A DIGITAL MATHEMATICAL PERFORMANCE TO SUPPORT THE PHYSICAL EMBODIMENT OF BALANCED EQUATIONS

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A "digital mathematical performance" is created in form of a skit to help both teachers and students learn the gesture of balanced and unbalanced equations. Similar to the use of physical balances, the developed skit introduces new metaphors to solving equations – specifically, the metaphors of loading and unloading and of falling out of balance. A new discourse may be required to understand the role of mathematical performances in learning mathematics. The authors draw from the complexity thinking discourse that takes abstract concepts to be emergent from a variety of examples, and from varied interpretations. From this perspective, mathematical performances might be as necessary as richness in representations, multiplicity in interpretations, variance in individual instances, and extension to technological media for the emergence and progress of mathematical concepts, and intuitions.

INTRODUCTION

Research has demonstrated that students can learn a new topic better when it is presented with relevant gestures than when it is presented through speech alone (Goldin-Meadow, Kim, and Singer, 1999). These gestures may express information that is beyond the information conveyed in speech (Kelly, Singer, Hicks, and Goldin-Meadow, 2002) and "other, more codified forms of communication" (p. 419, Goldin-Meadow, 1999). Gesture thus provides a powerful communication medium in which students and teachers can describe and discuss emergent, abstract concepts.

Our overall goal is to use directed role playing to encourage students to analyze and learn from physical experiences that have useful interactions with specific mathematical concepts. Students are led through these "kinesthetic explorations [which] directly involve bodily understanding" (p. 63, Roschelle, Kaput, and Stroup, 2000) by using a digital mathematical performance (Gadanidis, 2006). This performance has been designed to create actions and interactions that can be related to the mathematical concept of "balanced equations".

The topic of "balanced equations" relates to the solution of an equation for an unknown. During the solution of an equation, it is important to keep both sides of the equation "balanced". The example equation that will be discussed in depth is $2x + 1 = x + 3$. To isolate for x in this equation, one x and one unit is usually removed from each side of the equation. One of the authors has often gestured to create "arm scales" to demonstrate the concept of "balanced equations". The "arm scales" gesture has both palms facing up at about chest level. Moving the right hand up and down inversely with the left hand creates the visualization of a pan balance (see Figure 1).

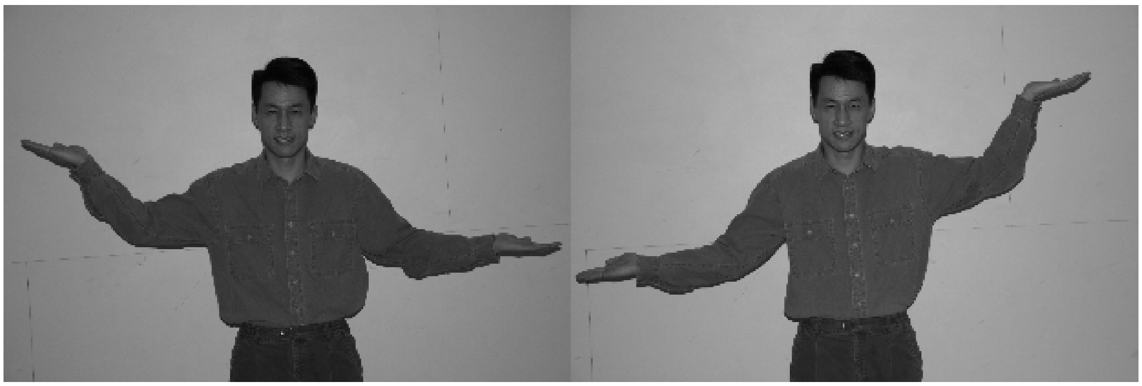

Figure 1: The "arm scales" gesture

Extending from the concept of balanced equations, there is the common student mistake of "imbalanced" equations – students often do not keep the two sides of the equation balanced while they are solving for the unknown. A new gesture for imbalanced equations has been developed to help students to circumnavigate this obstacle (). This gesture consists of (for example) the right hand going down, the left hand going up, leaning radically to the right, and hopping up and down on the right foot. The imbalanced equations are embodied by the off-balance person who is creating the visualization of imbalanced scales (see Figure 2).

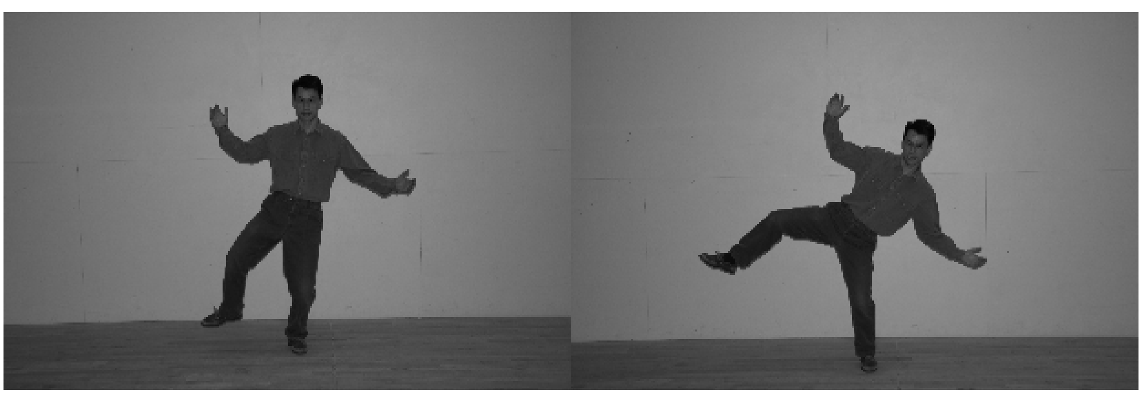

Figure 2: The "imbalanced equations" gesture.

The "digital mathematical performance" is created to help both teachers and students learn the new gesture. Our interpretation of a digital mathematical performance is a mathematical performance that has been recorded onto a digital format.[1] Our performance is the enactment of a skit in which a person is carrying several items in each hand. Due to the weight of each item, it is important for this person to be carrying the same amount of each item in each hand; otherwise, the person will be imbalanced. The process of unloading the person is similar to the process of solving for the unknown – items are removed from each hand/side of the equation. The skit demonstrates several correct actions which leave the person/equations balanced and several incorrect actions which cause the person/equations to become imbalanced.

Similar to the use of physical balances, the developed skit introduces new metaphors to solving equations – specifically, the *metaphors* of loading and unloading and of falling out of balance. Students who have watched and/or performed the skit should gain a unique *visualization* of a balanced or imbalanced equation and of falling out of or getting back into balance. The skit also allows more sensory modalities (e.g. auditory, tactile, and bodily orientation) to be integrated into the teaching and learning process.

Additional goals of the developed mathematical performance are to make mathematics fun, to engage the students into active learning, and to separate the conceptual and symbolic components of the mathematics. For example, the physical embodiment of imbalanced equations is intentionally comedic. Rather than just having one arm go up and one arm go down, the physical exaggeration of leaning to one side and hopping up and down is intended to be both humourous and memorable.

Student enactment of the skit provides an excellent opportunity to engage students in active learning that encourages student ownership of their own learning processes. The learning environment provided by role play allows students to leverage and build upon their own personal experiences as they learn the new mathematical concepts. Further, these mathematical concepts are learned in a more non-threatening/non-symbolic way (relative to traditional classroom/chalkboard instruction).

BACKGROUND

Many teachers appear to view concrete and virtual models and other learning media as merely tangible bases or visual illustrations of abstract ideas that exist "out there"

[1] See http://www.atkinson.yorku.ca/~sychen/research/math/DMP.html for the video clips.

(Towers and Davis, 2002). Innovative learning aids such as visuals and manipulative materials are often referred to as props. However, these “props” can facilitate recall for weaker students and offer diversity in interpretations which reinforces conceptual understanding of harder topics. These learning aids represent literal metaphors or analogies that offer many ways of expressing the same thing…and more.

In our experience, pre-service teachers quickly embrace innovative approaches as attempts to reach out to different learning styles (e.g. tactile, visual, aural, etc). Thus, the use of multiple embodiments fits with the desire to make mathematics more interesting and accessible to all students. However, we are also aware that the above justification does not refer to any specific issue or component of a mathematical concept that necessarily requires the use of an innovative teaching method.

Towers and Davis (2002) maintain that while many reform methods have fit well with populist notions such as child-centered learning, individuated learning styles, or multiple intelligence theory, they have not affected how teachers view mathematical concepts in their teaching practice. Novel ways of teaching and learning that emerge from current research get trivialized when they are placed in the context of existing student-centered discourses.

A new discourse may be required to understand the role of mathematical performances in learning mathematics. The complexity thinking discourse takes abstract concepts to be emergent entities (Namukasa, 2005). Specifically, abstract concepts emerge from human experiences, from a variety of examples, and from varied interpretations. They emerge from actions and interactions as stabilities or patterns that arise with time. From the recursive coordination of human actions and interactions, these regular and lawful entities arise.

Like human habits, abstract concepts are the meta-stabilities among actions and interactions. In von Foerster (2003) and Bateson’s (1979) language, algebraic concepts are the entities that stabilize as people talk, act, and converse about balances, equations, and equalities (to give some examples). After they arise, there is a danger to view them as if they existed before and independent of the actions. When concepts and objects emerge, discernible boundaries, preciseness, and logical structures emerge with them. However, to the learner who is encountering the concepts for the first time, these emergent concepts and objects may not necessarily pre-exist. Thus, there are potential benefits in re-enacting some of the possible actions and interactions which may have led to the stabilized concepts.

THE SKIT

An actor has been sent to the store to buy some coffee for the group. This person returns with a heavy load of one bag of coffee cups and three additional coffee cups in one hand and two bags of coffee cups and one additional coffee cup in the other hand. This person has been carrying this heavy load for quite some time and needs help to unload.

To help the first actor, a second actor grabs the three loose coffee cups from one side and proceeds to lift them. The first actor raises the hand which had the three coffee cups and starts to fall over to the side which had only one coffee cup – performing the new gesture for imbalanced equations. The action of removing unequal amounts from each side of balanced equations is a common student error, and the skit demonstrates this error with a comical and visual representation of imbalanced equations.

After replacing the cups to rebalance the first actor, another actor now helps out by removing one cup of coffee from each hand. The actor movers both hands up and down showing the gesture for balanced equations, and it is discussed how the same action has to be performed to both sides to keep the sides/equations balanced. The second actor now claims to understand and removes one bag from one hand and one cup from the other hand. This action represents another common student error in that the unknown has no physical significance in many algebraic equations – it is viewed as one unit. The first actor again performs the gesture for imbalanced equations to demonstrate and reinforce this concept – the same action has not been performed on both sides/equations.

Finally, one bag of coffee cups is removed from each hand. The question is now raised of how many cups of coffee did the first actor actually get. To answer this question, it is necessary to know how many cups of coffee are in each bag of coffee cups (or to solve for the unknown in the equations). Since two cups of coffee balance one bag of coffee cups, it can be determined that each bag of coffee cups contains two cups of coffee.

DISCUSSION

In teaching any mathematical concept, it should be recognized that two things are being taught – the mathematical concept and the symbolic notation used to mark or represent that concept. It has previously been noted that the difficulties that students have in learning algebraic concepts can be with the symbolic notations rather than the algebra (Hewitt, 2001). Therefore, there can be advantages to teaching the (algebraic) concept before and independent of the symbolic notation.

The developed skit uses role playing to create an opportunity for students to learn about the concepts of balanced/imbalanced equations and the solution of equations. During these role-playing activities, students can develop their algebraic thinking and understanding before proceeding to the associated symbolic notations. In addition to being potentially easier for the student, this teaching method may also make it easier for a teacher to determine if a student's difficulties involve the algebraic concepts or the symbolic notation.

A broader goal of mathematical performances and other teaching activities that involve metaphors and visualizations is to reshape the students' mathematical worlds. For example, from seeing the consequence of falling out of balance a student might understand the consequences of not following the rule "what you do on one side of the equation has to be done on the other so as to keep the balance." The same student might understand equality in terms of balancing.

That "enacted concepts" and "objects as emergent wholes" arise from the interaction of many local actions and interactions illustrates a discourse that is at the intersection of a focus on students and on content. Students are complex systems and so are the knowledge systems that they enact. Waldrop (1992) claims that the most crucial thing we have to achieve to understand complex systems like insights and concepts is to see how they emerged and how they evolved from many interacting and recurring instances. The emergent nature of these dynamic systems implies that they are constantly recombining and changing shape in relation to other interacting factors. Similarly, mathematical concepts have neither objective existence nor personal sense-making, but they arise through intersection with contexts, artifacts, performances, and the community of learners (Kieren, 1995).

Ideally, mathematical performances act as dynamic attractors of mathematical concepts. This view resonates with the hermeneutic view that the thing-in-itself (in this case the mathematical objects and concepts) is nothing but the continuity with which the various perceptual/performance perspectives shade into one another (Gadamer, 1992). It is the nature of the emergence of abstract concepts from human activity, actions, and interactions that first and foremost supports such innovative teaching methods. From this perspective, mathematical performances might be asnecessary as richness in representations, multiplicity in interpretations, variance in individual instances, and extension to technological media for the emergence and progress of mathematical concepts, ideas, and intuitions.

References

Bateson, G. (1979/1980) Mind and Nature: A necessary Unity. Bantaman Books.

Gadamer, H.-G. (1975/1992). Truth and method (2nd revised, edition). Translated by Weinsheimer, J. and Marshall, D. G. Crossroad.

Gadanidis, G. (2006) “Exploring Digital Mathematical Performance in an Online Teacher Education Setting.” In Proceedings of Society for Information Technology and Teacher Education International Conference 2006. pgs 3726-3731.

Goldin-Meadow, S. (1999). “The role of gesture in communication and thinking.” Trends in Cognitive Sciences, 3(11): 419-429.

Goldin-Meadow, S. (2004). “Gesture’s role in the learning process.” Theory into Practice, 43(4): 314-321.

Goldin-Meadow, S., Kim, S., and Singer, M. (1999) “What the teacher's hands tell the student's mind about math.” Journal of Educational Psychology, 91: 720-730.

Hewitt, D. (2001) “On learning to adopt formal algebraic notation.” In Proceedings of the 12th Study conference of the International Commission on Mathematical Instruction: The Future of the Teaching and Learning of Algebra. Volume 1 pgs 305-312.

Kelly, D. S., Singer, M., Hicks J., and Goldin-Meadow, S. (2002). “A helping hand in assessing children’s knowledge: Instructing adults to attend to gesture.” Cognition and Instruction, 20(1): 1-26.

Kieren, T. E. (1995) “Teaching in the middle.” Presented at the Gage Conference for Teachers. Queens University, Kingston, Canada.

Namukasa, I. K. (2005) Attending in mathematics: A dynamic view about students’ thinking. PhD thesis, University of Alberta.

Roschelle, J.M., Kaput, J.J., and Stroup, W. (2000) “SimCalc: Accelerating students’ engagement with the mathematics of change. In Jacobson, M. and Kozma, R. (eds) Innovations in science and mathematics education: Advanced designs for technologies of learning. pgs 47–75. Erlbaum.

Towers, J. and Davis, B. (2002) “Structuring Occasions.” Educational Studies in Mathematics. 49: 313-340.

von Foerster, H. (2003) Understanding understanding. Essays on cybernetics and cognition. Springer.

Waldrop, M. M. (1992) Complexity: The emerging science at the edge of order and chaos. Touchstone.