Standard Model Solution 6: Strange and Charming

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1 Weak Decays of D Mesons

We can understand some of the properties of heavy quark mesons using the approximate $SU(3)$ flavor symmetry of the light quarks. In particular we will find relationships between some of the semileptonic partial decay widths of the D mesons. The D mesons are pseudoscalar mesons made of a charm quark and a light antiquark (or a light quark and anticharm quark).

a) The weak decay of the charm quark ($c \to se^+\nu$) can be described by the following Fermi Lagrangian:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{cs}^* \bar{s}\gamma^\mu(1-\gamma^5)c\bar{e}\gamma_\mu(1-\gamma^5)\nu_e \tag{1}$$

This interaction breaks the $SU(3)$ flavor symmetry of the light quarks. By figuring out how this interaction breaks the flavor symmetry, we can determine the interactions of the D mesons with the light quark pseudoscalar mesons. We can rewrite (1) as:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{cs}^* \bar{q}X\gamma^\mu(1-\gamma^5)c\bar{e}\gamma_\mu(1-\gamma^5)\nu_e \tag{2}$$

where q is the light quark triplet of $SU(3)_{\text{flavor}}$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{3}$$

and X is the *constant* vector

$$X = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

We will ignore the fact that X is a constant for now (we will remember again in part (d)), and pretend that it transforms under $SU(3)_{\text{flavor}}$. In other words, we will treat X as a *spurion*.

This will allow us to determine relations among the interactions of the D mesons with the light pseudoscalar mesons, because the breaking of the $SU(3)_{\text{flavor}}$ symmetry by weak interactions is determined by X. How should X transform if we want (2) to be invariant under $SU(3)_{\text{flavor}}$?

b) There are three D mesons containing an anticharm quark: $\bar{D}^0 = \bar{c}u$, $D^- = \bar{c}d$, and $D_S^- = \bar{c}s$. What representation of $SU(3)_{\text{flavor}}$ do these three D mesons live in?

c) Recall that the pseudoscalar mesons M form an octet (i.e. adjoint) of $SU(3)_{\text{flavor}}$. Still pretending that X transforms to make (2) invariant under $SU(3)_{\text{flavor}}$ (as in part (a)), what $SU(3)_{\text{flavor}}$ invariant combinations can you make out of X, M, D, and the leptonic weak current $\bar{e}\gamma_\mu(1-\gamma^5)\nu_e$?

d) Now we will remember that X is actually a constant. By expanding out the term you found in the previous part, argue that

$$\Gamma(\bar{D}^0 \to K^+ e\bar{\nu}_e) = \Gamma(D^- \to K^0 e\bar{\nu}_e) = \frac{3}{2}\Gamma(D_S^- \to \eta e\bar{\nu}_e) \tag{5}$$

Recall that the pseudoscalar meson octet is:

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta}{\sqrt{6}} \end{pmatrix} \tag{6}$$

e) Compare your result with the following data (taken from the PDG):

Particle	Lifetime (10^{-15} s)	Decay Channel	Branching Ratio (%)
D^-	1040 ± 7	$K^0 e^- \bar{\nu}_e$	8.83 ± 0.22
$\bar{D}^0$	410.1 ± 1.5	$K^+ e^- \bar{\nu}_e$	3.57 ± 0.06
D_S^-	500 ± 7	$\eta e^- \bar{\nu}_e$	2.67 ± 0.29

(7)

Solution:

a) 3. As $\bar{q}X$ needs to be a singlet.

b) also 3. As $\bar{c}$ is a singlet. D mesons transform as the associate the quark tripet.

c) This is clear $\bar{3}Adj3$ =singlet. So the term we can write down is $X^\dagger M D$.

d) We have

$$X^\dagger M D = (0,0,1)\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta}{\sqrt{6}} \end{pmatrix}\begin{pmatrix} \bar{D}^0 \\ D^- \\ D_S^- \end{pmatrix} \tag{8}$$

$$= \left(K^-, \bar{K}^0, \frac{-2\eta}{\sqrt{6}}\right)\begin{pmatrix} \bar{D}^0 \\ D^- \\ D_S^- \end{pmatrix}$$

$$= K^-\bar{D}^0 + \bar{K}^0 D^- + \frac{-2\eta}{\sqrt{6}} D_S^-$$

The decay rate ratio will be proportional to the square of the coefficient, so it is $1:1:\frac{2}{3}$.

e) Recall

$$\Gamma_{tot} = \frac{1}{\tau}$$

We have

$$\begin{aligned}\frac{\Gamma(\bar{D}^0 \to K^+ e\bar{\nu}_e)}{\Gamma(D^- \to K^0 e\bar{\nu}_e)} &= \frac{\Gamma^0_{tot} Br^0}{\Gamma^-_{tot} Br^-} \\ &= \frac{\tau^- Br^0}{\tau^0 Br^-} \\ &= \frac{1040 * 3.57\%}{410.1 * 8.83\%} = 1.025\end{aligned} \tag{9}$$

and

$$\begin{aligned}\frac{\Gamma(D^- \to K^0 e\bar{\nu}_e)}{\Gamma(D_S^- \to \eta e\bar{\nu}_e)} &= \frac{\Gamma^-_{tot} Br^-}{\Gamma^S_{tot} Br^S} \\ &= \frac{\tau^S Br^-}{\tau^- Br^S} \\ &= \frac{500 * 8.83\%}{1040 * 2.67\%} = 1.59\end{aligned} \tag{10}$$

2 Baryon Mass Splittings

In this problem we will derive the Gell-Mann-Okubo mass formula, which clearly shows that although strange quark is a bit heavy, the $SU(3)$ flavor symmetry is still pretty predictive.

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

There is no singlet baryon state due to the anti-symmetric of the wave function. (why?) The Lagrangian is given by

$$\mathcal{L} = \text{Tr}[\bar{B}(i\partial\!\!\!/ - \mu_0)B] - \alpha\text{Tr}[\bar{B}\Delta_s B] - \beta\text{Tr}[\Delta_S \bar{B}B]$$

where

$$\Delta_S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Solution: See the notebook. From the notebook, we can read off,

$$\begin{aligned} m_{p,n} &= \mu_0 + \alpha m_s \\ m_\Lambda &= \mu_0 + \frac{2}{3}\alpha m_s + \frac{2}{3}\beta m_s \\ m_\Sigma &= \mu_0 \\ m_\Xi &= \mu_0 + \beta m_s \end{aligned} \tag{11}$$

which implies

$$\frac{m_\Sigma + 3m_\Lambda}{2} = \mu_0 + \alpha m_s + \beta m_s = m_\Sigma + m_\Xi \tag{12}$$

This is the Gell-Mann-Okubo mass formula. Works to better than 1%.