime}\text{s} \tag{13}$$

$$\mathrm{tr}\, \gamma^\mu \gamma^\nu = 4g^{\mu\nu} \tag{14}$$

$$\mathrm{tr}\, \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\rho = 4\left(g^{\mu\nu} g^{\lambda\rho} + g^{\mu\rho} g^{\nu\lambda} - g^{\mu\lambda} g^{\nu\rho}\right) \tag{15}$$

$$\mathrm{tr} \gamma^\mu \gamma^\nu \gamma^5 = 0 \tag{16}$$

$$\mathrm{tr} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5 = -4i\epsilon^{\mu\nu\rho\sigma} \tag{17}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{CM}\, 2\to 2\, \mathrm{massless}} = \frac{|\mathcal{M}|^2}{64\pi^2 E_{cm}^2} \tag{18}$$