Strange and Charming

21 January 2016

1 Weak Decays of D Mesons

We can understand some of the properties of heavy quark mesons using the approximate $SU(3)$ flavor symmetry of the light quarks. In particular we will find relationships between some of the semileptonic partial decay widths of the D mesons. The D mesons are pseudoscalar mesons made of a charm quark and a light antiquark (or a light quark and anticharm quark).

a) The weak decay of the charm quark ($c \to s e^+ \nu$) can be described by the following Fermi Lagrangian:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{cs}^* \bar{s} \gamma^\mu (1 - \gamma^5) c \bar{e} \gamma_\mu (1 - \gamma^5) \nu_e \tag{1}$$

This interaction breaks the $SU(3)$ flavor symmetry of the light quarks. By figuring out how this interaction breaks the flavor symmetry, we can determine the interactions of the D mesons with the light quark pseudoscalar mesons. We can rewrite (**??**) as:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{cs}^* \bar{q} X \gamma^\mu (1 - \gamma^5) c \bar{e} \gamma_\mu (1 - \gamma^5) \nu_e \tag{2}$$

where q is the light quark triplet of $SU(3)_{\text{flavor}}$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{3}$$

and X is the *constant* vector

$$X = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

We will ignore the fact that X is a constant for now (we will remember again in part (d)), and pretend that it transforms under $SU(3)_{\text{flavor}}$. In other words, we will treat X as a *spurion*. This will allow us to determine relations among the interactions of the D mesons with the light pseudoscalar mesons, because the breaking of the $SU(3)_{\text{flavor}}$ symmetry by weak interactions is determined by X. How should X transform if we want (**??**) to be invariant under $SU(3)_{\text{flavor}}$?

b) There are three D mesons containing an anticharm quark: $\bar{D}^0 = \bar{c}u$, $D^- = \bar{c}d$, and $D_S^- = \bar{c}s$. What representation of $SU(3)_{\text{flavor}}$ do these three D mesons live in?

c) Recall that the pseudoscalar mesons M form an octet (i.e. adjoint) of $SU(3)_{\text{flavor}}$. Still pretending that X transforms to make (**??**) invariant under $SU(3)_{\text{flavor}}$ (as in part (a)), what $SU(3)_{\text{flavor}}$ invariant combinations can you make out of X, M, D, and the leptonic weak current $\bar{e}\gamma_\mu(1-\gamma^5)\nu_e$?

d) Now we will remember that X is actually a constant. By expanding out the term you found in the previous part, argue that

$$\Gamma(\bar{D}^0 \to K^+ e\bar{\nu}_e) = \Gamma(D^- \to K^0 e\bar{\nu}_e) = \frac{3}{2}\Gamma(D_S^- \to \eta e\bar{\nu}_e) \tag{5}$$

Recall that the pseudoscalar meson octet is:

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta}{\sqrt{6}} \end{pmatrix} \tag{6}$$

e) Compare your result with the following data (taken from the PDG):

Particle	Lifetime (10^{-15} s)	Decay Channel	Branching Ratio (%)
D^-	1040 ± 7	$K^0 e^- \bar{\nu}_e$	8.83 ± 0.22
$\bar{D}^0$	410.1 ± 1.5	$K^+ e^- \bar{\nu}_e$	3.57 ± 0.06
D_S^-	500 ± 7	$\eta e^- \bar{\nu}_e$	2.67 ± 0.29

(7)

2 Baryon Mass Splittings

Disclaimer: this might sound very scary, but this is much simpler than what Sean did in class.

Important note: we consider $\bar{B}$, the anti-baryons to be transforming in a distinctive octet.

In this problem we will derive the Gell-Mann-Okubo mass formula, which clearly shows that the $SU(3)$ flavor symmetry, although badly broken, is very predictive. The lowest octet baryon can be parameterize in the following way,

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

a) Comment on why there is no singlet baryon state based on what you learned from the wave function of a baryon from the last tutorial.

b) Let us suppose the Lagrangian is given by

$$\mathcal{L} = \text{Tr}[\bar{B}(i\partial\!\!\!/ - \mu_0)B] - \alpha \text{Mysterious term 1} - \beta \text{Mysterious term 2} \tag{8}$$

What are the two mysterious terms that will give baryon mass and that are allowed by the symmetry using the same mass splitting matrix you see in class that transforms as an adjoint under $SU(3)$ flavor symmetry?

$$\Delta_s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{9}$$

Hint: Look at the μ_0 mass term to write $\bar{B}$ as another 3×3 matrix that transforms as an adjoint. The idea behind this is that $\bar{B}$ should be the distinct octet where the anti-baryons live.

c) Expand the Lagrangian out, you should get something like,

$$\begin{aligned} m_{p,n} &= m_{p,n}(\mu_0, \alpha, \beta, m_s) \qquad (10) \\ m_\Lambda &= m_\Lambda(\mu_0, \alpha, \beta, m_s) \\ m_\Sigma &= m_\Sigma(\mu_0, \alpha, \beta, m_s) \\ m_\Xi &= m_\Xi(\mu_0, \alpha, \beta, m_s) \end{aligned}$$

d) From here, derive Gell-Mann-Okubo mass formula

$$\frac{m_\Sigma + 3m_\Lambda}{2} = m_N + m_\Xi \tag{11}$$

e) Use the following masses (from the PDG) to see how well the Gell-Mann-Okubo formula works (in MeV):

$$\begin{aligned} m_p &= 938.3 && (12) \\ m_n &= 939.6 && (13) \\ m_{\Sigma^+} &= 1189.4 && (14) \\ m_{\Sigma^0} &= 1192.6 && (15) \\ m_{\Sigma^-} &= 1197.4 && (16) \\ m_\Lambda &= 1115.7 && (17) \\ m_{\Xi^0} &= 1314.9(2) && (18) \\ m_{\Xi^-} &= 1321.7 && (19) \\ & && (20) \end{aligned}$$