Liquidity Constraints, International Trade, and Optimal Monetary Policy

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Abstract

The availability of liquidity matters for an economy’s production and trade as firms need working capital to finance their operations. This paper studies the interaction between trade and capital flows operating through the liquidity allocations in the financial markets using a small-open-economy, overlapping-generations model. Working capital requirements distort the intratemporal consumption allocations. International capital inflows help easing liquidity in the domestic credit market, facilitating trade and improving the intratemporal allocation, while distorting the intertemporal allocation of the economy. We show how the government can use the Friedman rule and differentiated consumption taxes to address the tradeoff between the intratemporal and intertemporal distortions and achieve the second best optimum. Imposing a higher tax rate on imports can reduce the international borrowing to imports ratio and enhance the efficiency in using capital inflows to facilitate trade flows.

Keywords: liquidity constraints, intratemporal and intertemporal trade, excessive borrowing, Friedman rule, small open economy.

JEL Classification: E52, E63, F41

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1 Introduction

When an economy takes part in trading in international markets, the connection between its intratemporal and intertemporal allocations depends on the relationship between its international trade and capital flows. Although the standard balance-of-payments equation relates the net trade balance to net capital flows, not much is known about the interaction between the trade and capital flows. This paper emphasizes the financial aspects of international transactions and the role of liquidity constraints in the determination of international trade flows, and explores the impacts of international capital inflows on the availability and allocation of liquidity in the domestic credit market of a small open economy. Two main contributions are presented. First, the investigation into the interaction between international trade and finance enables a better understanding of the need and efficiency of using international borrowing in facilitating international trade flows. We show how, in the presence of working capital requirements, the values of exports and imports may be related to international borrowing individually and how the international borrowing to imports ratio depends on the policy variables. Second, we provide new insights into the design of optimal policy in a small open economy to address the issue of excessive borrowing. Our study illustrate how the impacts of a reduction in the domestic interest rate on the intratemporal and intertemporal distortions depend on the availability of other policy instruments, indicating the needs to re-examine not only the connection between the optimality of the Friedman rule and the uniform taxation of consumption but also the desirability of taxes on imports in an intertemporal small open economy.

Because of the time lags between production and the receipts from sales, firms often face liquidity constraints and rely on bank loans to finance their working capital. The time lags tend to be longer for international transactions, financial frictions can therefore severely restrict a country’s international trade flows. A recent literature on trade and finance has identified the negative impacts of domestic credit market frictions on the ability of firms to exports and on countries’ trade flows. Manova (2010) surveys the empirical and theoretical research, noting that this literature has often ignored the international financial markets and capital mobility.\footnote{Beck (2003), Svaleryd and Vlachos (2005), and Manova (2008) present strong evidence that domestic credit frictions are important in determining countries' trade patterns. Kletzer and Bardhan (1987), Beck (2002), Matsuyama (2005), and Manova (2010) provide the theoretical frameworks to rationalize this link between trade and finance.} Given the interconnection of the domestic and international financial markets, it is
of interest to study how the availability and allocation of liquidity in the domestic financial market depend on the flows of international capital.

As documented in the World Bank’s *Global Development Finance* (2012), most developing countries’ short-term external debts were trade-related, and their movements closely reflected the changes in the countries’ imports.\(^2\) A few recent theoretical papers have studied the relationship between international trade and capital flows in the presence of financial frictions. Antrás and Caballero (2009) construct a world economy consisting of two countries with different financial development and two sectors with different financial dependence to study the effects of trade integration on international capital flows.\(^3\) Ju and Wei (2011) incorporate a financial contract model of Holmstrom and Tirole (1997) into the standard Heckscher-Ohlin framework and examine how factor endowments and the quality of financial institutions jointly determine the patterns of production and trade of an economy.\(^4\),\(^5\) These studies focus on the role of *physical capital* in the real economy and model international capital flows as driven by the allocations of physical capital across countries. The short-term working capital needs associated with trade are absent in their models. This paper is filling the gap by highlighting the role of *financial capital* in facilitating international trade and investigating the efficiency of using international capital inflows to help meeting the liquidity needs of domestic borrowers and enhancing their economic activities, while potentially affecting the effectiveness of domestic monetary policy.

Our study emphasizes the role of liquidity in facilitating transactions and the function of financial markets in providing liquidity. In a model economy populated with two-period-lived overlapping generations, liquidity constraints are imposed on not only consumption but also production and international trade.\(^6\) Consumers must settle their consumption purchases by

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\(^2\)For the 129 developing countries included in the report, the short-term debts constituted 25% of the total external debt stock at the end of 2010, the short-term debts to imports ratio was 17.2%, and their net short-term external debt flows reached 268.5 billions of U.S. dollars in 2010.

\(^3\)Antrás and Caballero (2009) show that trade integration can lead the financially less developed country to export the product of the financially less dependent sector, resulting in an increase in the return to capital in the country and inducing capital inflows from the financially more developed country. It is noted that their dynamic model adopts the overlapping generation structure used in Caballero, Farhi, and Gourinchas (2008).

\(^4\)Ju and Wei (2011) illustrate that an economy with low-quality institutions will specialize in producing the labor-intensive good and experience capital outflows if international capital flows are allowed.

\(^5\)Jin (2012) also studies the role of industrial structure in determining capital flows in a two-country overlapping generations model with factor-proportions-based trade. The model predicts a tendency for capital to flow toward countries that become more specialized in capital-intensive goods, however, it does not model financial frictions.

\(^6\)In this paper, the term “liquidity” refers to assets that can be used as a means of payment, including both government-issued money (outside money) and chequing account balances at financial intermediaries (inside money).
cheques drawn against predetermined chequing account balances. Firms and importers borrow from the domestic financial intermediaries in order to meet their working capital needs. The heterogeneity in their dependence of external finance is modeled by using different working capital requirements when specifying the liquidity constraints. Financial intermediaries can channel liquidity from the international credit market to the domestic credit market. Because of limited participation in financial markets, domestic depositors and borrowers are affected asymmetrically. A decrease in the world interest rate induces capital inflows and increases production and trade as borrowers obtain more working capital, while consumption tends to fall as depositors’ interest income decreases. We use an overlapping generations structure to capture the limited participation idea and to model the distributional effects. The asymmetric access of liquidity is the key element in the analysis. As Bernanke and Gertler (1989) illustrated, the overlapping generations framework makes tractable the incorporation of the heterogeneity among borrowers and lenders, the “generations” should be interpreted as the entry and exit of firms from credit markets, and a “period” should be considered as the length of a typical loan contract. This setup allows us to extend the existing literature that studies monetary policy in a small open economy to explore the distributional effects of policies. As Carlstrom and Fuerst (2002), Schmitt-Grohé and Uribe (2003 and 2010), Cunha (2008), and Lai and Chin (2010) consider small open economies with infinitely-lived representative households, the distributional effects have been omitted in their studies.

The overlapping generations model can be thought of as a kind of the heterogeneous-agent limited-participation model proposed by Grossman and Weiss (1983) and Rotemberg (1984). As Williamson (2005) points out, the distributional effects of monetary policy may indeed be very important in the macroeconomy. The limited participation literature has focused on the liquidity effects of monetary policy on asset prices, while the implications of limited participation on the distributional effects of monetary policy has not received much attention.

Bernanke and Gertler (1989) use a model with two-period lived overlapping generations to study the role of credit market frictions in short-term business fluctuations. Galváni (1988, 2007), Bhattacharya and Haslag (2001), Smith (2003), and Bhattacharya, Haslag, and Russell (2005) study optimal monetary policy in closed-economy, overlapping generations models and show that the Friedman rule will be optimal if generation-specific lump-sum taxes and transfers are feasible.

The assumption of a small open economy allows us to leave out the complications arising from the strategic interactions among countries in the international marketplace. Some examples of studies of the strategic interactions of governments in two-country models are Benigno and Benigno (2003), Cooley and Quadrini (2003), Corsetti and Pesenti (2005), and Arseneau (2007).

Carlstrom and Fuerst (2002) model a small open economy that is subject to the requirement of purchasing foreign goods using the foreign currency accumulated in advance and faces an exogenous positive foreign interest rate, they show that the second-best optimal domestic interest rate is positive. As international borrowing is not allowed, trade is always balanced in their model. Schmitt-Grohé and Uribe (2003) highlight the role of complete international asset markets in diversifying the country-specific risk and show that the Friedman rule is optimal when consumption tax is available. Cunha (2008) shows that the availability of taxes
The operation of the domestic financial intermediaries results in two crucial liquidity flows in the domestic economy. First, using funds collected from domestic savers and borrowed abroad, financial intermediaries allocate loans to domestic borrowers (firms and importers) to facilitate their economic activities, helping them to meet their working capital requirements. Second, financial intermediaries accept demand deposits and issue inside money, allowing depositors to use cheques to pay for their consumption purchases. These two liquidity flows are related as they represent entries in the balance sheets of financial intermediaries. Their interactions determine the equilibrium allocation of the economy, establishing a link between the economy’s trade and capital flows operating through the liquidity allocations in the financial markets.

The liquidity constraints on production and trade effectively impede the channeling of the economy’s proceeds from exports to finance its imports in the same period.\textsuperscript{11} It results in a relation between the economy’s flows of exports and imports and its new international debt, imposing a restriction on the economy’s trade flows that is in addition to the standard balance-of-payments equation, distorting the intratemporal consumption allocation. International capital inflows help easing the constraints on the values of exports and imports, improving the \textit{intragtemporal} consumption allocation, while introducing a distortion to the \textit{inter}temporal consumption allocation of the economy if the world interest rate is positive. A positive world interest rate represents not only a private cost of using the liquidity in the international credit market but also a social cost of welfare redistribution among the heterogenous agents in the domestic economy. Hence, the domestic government has to choose the optimal policy combination to affect the interaction between trade and capital flows so as to address both the intratemporal and intertemporal distortions.

Given that some of the distortions resulting from the liquidity constraints take the form of quantitative constraints on the values of exports and imports, and that they are not wedges in the optimal conditions, it is necessary for the domestic government to distribute lump-sum transfers to the young, financed by its tax revenue, to address the quantitative constraints on consumption of the non-tradables is a sufficient condition for the Friedman rule to be optimal. Schmitt-Grohé and Uribe (2010) illustrate the optimality of the Friedman rule with distortionary taxation when there is no untaxed income or foreign demand for domestic currency. Lai and Chin (2010) show that in the presence of a perfect world capital market, Friedman rule can be optimal in a small open economy with endogenous growth.

\textsuperscript{11}This captures the fact that exports and imports are conducted by different parties in the economy, and the timing/informational frictions prevent the economy from mobilizing the proceeds from exports to settle the payments for imports right away.
and redirect the liquidity flows in the domestic credit market. Following the rationale in the literature on optimal money growth in closed economies, we find that, as long as the generation-specific lump-sum transfers are set optimally, the Friedman rule and uniform consumption taxation could be parts of the optimal policy combination in this small open economy, achieving the first-best socially optimal allocation via international trade, while eliminating completely the need of international borrowing.\textsuperscript{12} Although this result is unsurprising, it serves as a benchmark for comparison with the more interesting and relevant cases in which the lump-sum transfers are not feasible.

When the government has only the domestic interest rate and the distortionary taxes at its disposal, these policy instruments are imperfect substitutes of the lump-sum transfers in dealing with the intertemporal distortions. A decrease in the domestic interest rate, in combination with the corresponding optimal differential consumption tax rates, can help relieving the distortion in the intertemporal consumption allocation, while worsening the intratemporal distortions. Facing the tradeoff between the intratemporal and intertemporal distortions, the domestic government finds it optimal to implement the Friedman rule in this second-best environment and use some international borrowing to help facilitate the trade flows. As a result, there will be a positive relation between international trade and capital flows. A higher world interest rate implies a higher welfare cost of the intertemporal distortions from international borrowing. As the Friedman rule has been adopted and the domestic interest rate cannot go lower, the government should adjust its optimal values of consumption tax rates so as to induce the economy to economize on the use of foreign finance, reducing not only the value of imports but also the quantity of international borrowing used per unit of imports. These findings highlight the role of government policy in affecting the relationship between international trade and financial capital flows.

The availability of policy instruments not only limits the domestic government’s ability in addressing the economy’s excessive international borrowing issue but also determines the

relationship between the domestic interest rate and the intertemporal and intratemporal distortions. If differential consumption taxes are not allowed, a decrease in the domestic interest rate, together with the corresponding optimal uniform consumption tax rate, will worsen the intertemporal distortions, while improving the intratemporal distortions. Hence, it will be optimal to adopt a positive domestic interest rate. The resulting suboptimal utilization of capital inflows implies a higher international borrowing to imports ratio and lower economic welfare.

In contrast to Schmitt-Grohé and Uribe (2003) and Cunha (2008) that find an association between uniform consumption taxation and the optimality of the Friedman rule, we show that the Friedman rule will be optimal in the second-best environment only if the government adopts a set of differentiated consumption taxes, effectively taxing the imported goods at a rate higher than that on the domestically produced goods. Interestingly, in spite of the higher tax on the imported goods, the optimal policy combination not only promotes the economy’s trade flows (both exports and imports) but also improves its efficiency in using international borrowing to facilitate trade (reduces the ratio of international borrowing to imports). The intuition is that, although the government cannot address the quantitative constraints on exports and imports directly in the absence of the lump-sum transfers, the differentiated consumption tax rates and the domestic interest rate give the government sufficient policy instruments for all the wedges between the marginal rates of substitution and the marginal rates of transformation that it wants to affect, allowing for a better allocation of liquidity in the economy and a reduction in its reliance on foreign finance, and therefore achieving the second-best consumption allocation. Contrary to the trade literature abstracting from intertemporal considerations, this result highlights the importance of the connection between intratemporal and intertemporal allocations in determining the effects of taxes on imports.

The remainder of the paper is organized as follows. Section 2 presents the model. The properties of the competitive equilibrium are described in Section 3. Section 4 discusses how the equilibrium relation between trade and capital flows depends on the availability of various policy instruments. Section 5 concludes the paper.

2 The Model

Consider an overlapping generations model of a small open economy with liquidity constraints. The world economy consists of a small open economy (the domestic economy) and the rest of
the world (the foreign economy). The rest of the world is treated as exogenous. Time is discrete
and the horizon is infinite. The small open economy is inhabited by an infinite sequence of
two-period-lived overlapping generations, with the measure of each generation normalized to
one. There is no uncertainty. Households of each generation have perfect foresight, they
engage in production activities when young, and consume when old. Labor is internationally
immobile. There are two tradable consumption goods – a domestically produced good (good
x) and an imported good (good y). The production function of good x requires input of labor
only.\(^{13}\)

\[
Q_{xt} = A_{xt} L_{xt}.
\]

(1)

The output of good x, \(Q_{xt}\), depends on the productivity parameter \(A_{xt}\) and labor input \(L_{xt}\).
The assumption of constant returns to scale in the production of good x is for simplicity.

The world economy is a monetary economy, payments of transactions can be settled either
with cash or cheques.\(^{14}\) Firms and importers face working capital constraints but lack internal
funds; they must borrow from the domestic financial intermediaries to finance the fractional
advance payments. The old must save in the previous period for consuming in the current
period so that their consumption purchases are paid by cheques drawn against their prede-
termined chequing account balances. Economic agents of the small open economy can trade
assets and goods with the rest of the world, taking as given the world nominal interest rate
on the one-period, foreign-currency-denominated bonds, \(i_{wt}\), the price of the foreign currency
in terms of the domestic currency in the foreign exchange market, \(e_{t}\), and the foreign prices of
good x and good y in the world goods market, \(P_{x}^{f}\) and \(P_{y}^{f}\), respectively. Although prices in
the world goods market are set in units of the foreign currency, international trade in goods
is carried out using the buyers’ currency.\(^{15}\)

In each period, the small open economy is composed of a benevolent government (a so-
cial planner) and four types of private economic agents: young households (workers and
entrepreneurs), old households (consumers), importers, and financial intermediaries. For con-
venience, we normalize the number of each type of private economic agents to one.\(^{16}\) Economic

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\(^{13}\)We abstract from physical capital in the production process so as to focus on financial capital flows.

\(^{14}\)The introduction of chequing account deposits makes intergenerational loans possible in an overlapping
generations setup.

\(^{15}\)That is, the foreign buyers use foreign currency to pay for their purchases of good x, and the foreign
suppliers of good y are willing to accept domestic currency as payments. This assumption is for simplifying
the timing of events and economizing on the notations. By assuming that the domestic economy receives
foreign currency from exporting good x and pays domestic currency for its imports of good y, the foreign
exchange market only has to open once each period (at the end of each period).

\(^{16}\)It is noted that with the assumption of perfect competition, importers and financial intermediaries earn
zero profits in equilibrium, so that the ownership structures of the importing firms and intermediaries can be
ignored.
agents act as price-takers in perfectly competitive markets and perform different tasks in the economy. Because of limited participation in financial markets, only the borrowers (the entrepreneurs and importers) have access to the liquidity in the financial markets. The economic activities of domestic depositors and borrowers are affected asymmetrically by international capital flows. Their economic behaviors are described as follows. The timing of events is summarized in Figure 1.

2.1 Households

In each period, a new generation of two-period-lived households is born. Each household consists of two members: a worker and an entrepreneur. When young, each worker is endowed with a single unit of time that can be allocated between leisure and working, and each entrepreneur has access to the production technology that produces good $x$ and can operate a domestic firm. When old, households have no endowment and consume their accumulated wealth at the end of their lives. The representative household born in period $t$ has preferences characterized by the utility function,

$$U_t = U(h_t, C_{t+1}) = -\frac{h_t^{1+\eta}}{1 + \eta} + \beta C_{t+1},$$

where $C_{t+1} = C_{xt+1}^a C_{yt+1}^{1-a}, \quad 0 < a < 1, \quad \eta > 0, \quad \text{and} \quad 0 < \beta < 1.$

The household’s lifetime utility, $U_t$, depends on the effort that the worker supplies to the labor market when young, $h_t$, and the household’s consumption of the composite good when old, $C_{t+1}$. The parameter $\eta$ is the elasticity of marginal disutility of work effort. $\beta$ is the subjective discount factor of the household. The composite consumption consists of the consumption of goods $x$ and $y$, $C_{jt+1}, j = x, y$. The household’s expenditure share on good $x$ is measured by $a$. Each household of generation $t$ saves all the earnings of the worker and the entrepreneur to finance the consumption in period $t + 1$.

At the beginning of period $t$, members of each young household separate and go to various markets to perform their tasks. The worker goes to the domestic labor market and supplies $h_t$ units of time at the market wage rate of $W_t$ units of domestic currency. The entrepreneur goes to the domestic loan market to obtain a working capital loan to finance the production

\[^{17}\text{We follow the literature to assume that the utility function is homothetic in the consumption goods and separable in leisure. Chari and Kehoe (1999) show that, with these properties of preferences, both uniform consumption taxation and the Friedman rule are optimal as the relative prices between consumption of different goods are not distorted. In addition, in order to simplify the saving decision, households are assumed to consume only when old. The utility function is assumed to be linear in } C_{t+1} \text{ to allow for closed-form analytical solutions. The assumption of a constant expenditure share of each good helps simplify the analysis.}\]
of good $x$. Because of the timing of events, the young household cannot channel the labor income of its worker to meet the liquidity needs of its entrepreneur during the period.

The representative entrepreneur maximizes the profit of the domestic firm by borrowing $b_{xt}$ units of domestic currency to finance the working capital for the production of $C^{s}_{xt}$ units of good $x$ for the domestic market and $EX_{t}$ units of good $x$ for export. Let $\psi_{x}$ and $\psi_{EX}$ be the fractions of its wage bills that the firm must pay in advance of the production process, where $0 \leq \psi_{x} < \psi_{EX} \leq 1$. Because of the longer time lags between production and receipt of export revenue, the fraction of the wage bill required to be paid in advance for the production of exports, $\psi_{EX}$, is assumed to be higher than that of the production for the domestic market, $\psi_{x}$.

Taking as given the nominal interest rate on the domestic-currency-denominated loans, $i_{t}$, the nominal wage rate, $W_{t}$, the foreign price of good $x$, $P_{fxt}$, the foreign exchange rate, $e_{t}$, and the domestic price of good $x$, $P_{xt}$, and following the production technology given by (1), the representative entrepreneur's decision problem is given by

$$\max_{b_{xt}, \text{C}^{s}_{xt}, \text{EX}_{t}} \left\{ P_{xt}C^{s}_{xt} + e_{t}P_{fxt}\text{EX}_{t} + \left[ b_{xt} - \psi_{x} W_{t} \frac{C^{s}_{xt}}{A_{xt}} - \psi_{EX} W_{t} \frac{\text{EX}_{t}}{A_{xt}} \right] - b_{xt}(1 + i_{t}) - (1 - \psi_{x}) W_{t} \frac{C^{s}_{xt}}{A_{xt}} - (1 - \psi_{EX}) W_{t} \frac{\text{EX}_{t}}{A_{xt}} \right\},$$

subject to the liquidity constraint that specifies the working capital needs of the firm,

$$\psi_{x} W_{t} \frac{C^{s}_{xt}}{A_{xt}} + \psi_{EX} W_{t} \frac{\text{EX}_{t}}{A_{xt}} \leq b_{xt}. \quad (3)$$

The entrepreneur will supply to a market if the effective marginal revenue product of labor is equal to the wage rate. The linear production technology implies that the entrepreneur earns zero profit.

At the end of period $t$, the worker and entrepreneur return home and pool their earnings. Each young household receives a nominal lump-sum transfer, $\Gamma^{Y}_{t}$, from the domestic government, and then deposits its saving to a domestic financial intermediary.
In period \( t + 1 \), each household of generation \( t \) receives a nominal lump-sum transfer, \( \Gamma_{t+1}^O \), from the government, and purchases goods for consumption using cheques drawn against its deposit balance at the domestic financial intermediary.

Given the assumption of perfect foresight, the optimization problem facing each young household of generation \( t \) at the beginning of period \( t \) is as follows.

\[
\max_{h_t, D_{t+1}, C_{xt+1}, C_{yt+1}} \left\{ -\frac{h_t^{1+\eta}}{1+\eta} + \beta C_{xt+1}^a C_{yt+1}^{1-a} \right\},
\]

subject to

\[
D_{t+1} \leq W_t h_t + \Gamma_t^Y, \tag{5}
\]

and

\[
(1 + \tau_{xt+1}^c) P_{xt+1} C_{xt+1} + (1 + \tau_{yt+1}^c) P_{yt+1} C_{yt+1} \leq D_{t+1}(1 + i_{dt+1}) + \Gamma_{t+1}^O, \tag{6}
\]

where \( W_t h_t \) is the worker’s wage income, \( \Gamma_t^Y \) is the nominal lump-sum transfer received when young, and \( D_{t+1} \) is the one-period, domestic-currency-denominated bank deposits made by the household at the end of period \( t \). The household consumes \( C_{jt+1} \) in period \( t + 1 \), taking as given the nominal deposit interest rate, \( i_{dt+1} \), the nominal lump-sum transfer received when old, \( \Gamma_{t+1}^O \), the nominal prices of goods, \( P_{jt+1} \), and the consumption tax rates, \( \tau_{jc}^c, j = x, y \).

The first-order conditions of the household’s problem characterize its optimal intertemporal and intratemporal allocations, respectively.

\[
h_t^\eta = \left[ \frac{W_t (1 + i_{dt+1})}{(1 + \tau_{xt+1}^c) P_{xt+1}} \right] \beta a C_{xt+1}^{a-1} C_{yt+1}^{1-a}, \tag{7}
\]

and

\[
a (1 - a) C_{yt+1}^{1-a} C_{xt+1}^a = \frac{(1 + \tau_{xt+1}^c) P_{xt+1}}{(1 + \tau_{yt+1}^c) P_{yt+1}}. \tag{8}
\]

Equation (7) states the household’s optimal leisure-consumption tradeoff, equating the current marginal disutility of working to the discounted future marginal utility gain from consumption. If the worker supplies an extra unit of effort to the labor market, the additional wage income, \( W_t \), will be saved at the nominal interest rate \( i_{dt+1} \), allowing the household to consume an additional \( \frac{W_t (1 + i_{dt+1})}{(1 + \tau_{xt+1}^c) P_{xt+1}} \) units of good \( x \) in period \( t + 1 \). Equation (8) equates the household’s marginal rate of substitution between goods \( x \) and \( y \) to the effective relative good price.

Using budget constraint (6) and condition (8), we have the household’s optimal consumption demand when old.

\[
C_{xt+1} = a \frac{D_{t+1}(1 + i_{dt+1}) + \Gamma_{t+1}^O}{(1 + \tau_{xt+1}^c) P_{xt+1}} \quad \text{and} \quad C_{yt+1} = \frac{(1 - a)}{(1 + \tau_{yt+1}^c) P_{yt+1}}. \tag{9}
\]
By defining $P^\tau_{t+1} C_{t+1} = \sum_{j=x,y} (1 + \tau^c_{j,t+1}) P_{j,t+1} C_{j,t+1}$, we can derive the tax-included price of the composite consumption good in period $t + 1$ faced by the representative household, $P^\tau_{t+1}$

$$P^\tau_{t+1} = \left[ \frac{1}{a} \left( 1 + \tau^c_{x,t+1} \right) P_{x,t+1} \right]^a \left[ \frac{1}{1 - a} \left( 1 + \tau^c_{y,t+1} \right) P_{y,t+1} \right]^{1-a}. \quad (10)$$

### 2.2 Importers

All importers in the domestic economy are identical. The representative importer gets a working capital loan of $b_{yt}$ units of domestic currency from a domestic financial intermediary to finance the partial advance payment for purchasing $IM_t$ units of good $y$ from the world market, and sells them to the domestic consumers in order to maximize the nominal profit, taking as given the domestic nominal interest rate, $i_t$, the foreign exchange rate, $e_t$, and the nominal prices of good $y$ in terms of the domestic and foreign currencies, $P_{yt}$ and $P^f_{yt}$ respectively.

$$\max_{b_{yt},IM_t} \left\{ P_{yt} IM_t + \left[ b_{yt} - \psi_{IM} e_t P^f_{yt} IM_t \right] - b_{yt}(1 + i_t) - (1 - \psi_{IM}) e_t P^f_{yt} IM_t \right\},$$

subject to the liquidity constraint

$$\psi_{IM} e_t P^f_{yt} IM_t \leq b_{yt}, \quad (11)$$

where $\psi_{IM}$ is the exogenous required fraction of payment in advance, $0 \leq \psi_{IM} \leq 1$. The weaker the trading relation between the importer and foreign supplier is, the higher the fraction $\psi_{IM}$ will be.$^{18}$

At the end of the period, the importer will make the loan repayment to the domestic financial intermediary, $b_{yt}(1 + i_t)$, and the remainder payment to its foreign supplier, $(1 - \psi_{IM}) e_t P^f_{yt} IM_t$, after receiving the revenue from sale $P_{yt} IM_t$. As the foreign suppliers of good $y$ will exchange their receipts of domestic currency in the foreign exchange market at the rate $e_t$, perfect competition in the world market of good $y$ implies that the price of good $y$ in units of the domestic currency paid by the domestic importers to the foreign suppliers is $e_t P^f_{yt}$. $P_{yt}$ is the price of good $y$ in units of the domestic currency paid by the domestic importers to the foreign suppliers to the domestic importers. The first-order conditions of the importer imply an equalization of the marginal revenue, $P_{yt}$, and the effective marginal cost, $e_t P^f_{yt}(1 + \psi_{IM} i_t)$,

$$P_{yt} = e_t P^f_{yt}(1 + \psi_{IM} i_t), \quad (12)$$

and each importer earns zero profit.$^{18}$

$^{18}$With well-established trading relations, the foreign suppliers would be more willing to provide open account trade credit to the importers, implying a smaller value of $\psi_{IM}$.  

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2.3 Financial Intermediary

All financial intermediaries in the domestic economy are identical. Because of financial market segmentation, the domestic financial intermediaries play a unique role in the economy serving to intermediate between the domestic and foreign savers and the domestic borrowers. At the end of period \( t-1 \), each intermediary has access to funds from two sources so as to make loans to its borrowers in period \( t \). First, the intermediary accepts domestic-currency-denominated deposits from the domestic households of generation \( t-1 \), \( D_t \). Second, it borrows \( F_t \) units of foreign currency from the world credit market and then exchanges them for \( e_{t-1} F_t \) units of domestic currency in the foreign exchange market at the exchange rate, \( e_{t-1} \).\(^{19}\) The nominal interest rate on the one-period, foreign-currency-denominated loans in the world credit market is \( i_{wt} \). At the beginning of period \( t \), the representative intermediary allocates its loanable funds, \( D_t + e_{t-1} F_t \) units of domestic currency, across its borrowers. The loans to the domestic entrepreneurs and importers are denoted respectively by \( b_{xt} \) and \( b_{yt} \). At the end of period \( t \), the intermediary collects the loan repayments \( (b_{xt} + b_{yt})(1 + i_t) \), repays its foreign loans \( e_t F_t(1 + i_{wt}) \), and pays out the deposits accounts \( D_t(1 + i_{dt}) \).\(^{20}\) Taking as given the values of \( D_t, b_{xt}, b_{yt}, i_t, i_{wt}, e_t, \) and \( e_{t-1} \), the optimization problem facing the representative financial intermediary at the end of period \( t-1 \) is to choose \( F_t \) to maximize the return to the domestic depositors of generation \( t-1 \), \( D_t(1 + i_{dt}) \),

\[
D_t(1 + i_{dt}) = (b_{xt} + b_{yt})(1 + i_t) + [e_{t-1} F_t + D_t - b_{xt} - b_{yt}] - e_t F_t(1 + i_{wt})
\]

subject to its liquidity constraint

\[
 b_{xt} + b_{yt} \leq e_{t-1} F_t + D_t.
\] \(^{(14)}\)

The intermediary’s optimal choice of \( F_t \) must satisfy the first-order condition,

\[
 (1 + i_t) = \frac{e_t}{e_{t-1}} (1 + i_{wt}),
\]

where the marginal benefit of lending to its borrower a unit of domestic currency is equal to the marginal cost of obtaining a unit of domestic currency via borrowing in the world credit market. As there is no uncertainty in the model economy, equation \((15)\) is the (covered) interest rate parity condition. It should be noted that, this condition holds for the financial

\(^{19}\)The financial intermediaries do not hold any foreign currency for next period as it is assumed that the demands for loans faced by them are all in units of domestic currency.

\(^{20}\)We can assume that the representative intermediary accepting deposits \( D_t \) and borrowing \( F_t \) in period \( t-1 \) is established and collectively owned by its depositors (the young agents of generation \( t-1 \)), and that it will operate until the end of period \( t \) only. Depositors are allowed to use cheques drawn against their deposit account balances \( D_t(1 + i_{dt}) \) to finance their consumption expenditures. Cheques written during the period will be cleared at the end of the period. However, given the assumption of perfect competition in the financial markets, the ownership of the financial intermediary will be irrelevant to the analysis.
intermediaries only as they are the only economic units in the domestic economy that have direct access to both the domestic and international credit markets.

Let $\epsilon_t$ denote the nominal depreciation rate of the domestic currency in period $t$, $1 + \epsilon_t \equiv \frac{e_t}{e_{t-1}}$. The difference between the domestic and world nominal interest rates reflects the nominal depreciation rate of domestic currency, $\epsilon_t$.\footnote{Given the world interest rate $i_{wt}$, the domestic monetary policy determines the domestic nominal interest rate $i_t$ and pins down the nominal depreciation rate of domestic currency $\epsilon_t$. Imposing a tax rate on the interest payments on the domestic financial intermediaries' international borrowing, $\tau_{wt}$, can affect $\epsilon_t$, given $i_{wt}$ and $i_t$, and results in $(1 + i_t) = (1 + \epsilon_t) (1 + i_{wt} (1 + \tau_{wt}))$. Given that the tax only offers an extra degree of freedom to the government's optimal policy problem but has no impact on the real allocation, we assume $\tau_{wt} = 0$ for simplicity.} Substituting equations (14) and (15) into equation (13) shows that $i_{dt} = i_t$, it is optimal for the financial intermediary to set the interest rate on domestic deposits equal to the interest rate on domestic loans.\footnote{These two interest rates will be different if the financial intermediaries are subject to a fractional reserve requirement on deposit accounts. It is assumed that the government does not impose reserve requirements for simplicity.}

\section{2.4 The Government}

In a decentralized economy, the benevolent government uses macroeconomic policies to maximize the social welfare of the domestic country. Following Abel (1987), we measure social welfare by using the criterion function introduced by Samuelson (1967, 1968). In period 1, the government maximizes

$$U = (1 - B) \sum_{t=0}^{\infty} B^t U_t, \quad 0 < B \leq 1,$$

where $U_0 = \beta C_1$ is the utility of the initial old in period 1, the utility function of a household of generation $t$, $U_t$, $\forall t \geq 1$, is given by (2), and $B$ is the government's intergenerational discount factor. This specification of $U$ allows us to consider two alternative cases. The first case assumes that $0 < B < 1$ so that the government maximizes the sum of discounted utilities of representative households in current and future generations (the weighted average of utility of all generations) in the domestic economy; and for simplicity, we further assume that $B = \beta$. The second case assumes that the government maximizes the utility of the representative household in the steady state, where the criterion function (16) approaches the household's utility function given by (2) as $B \rightarrow 1$. In the following discussion, we will present the optimal allocation that maximizes the criterion function in (16). To obtain the optimal allocation that maximizes the steady state utility of the representative household given by (2), we can simply set $B = 1$.\footnote{Maximizing the steady state utility of a representative generation is the “golden rule” welfare criterion that is widely used for overlapping generations models in the literature. The golden rule allocation is the}
sum transfers \((T^m_t, T^Y_t, T^O_t)\) and the distortionary taxes \(\tau^c_{xt}\) and \(\tau^c_{yt}\) to affect the equilibrium allocation of the economy.

Under a flexible exchange rate regime, the small open economy can have monetary autonomy to set either the domestic nominal interest rate, \(i_t\), or the domestic money growth rate, \(\mu_t\), while the nominal exchange rate, \(e_t\), adjusts freely to clear the foreign exchange market. During period 1 after the intermediaries’ loan allocations are done, the government distributes the newly issued money, \(\mu_{t+1}\mathcal{M}_t\), to each young household as a lump-sum transfer, \(T^m_t\),

\[
T^m_t = \mu_{t+1}\mathcal{M}_t,
\]

where \(\mathcal{M}_t\) is the domestic money stock at the beginning of period 1 and \(1 + \mu_t\equiv \mathcal{M}_{t+1}/\mathcal{M}_t\) is the gross growth rate of money supply in period 1. Because of the timing of event, the domestic money newly issued in period 1 is distributed to the households of generation 1 only. As the households deposit this transfer into the financial intermediary at the end of period 1, we can interpret \(T^m_t\) as the government’s monetary injection into the domestic credit market for the use in period 1.\(^{24}\)

Let \(T_t\) denote the total tax revenue of the domestic government collected in period 1.

\[
T_t \equiv \tau^c_{xt}P_{xt}C_{xt} + \tau^c_{yt}P_{yt}C_{yt} = T^O_t + T^Y_t.
\]

In period 1, the government rebates its tax revenue \(T_t\) to the domestic households by distributing the generation-specific, nominal lump-sum transfers \(T^O_t\) and \(T^Y_t\) to the old and the young, respectively. Hence, the nominal lump-sum transfer distributed by the government to each domestic young household in period 1 is from two sources, \(\Gamma^Y_t = T^m_t + T^Y_t\), the newly issued money and the lump-sum transfer of some of the government’s nominal tax revenue. The only lump-sum transfer received by each domestic old household in period 1 is the lump-sum transfer of the government’s remaining nominal tax revenue, \(\Gamma^O_t = T^O_t = T_t - T^Y_t\). In the case with \(\tau^c_{xt} = \tau^c_{yt} = 0\), the lump-sum transfer to the young, \(T^Y_t\), will be financed solely by a lump-sum tax on the old, \(T^O_t = -T^Y_t\).\(^{25}\)

For analytical convenience, we define \(\gamma_t \equiv T^Y_t / \mathcal{M}_t\) so as to provide a measure of the relative size of the lump-sum transfer of tax revenue received by the young \(T^Y_t\) and the stock stationary, feasible allocation that maximizes the utility of the future generations.

\(^{24}\)Alternatively, we can interpret \(T^m_t\) as the government’s direct monetary injection to the financial intermediary at the end of period 1, the intermediary’s resulting profit \(T^m_t(1 + i_t+1)\) will be paid out to its owners (households of generation 1) in period 1 + 1.

\(^{25}\)Equation (7) shows the government can affect the intertemporal leisure-consumption tradeoff by using either the domestic interest rate, \(i_t\), and/or the consumption tax rates, \(\tau^c_{jt+1}, j=n,x,y\). There is no need to impose a tax rate on the income of the young. As given by equation (18), the allocation of \(T_t\) between \(T^Y_t\) and \(T^O_t\) can simply be interpreted as a redistribution of wealth from the old to the young. In the next section, we will illustrate that the division of \(T_t\) between \(T^Y_t\) and \(T^O_t\) matters to the determination of the competitive equilibrium.
of domestic money $\mathcal{M}_t$. It should be noted that in contrast to the domestic money supply growth rate $\mu_{t+1}$, the value of $\gamma_t$ does not result in any changes in $\mathcal{M}_{t+1}$. Equations (17) and (18) indicate that all of the policy variables cannot be set independently of one another. After the four independent policy instruments $i_t$, $\tau^c_{zt}$, $\tau^c_{yt}$, and $\gamma_t$, have been set, the values of $e_t$, $\mu_{t+1}$, $T^m_t$, $T^Y_t$, and $T^O_t$ can be determined endogenously.

3 Equilibrium Allocations

3.1 The Monetary Competitive Equilibrium

In the monetary competitive equilibrium of the small open economy, the following conditions must be satisfied. First, each household solves its optimization problem. Second, each importer and financial intermediary earn zero profit. Third, the government’s budget constraints, (17) and (18), are satisfied. Finally, all markets clear,

- labor market: $l^p_{xt} + l^{ex}_{xt} = h_t$,
- domestic loan market: $b_{xt} + b_{yt} = D_t + e_{t-1} F_t$,
- money market: $\mathcal{M}_t = M_t$,
- goods markets: $Q_{xt} = C_{xt} + EX_t$ and $IM_t = C_{yt}$,
- foreign exchange market: $e_t F_{t+1} + e_t P^f_{xt} EX_t = e_t F_t (1 + i_{wt}) + e_t P^f_{yt} IM_t$.

After all the transactions are cleared at the end of period $t-1$, the domestic financial intermediaries carry the entire domestic money stock, $\mathcal{M}_t$, into period $t$.

$$D_t + e_{t-1} F_t = \mathcal{M}_t = M_t = b_{xt} + b_{yt}. \quad (19)$$

They lend to the domestic firms and importers in period $t$. At the end of period $t$, loans repayments are made as illustrated in equation (13).

The foreign exchange market clearing condition is identical to the balance of payments equilibrium condition of the economy. The current account of the economy is the sum of the trade account (net exports), $P^f_{xt} EX_t - P^f_{yt} IM_t$, and the service account (net factor payments from abroad), $-i_{wt} F_t$. Balance of payments equilibrium requires that the sum of the current account balance, $P^f_{xt} EX_t - P^f_{yt} IM_t - i_{wt} F_t$, and the capital account balance, $F_{t+1} - F_t$, equals zero. The model allows for an investigation of the interaction between the international trade and capital flows. In contrast, Carlstrom and Fuerst (2002) assume away international borrowing so that trade is always balanced in their model.
3.2 Optimal Allocation in a Centralized, Non-monetary Economy

As a benchmark, we examine the efficient allocation in a centralized, non-monetary economy with no liquidity constraints. Suppose that the benevolent government centralizes the decisions of allocating resources in the domestic economy, and that international trade in goods and assets are measured in terms of good $x$. Taking the world relative price of good $y$ in terms of good $x$, $p^y_{yt} ≡ P^y_{yt}/P^x_{xt}$, and the world real interest rate on the one-period loans, $r_{wt}$, as given, the government maximizes the social welfare of its residents characterized by (16), subject to the following feasibility constraint,

$$A_{xt} L_{xt} + F_{t+1} = C_{xt} + p^y_{yt} C_{yt} + F_t (1 + r_{wt}),$$

(20)

where $L_{xt}$ is the labor input to the production of good $x$, and $F_{t+1}$ is the small open economy’s international borrowing from the world market in period $t$ (measured in terms of good $x$). Assume that the domestic economy has no initial debt, $F_1 = 0$. As there is no Ponzi game, $\lim_{t \to \infty} \frac{F_{t+1}}{\Pi_{j=1}^{t}(1+r_{wj})} = 0$, we can rewrite constraint (20) to obtain the economy’s intertemporal feasibility constraint.

$$\sum_{t=1}^{\infty} \frac{A_{xt} L_{xt} - C_{xt} - p^y_{yt} C_{yt}}{\Pi_{j=1}^{t}(1+r_{wj})} = 0.$$  

(21)

As presented in Appendix A, maximizing the social welfare function (16) subject to (21) yields the following conditions that characterize the Pareto efficient (first-best) allocation of the economy.\footnote{As will be discussed below, these three conditions help to identify the distortions in the monetary model with liquidity constraints and to develop optimal policies that address these distortions.}

First, the optimal leisure-consumption allocation in the domestic economy is achieved by equating the marginal disutility of working of a young household and the marginal utility gain from consumption of an old household in each period.

$$B h_t^n = \beta \left( \frac{a C_t}{C_{xt}} \right) A_{xt},$$

(22)

where $B h_t^n$ is the social marginal disutility of working of the young household in period $t$, $A_{xt}$ is the marginal product of labor in the production of good $x$, and $\beta \left( \frac{a C_t}{C_{xt}} \right)$ is the social (and private) marginal utility of consuming one unit of good $x$ by the old household in period $t$.

Second, the domestic economy’s optimal intratemporal consumption allocation requires that the marginal rate of substitution of good $y$ for good $x$ is equal to the world relative price of good $y$ in terms of good $x$.

$$\left(1 - \frac{a}{a} \right) \frac{C_{xt}}{C_{yt}} = p^y_{yt}.$$  

(23)
Third, the optimal intertemporal consumption allocation of the domestic economy satisfies

\[
F_{t+1} \left[ \frac{\beta a C_t}{C_{xt}} - B(1 + r_{t+1}) \left( \frac{\beta a C_{t+1}}{C_{xt+1}} \right) \right] = 0.
\]  (24)

Assume that \( r_{wt} = r_w \). In the case with \( B(1 + r_{w}) = 1 \), it is optimal to equalize the marginal utilities of consumption across generations, \( \frac{\beta a C_t}{C_{xt}} = \frac{\beta a C_{t+1}}{C_{xt+1}} \), by having \( C_t = C_{t+1} \). In the case with \( B = 1 \), maximizing the utility of the representative household in the steady state also implies a constant consumption path \( C_t = C_{t+1} \). In both cases, as the small open economy does not gain from accessing the liquidity in international financial market in the non-monetary environment, its optimal international borrowing is zero, \( F_{t+1} = 0 \), \( \forall t \geq 1 \); and trade in goods is balanced in each period.

\[
A_{xt} L_{xt} - C_{xt} = p_{yt}^f C_{yt}, \quad \Rightarrow \quad EX_t = p_{yt}^f IM_t \quad \text{and} \quad F_{t+1} = 0, \quad \forall t \geq 1.
\]  (25)

The socially optimal allocation \( (L_x^*, C_x^*, C_y^*) \) that maximizes the criterion function (16) in the case with time-invariant values of \( A_{xt}, p_{yt}^f \), and \( r_{wt} \) is presented in Appendix A. Each initial old household receives \( U_0^* = \beta C_x^* \), while the representative household in each generation \( t, t \geq 1 \), receives \( U^* = \frac{h^{1+\eta}}{1+\eta} + \beta C^* \), where \( h^* = L_x^* \) and \( C^* = C_x^a C_y^{1-a} \). For simplicity, we will use \( (h^*, C^*, U^*) \) to summarize this optimal allocation.

### 3.3 Properties of the Monetary Competitive Equilibrium

Given the overlapping generations setting, the imposition of liquidity constraints on transactions, and the presence of distortionary taxes, the monetary competitive equilibrium described in Section 3.1 needs not be socially optimal. By comparing with the conditions that characterize the socially optimal allocation in Section 3.2, we can identify three kinds of inefficiencies prevailing in the monetary competitive equilibrium.

Firstly, each household faces an intertemporal tradeoff between leisure and consumption as it works when young and consumes when old. The current-period marginal disutility of working is equal to the discounted, next-period marginal utility gain from consuming the wage income saved in the current period. From equations (4) and (7), we get

\[
h_t = \frac{e_t P_{xt}^f A_{xt}}{1 + \psi_{xt+1}^c t} \left[ \frac{(1 + i_{t+1})(1 + \psi_{xt+1}^c t)}{(1 + \psi_{xt+1}^c t)} e_{t+1} P_{xt+1}^f (1 + \psi_{xt+1}^c t) \right] \left( \frac{a \beta C_{t+1}}{C_{xt+1}} \right),
\]  (7')

where the wage income from supplying an additional unit of work effort \( W_t = \frac{e_t P_{xt}^f A_{xt}}{1 + \psi_{xt+1}^c t} \) is saved at the market gross interest rate \( 1 + i_{t+1} \), and will be used to purchase good \( x \) at the effective price \( (1 + \psi_{xt+1}^c t) P_{xt+1} \), where \( P_{xt+1} = \frac{(1 + \psi_{xt+1}^c t)}{1 + \psi_{xt+1}^c t} e_{t+1} P_{xt+1}^f \). In contrast, as illustrated in equation (22), the benevolent government faces an intratemporal but intergenerational
tradeoff between the leisure of generation $t$ and the period-$t$ consumption of generation $t-1$. In order to compare the representative household’s optimal condition (7') with the benevolent government’s optimal condition (22), we obtain $a P^r_t C_t = (1 + \tau^c_t) \left(1 + \psi x_{i_t} i_t + \psi_{EX} i_t\right) e_t P^f_t C_{xt}$ from equations (4) and (8), and use the expression for $P^r_{t+1}$ given by (10) to rewrite equation (7') as follows.

\[ Bh^n = \beta \left(\frac{a C_t}{C_{xt}}\right) A_{xt} \Omega_{c,ht}, \quad \text{where} \quad \Omega_{c,ht} \equiv \frac{BP^r_t (1 + i_{t+1})}{(1 + \tau^c_{xt})(1 + \psi x_{i_t}) P^r_{t+1}}. \]  

(26)

The equilibrium leisure-consumption allocation will be inefficient if the private tradeoff facing each household is different from the social tradeoff considered by the benevolent government, that is, if the wedge $\Omega_{c,ht}$ in equation (26) is not equal to one. The presence of this wedge can be explained as follows. First, in the overlapping generations setting, when the benevolent government maximizes the weighted sum of utilities of all generations, $B < 1$, it can make intergenerational arrangements to achieve the socially optimal allocation. Second, the cost of financing the hiring of workers creates a gap between the real wage and the marginal revenue product of worker, represented by $(1 + \psi x_{i_t})$. Third, the term $(1 + \tau^c_{xt})$ reflects the gap between the nominal price of good $x$ facing the firms and the effective nominal price paid by the consumers resulting from the consumption tax. Fourth, as households save when young to finance their consumption when old, the ratio $P^r_t (1 + i_{t+1})/P^r_{t+1}$ captures how the purchasing power of their funds has changed over time.

Secondly, there will be a wedge between the marginal rate of substitution of good $y$ for good $x$ and the world relative price of good $y$ in terms of good $x$, $\Omega_{x,yt}$, if the tax rates on the consumption of goods $x$ and $y$ are not identical, or if the domestic interest rate $i_t$ is positive. The presence of the domestic interest rate results from the working capital requirements on production and imports.

\[ \left(1 - \frac{a}{a}\right) C_{xt} C_{yt} = \left(\frac{P^f_{yt}}{P^f_{xt}}\right) \Omega_{x,yt}, \quad \text{where} \quad \Omega_{x,yt} \equiv \frac{(1 + \tau^c_{yt})(1 + \psi_{IM} i_t)(1 + \psi_{EX} i_t)(1 + \psi x_{i_t})}{(1 + \tau^c_{xt})(1 + \psi x_{i_t})}. \]  

(27)

The intratemporal consumption allocation will be inefficient if the wedge $\Omega_{x,yt} \neq 1$.

Thirdly, comparing condition (25) in the centralized, non-monetary economy and the following balance of payments equation shows that a non-zero trade balance indicates an inefficiency in the intertemporal consumption allocation of the domestic economy.\(^{27}\)

\(^{27}\)We do not mean that international borrowing is inefficient in general, it is only for analytical convenience that by construction, there are no gains from international borrowing and lending in the social optimum, and thus the first-best level of international borrowing is equal to zero in every period in this deterministic environment. As is well-known that, in a stochastic environment, an economy can improve its economic welfare by using the external debt to smooth consumption over time.
International borrowing helps easing the constraints on exports and imports, while introducing a distortion to the intertemporal consumption allocation to the economy. The interest cost $i_{wt}F_t$ represents the resources transferred to the rest of the world from the domestic economy for its use of the liquidity provided by the international credit market.

The balance of payments equation describes the relation between the net trade flows and net capital flows of the economy, while offering little information regarding how international capital flows help facilitating exports and imports respectively. Some insight can be obtained by examining the balance sheet of the financial intermediaries. Financial intermediaries not only channel liquidity to the firms and importers to facilitate production and international trade, but also provide liquidity to the old households for their consumption purchases. These two purposes call for different quantities of liquidity, while they are related due to the intermediaries' balance sheets. This relation highlights the role of international capital flows in meeting the liquidity needs of the domestic economy. Using equations (3), (4) and (11)-(15), we obtain the liquidity allocation for financing the working capital of the firms and importers.

$$P_{xt}^f \text{ EX}_t - P_{yt}^I \text{ IM}_t = F_t (1 + i_{wt}) - F_{t+1}. \quad (28)$$

Equation (6) describes the liquidity supply to the household for their consumption expenditures.

$$D_t + e_{t-1}F_t = \psi_x \left[ \frac{P_{xt}^f C_{xt}}{1 + \psi_x t_t} \right] + \psi_{\text{EX}} \left[ \frac{e_t P_{xt}^f \text{ EX}_t}{1 + \psi_{\text{EX}} t_t} \right] + \psi_{\text{IM}} \left[ \frac{P_{yt}^I \text{ IM}_t}{1 + \psi_{\text{IM}} t_t} \right]. \quad (14')$$

Equation (29) describes the liquidity supply to the household for their consumption expenditures.

$$D_t (1 + i_t) + \Gamma_t^O = (1 + \tau_{x}^c) P_{xt}^f C_{xt} + (1 + \tau_{y}^c) P_{yt}^I C_{yt}. \quad (29)$$

Given the values of the domestic policy variables, the equilibrium liquidity flows in the domestic economy will ensure that the liquidity allocated to every economic agent is consistent with each other, resulting in the following equation,

$$e_{t-1}F_t = \psi_{\text{EX}} \left[ \frac{e_t P_{xt}^f \text{ EX}_t}{1 + \psi_{\text{EX}} t_t} \right] - \frac{1}{1 + i_t} \left\{ (1 - \psi_x) \left[ \frac{P_{xt}^f C_{xt}}{1 + \psi_x t_t} \right] + (1 - \psi_{\text{IM}}) e_t P_{yt}^I \text{ IM}_t - \left( \Gamma_t^O - T_t \right) \right\}. \quad (29)$$

where $T_t \equiv \tau_x^c P_{xt}^f C_{xt} + \tau_y^c P_{yt}^I C_{yt} = T_t^Y + T_t^O$ and $\Gamma_t^O = T_t^O$. Hence, in addition to the balance of payments equation, the presence of liquidity constraints results in a relation between the economy’s flows of exports and imports and its new international debt. The higher the fractions of payments required to be paid in advance, $\psi_x$, $\psi_{\text{EX}}$, and $\psi_{\text{IM}}$, are, the more the financial intermediary needs to borrow from the international loan market to accommodate the higher demands for working capital of firms and importers. A higher net transfer received
by the old, $\Gamma_t^O - T_t$, increases the liquidity available to the old for their acquisition of the consumption goods, leading the firms and importers to increase their demands for working capital and the financial intermediaries to borrow more from abroad.

Using equations (15), (18), (28), and (29), we have

$$e_t P_{xt}^f \text{EX}_t = (1 - \psi_{\text{EX}}) \left[ \frac{e_t P_{xt}^f \text{EX}_t}{1 + \psi_{\text{EX}} e_t} \right] + (1 - \psi_x) \left[ \frac{P_{xt} C_{xt}}{1 + \psi_{xt} t} \right] + (1 - \psi_{\text{IM}}) e_t P_{yt}^f \text{IM}_t + e_t (1 + i_{wt}) F_t + T_t^Y. \quad (29')$$

The economy uses its proceeds from exports to settle the remaining balances $(1 - \psi_{\text{EX}}) \left[ \frac{e_t P_{xt}^f \text{EX}_t}{1 + \psi_{\text{EX}} e_t} \right] + (1 - \psi_x) \left[ \frac{P_{xt} C_{xt}}{1 + \psi_{xt} t} \right] + (1 - \psi_{\text{IM}}) e_t P_{yt}^f \text{IM}_t$, to repay its international debt, $e_t (1 + i_{wt}) F_t$, and to finance its lump-sum transfer $T_t^Y = T_t - T_t^O$ to the young. The requirement of making a partial advance payment on its imports effectively prevents the economy’s current imports, $e_t P_{xt}^f \text{EX}_t$, from being paid in full by using the proceeds from its current exports, $e_t P_{xt}^f \text{EX}_t$.

The impacts of the financial frictions on the trade and capital flows can be highlighted by studying the special case in which $\psi_x = \psi_{\text{EX}} = \psi_{\text{IM}} = 1$ and no intergenerational transfer via taxes, $\mathcal{T}_t = \mathcal{T}_t^O$ and $\mathcal{T}_t^Y = 0$. In this case, $P_{yt}^f \text{IM}_t = F_{t+1}$ and $P_{xt}^f \text{EX}_t = (1 + i_{wt}) F_t$, exports and imports are linked individually to the international capital flows. The small open economy must rely on new international debt to help facilitate the imports of good $y$, while using the proceeds from exports of good $x$ to service the existing international debt.

The presence of the lump-sum transfer allows for the existence of a monetary competitive equilibrium with no international capital flows and therefore the possibility of achieving the socially optimal allocation. The lump-sum transfer to the young $\mathcal{T}_t^Y$ represents a redistribution of liquidity from the old to the young, reducing the old’s consumption expenditure and therefore reducing the need for international capital inflows to facilitate production of good $x$ and imports of good $y$. As will be shown in Section 4, in the case with $i_{wt} > 0$, to eliminate the distortion to the intertemporal allocation, the intergenerational transfer $\mathcal{T}_t^Y$ has to be set at an appropriate level to let $F_{t+1} = 0$, while the lump-sum transfer of newly printed money, $\mathcal{T}_t^m$, cannot help eliminating this inefficiency.

**Solving for the Monetary Competitive Equilibrium**

To simplify the notations, we define $m_t \equiv \mathcal{M}_t / e_t$ and $z_t \equiv P_{yt}^f C_{yt}$. Both the values of $m_t$ and $z_t$ are measured in units of the foreign currency. Recall that $\gamma_t \equiv \mathcal{T}_t^Y / \mathcal{M}_t$. As presented in Appendix B, we can derive the equilibrium values of $\mu_t$, $h_t$, $C_t$, and $F_{t+1}$ as functions of $m_t$ and $z_t$, the productivity parameter, $A_{xt}$, the policy variables, $i_t$, $\Gamma_t$, $\tau_{xt}^e$, and $\tau_{yt}^e$, the world prices, $P_{xt}^f$, $P_{yt}^f$, and $i_{wt}$, as well as the predetermined values of $F_t$ and $m_{t-1}$.

$$1 + \mu_t = \left( \frac{1 + i_t}{1 + i_{wt}} \right) \frac{m_t}{m_{t-1}}. \quad (30)$$
\[ h_t = \frac{1 + \psi_{\text{EX}}i_t}{\psi_{\text{EX}} A_{xt} P_{zt}^f} \left[ m_t - \left( \frac{a(\psi_x - \psi_{\text{EX}})(1 + \tau_{yt})}{(1-a)(1+\tau_{zt})} + \psi_{\text{IM}} \right) z_t \right], \quad (31) \]

\[ C_t = \left[ \frac{a(1 + \psi_{\text{EX}} i_t)(1 + \psi_{\text{IM}} i_t)(1 + \tau_{yt}) P_{yt}^f}{(1-a)(1+\psi_x i_t)(1+\tau_{zt}) P_{zt}^f} \right]^a z_t \quad \text{and} \]

\[ F_{t+1} = z_t \left[ 1 + \left( \begin{array}{c} \psi_x - \psi_{\text{EX}} \\ \psi_{\text{EX}} \end{array} \right) \left( \begin{array}{c} a(1 + \tau_{yt}) \\ (1-a)(1+\tau_{zt}) + \psi_{\text{IM}} - \psi_{\text{EX}} \end{array} \right) - m_t \left( \frac{1 - \psi_{\text{EX}}}{\psi_{\text{EX}} + \gamma_t} \right). \quad (33) \]

We can then solve for the equilibrium values of \( m_t \) and \( z_t \) by using (34) and (35).

\[ m_t = \frac{1}{1 + \psi_{\text{EX}} i_t - \gamma_t} \left( \begin{array}{c} 1 + \psi_{\text{IM}} i_t \end{array} \right) \left[ \frac{a(1 + \tau_{yt})}{(1-a)(1+\tau_{zt})} + 1 \right] z_t + F_t(1 + i_{wt}) \quad \text{for all } t \geq 1. \quad (34) \]

\[ F_{t+1} = z_t \left[ 1 + \left( \begin{array}{c} \psi_x - \psi_{\text{EX}} \\ \psi_{\text{EX}} \end{array} \right) \left( \begin{array}{c} a(1 + \tau_{yt}) \\ (1-a)(1+\tau_{zt}) + \psi_{\text{IM}} - \psi_{\text{EX}} \end{array} \right) - m_t \left( \frac{1 - \psi_{\text{EX}}}{\psi_{\text{EX}} + \gamma_t} \right). \quad (33) \]

With \( F_1 = 0 \), equations (30) – (35) describe the equilibrium dynamics of \( i_t, h_t, C_t, F_{t+1}, m_t, \) and \( z_t \) for all \( t \geq 1. \)

4 Stationary Monetary Competitive Equilibrium and Economic Welfare

In the following, we study the stationary competitive equilibrium in the monetary economy with liquidity constraints. Assume that the world goods prices and interest rate, the domestic productivity parameters, and the government’s policy variables are time-invariant, \( P_{zt}^f = P_{zt}^f, P_{yt}^f = P_{yt}^f, i_{wt} = i_w, A_{xt} = A, i_t = i, \gamma_t = \gamma, \tau_{zt}^c = \tau_{zt}^c, \) and \( \tau_{yt}^c = \tau_{yt}^c. \) In a stationary equilibrium, the domestic real variables and nominal variables measured in terms of the foreign currency are constant over time, that is, \( h_t = h, \ EX_t = EX, C_{zt} = C, C_{yt} = C_y, P_{yt}^f C_{yt} = z_t = z, \ P_{zt}^f / e_t = p^r, \ M_t / e_t = m, \mu_t = \mu, \) and \( F_{t+1} = F. \)

\[ \text{In a stationary equilibrium, all of the domestic nominal variables measured in terms of the domestic currency grow at the same rate as the domestic money growth rate, } \mu. \] In other words, both the domestic inflation rate and the depreciation rate of the domestic currency are equal to \( \mu. \)
4.1 Stationary Monetary Competitive Equilibrium with Liquidity Constraints

As shown in Appendix B, by applying the stationary assumption to the equilibrium conditions presented in Section 3.1, we can derive the stationary monetary equilibrium allocation of the small open economy facing liquidity constraints. Define the following variables,

\[
\Lambda_1 = \frac{A_x a \beta}{B} \left( \frac{(1-a)P_f^x}{a P_f^y} \right)^{1-a} \Omega_{ch} \Omega_{x,y}^{a-1},
\]

\[
\Lambda_2 = \frac{a \beta (1+i_w)}{(1+\psi_{EX} i)},
\]

\[
\Lambda_3 = \frac{(1+\psi_{EX} i)}{[1+\psi_{EX} i+(1-(1-\gamma)\psi_{EX})i_w]} \left[ \frac{a}{1-a} \left[ 1 - \frac{(1-\gamma)(\psi_{EX}-\psi) i_w}{(1+\psi_{EX} i)} \right] + \frac{[1+(1-\gamma)\psi_{IM} i_w]}{\Omega_{x,y}} \right] \frac{a \beta \Omega_{ch}}{B}.
\]

The stationary equilibrium values of work efforts, consumption, utility, imports, and international borrowing are given by

\[
h = \Lambda_1 \frac{\psi}{\beta},
\]

\[
C = \frac{a}{1-a} \left[ \Lambda_1^{1+\eta} \Lambda_3^{-\eta} \right]^{\frac{1}{\beta}},
\]

\[
U = \Lambda_1 \frac{1+\eta}{\beta} \left[ \Lambda_1^{1+\eta} \Lambda_3^{-\eta} \right]^{\frac{1}{\beta}},
\]

\[
z = \frac{P_f^x A_x}{(1+\tau_y^c)(1+\psi_{IM} i)} \left[ \Lambda_1 \Lambda_2 \eta \Lambda_3^{-\eta} \right]^{\frac{1}{\beta}},
\]

\[
F = \left[ \frac{(1-\gamma)\psi_{IM}(1+\psi_{EX} i) - [1-\psi_{EX} + \gamma \psi_{EX}] - \frac{a}{1-a} [1-\psi_x + \gamma \psi_x] \Omega_{x,y}}{1+\psi_{EX} i+(1-(1-\gamma)\psi_{EX})i_w} \right] z.
\]

and the stationary equilibrium value of exports in units of foreign currency is \( z + i_w F \). The economy’s foreign-currency values of imports \( z \) and international borrowing \( F \) are jointly determined by the domestic policy variables, \( i, \gamma, \tau_x^c, \) and \( \tau_y^c \), the world goods prices and interest rate, \( P_f^x, P_f^y \) and \( i_w \), and the productivity parameter, \( A_x \). The current account is zero in the steady state, however, its net foreign debt position and thus the trade account balance need not be zero. The economy runs a stationary trade surplus (deficit) to pay for the interest payments on its external debt (to spend the interest income from its external assets).\(^{29}\)

Equation (33') shows that the liquidity constraints and government policies can be a source of trade imbalance for the small open economy in this model. The sign of \( F \) depends on the sign

\(^{29}\)It is noted that both the current account and national saving equal zero in the stationary equilibrium.
of the term \[ (1 - \gamma)\psi_{\text{IM}}(1 + \psi_{\text{EX}} i) - [1 - \psi_{\text{EX}} + \gamma \psi_{\text{EX}}] - \frac{a}{1 - a} [1 - \psi x + \gamma \psi x] \Omega_{x,y}. \] Gale (1971) first demonstrates that in the presence of international lending and borrowing, general equilibrium with permanent trade imbalance is the rule rather than the exception. The present model shows how the presence of liquidity constraints results in the trade imbalance and illustrates how the direction and magnitude of the trade imbalance depend on the policy instruments in the domestic economy.\footnote{There are only real assets in Gale (1971). Fisher (1990) extends the analysis to the case with money as a store of value and shows that a country can have monetary equilibria with sustainable trade deficits. By contrast, our model emphasizes the role of the liquidity constraints in the determination of the trade balances and shows that a small open economy tends to run a sustained trade surplus when there are liquidity constraints on its economic activities.}

Consider the laissez-faire policy \( (\gamma = \mu = \tau^c_x = \tau^c_y = 0) \). The economy will have \( i = i_w \) and

\[
\text{sign } F = \text{sign} \left[ \frac{(1 + \psi_{\text{EX}} i_w)\psi_{\text{IM}} - (1 - \psi_{\text{EX}})}{(1 + \psi_{\text{IM}} i_w)} - \left( \frac{a}{1 - a} \right) \frac{(1 + \psi_{\text{EX}} i_w)(1 - \psi x)}{(1 + \psi x i_w)} \right].
\]

If the home households’ demand for imports are strong (its share of expenditure on good \( y \), \( 1 - a \), is high), and if the firms and importers are subject to high working capital requirements (the fractions \( \psi x, \psi_{\text{EX}}, \) and \( \psi_{\text{IM}} \) are high), then the home economy will need to borrow from the international credit market to help meeting its liquidity needs. The net foreign debt position \( F \) will be positive and the economy will have a sustained trade surplus.

From now on, we will focus our discussion on the cases in which the equilibrium value of \( F \) is positive. That is, the values of \( 1 - a, \psi x, \psi_{\text{EX}}, \) and \( \psi_{\text{IM}} \) are sufficiently high so that the home economy relies on international capital inflows to meet its liquidity needs, \( F > 0 \). Using the discussion in section 3.3 and equation (33'), we have the following propositions.

**Proposition 1:** In the stationary equilibrium, taking the world interest rate \( i_w \) as given, the quantity of international capital inflows needed to facilitate each unit of imports, as measured by the ratio \( F/z \), depends crucially on the domestic policy variables.

\[
\frac{F}{z} = \frac{(1 + \tau^y_x)(1 + \psi_{\text{IM}} i) (1 + \psi_{\text{EX}} i)(1 - \gamma)\psi_{\text{IM}} - [1 - \psi_{\text{EX}}(1 - \gamma)] \Omega_{x,y}}{(1 + \tau^c_y)\psi_{\text{EX}} i + (1 - (1 - \gamma)\psi_{\text{EX}} i) i_w} - \frac{a(1 + \psi_{\text{EX}} i)[1 - \psi x(1 - \gamma)]}{(1 - a)(1 + \tau^c_x)(1 + \psi x i)}. 
\]

Holding other policy variables constant, an increase in the lump-sum transfer of tax revenue distributed to the young, as captured by \( \gamma \), helps improving the economy’s efficiency in using foreign borrowing to facilitate its trade flows, \( \frac{\partial(F/z)}{\partial \gamma} < 0 \).

The quantity of international capital inflows needed to facilitate each unit of imports, measured by the ratio \( F/z \), can be interpreted as a measurement of the severity of the intertemporal distortion in consumption in the economy. For any given value of \( z \), the higher
the value of $F$, the higher the financing cost of imports, and the less efficient the economy’s intertemporal consumption allocation. Proposition 1 demonstrates the link between the international trade and capital flows operating through the liquidity allocations in the financial markets, pointing to an important role played by the domestic government in determining the efficiency of the economy’s utilization of capital inflows in facilitating its trade flows.31 Further discussions on the relationship between $F$ and $z$ are presented in Sections 4.2 and 4.3.

4.2 The Optimal Policy

We now study the optimal policy in a first-best environment where lump-sum taxes/transfers are available. For simplicity, our analysis focuses on time-invariant policy. As discussed in Section 3.3, the comparison of equations (22)-(24) with (26)-(28) identifies three kinds of distortions prevalent in the domestic economy,

- a distortion in the intratemporal leisure-consumption allocation if $\Omega_{c,h} \neq 1$,
- a distortion in the intratemporal consumption allocation if $\Omega_{x,y} \neq 1$, and
- a distortion in the intertemporal consumption allocation if $F \neq 0$.

In order to eliminate these distortions in the monetary competitive equilibrium and to achieve the socially optimal allocation discussed in section 3.2, we have three independent relations to pin down the optimal values of the four policy instruments $(i, \tau^c_x, \tau^c_y, \gamma)$.32

The domestic interest rate, $i$, and the consumption tax rates, $\tau^c_x$ and $\tau^c_y$, are chosen to eliminate the wedge between the private and social leisure-consumption tradeoff and the wedge between the marginal rate of substitution in consumption and the world relative price of goods,

$$
\Omega_{c,h} \equiv \frac{B(1 + i_w)}{(1 + \psi_x i)(1 + \tau^c_x)} = 1 \quad \text{and} \quad \Omega_{x,y} \equiv \frac{(1 + \tau^c_y)(1 + \psi_{IM} i)(1 + \psi_{EX} i)}{(1 + \tau^c_x)(1 + \psi_x i)} = 1.
$$

As there are three instruments to determine two wedges, there exists one extra degree of freedom, and one of the three instruments $i$, $\tau^c_x$, and $\tau^c_y$ can be set exogenously.

With the choices of $i$, $\tau^c_x$, and $\tau^c_y$, the lump-sum transfer to the young, measured by $\gamma$, is then chosen to achieve $F = 0$ so that the additional restriction on the values of exports

31The ratio of international capital inflows to imports, $F/z$, depends positively on $\tau^c_x$ but negatively on $\tau^c_y$ and $\gamma$. As shown in equation (28), a higher $\tau^c_x$ reduces $C_x$ but increases the export of good $x$, allowing the domestic economy to run a larger trade surplus to support a higher level of foreign debt. Using equation (29'), we can explain the following three relations. First, an increase in $\gamma$ indicates an increase in $T_Y^T$, reducing the economy’s reliance on capital inflows to facilitate its imports. Third, higher value of $\tau^c_y$ reduces $C_y$, the resulting decrease in the demand for liquidity from the importers implies a decrease in the economy’s demand for capital inflows.

32The domestic money growth rate $\mu$ is given by $(1 + i) = (1 + \mu)(1 + i_w)$.
and imports presented in equation (29') becomes irrelevant, eliminating the inefficiency from excessive international capital flows. Using equation (33'), \( F = 0 \), and \( \Omega_{xy} = 1 \), we get,

\[
\gamma = 1 - \frac{1}{a \psi_x + (1 - a)(\psi_{IM} + \psi_{EX})}.
\]

The optimal value of \( \gamma \) can be positive or negative; it serves to adjust the liquidity flows so as to give the old households the appropriate amount of liquidity for their consumption expenditure. A sufficient condition for \( \gamma > 0 \) is \( a \psi_x + (1 - a)(\psi_{IM} + \psi_{EX}) > 1 \). When the working capital requirements facing the firms and importers are sufficiently high, the domestic economy tends to use international capital inflows to help facilitating its production and trade. In order to eliminate the inefficiency from international borrowing and achieve the first-best allocation, the government should use the lump-sum transfer to redistribute the liquidity from the old to the young \( \gamma > 0 \).

It is interesting to note that the sum \( a \psi_x + (1 - a)(\psi_{IM} + \psi_{EX}) \) serves as a measure of the liquidity needs of the economy implied by the composition of the consumption basket of the domestic household. \( \psi_x \) represents the working capital requirement for the production of \( C_x \). \( \psi_{IM} + \psi_{EX} \) captures the liquidity needs for the acquisition of \( C_y \). In addition to the explicit working capital requirement on the importers, \( \psi_{IM} \), the import is subject to an implicit liquidity requirement \( \psi_{EX} \) because the economy needs to export to generate revenue so as to pay for its import ultimately. The weights of the two liquidity needs captured by \( \psi_x \) and \( \psi_{IM} + \psi_{EX} \) are governed by the expenditure shares of the two goods, \( a \) and \( 1 - a \).

For analytical convenience, we will use the monetary policy variable \( i \) to characterize different optimal policy combinations \((i, \tau^e_x, \tau^e_y, \gamma)\). With \( F_1 = 0 \), following the optimal policy can deliver the socially optimal allocation right away (there is no transitional dynamics to the steady state), allocating \((h^*, C^*, U^*)\) to each household born in period \( t \geq 1 \). We can then obtain the corresponding optimal values \( F^* \) and \( z^* \).

\[
F^* = 0, \quad \text{and} \quad z^* = (1 - a)P^f_x A_x \left[ \left( \frac{\beta}{B} \right) A_x P^f_x \left( \frac{a}{P^f_x} \right)^a \left( \frac{1 - a}{P^f_y} \right)^{1 - a} \right]^{\frac{\eta}{\eta - 1}}.
\]

The values of domestic exports and imports, measured in terms of the foreign currency, are both equal to \( z^* \). The international trade of the economy is balanced, and there is no need to use any international capital flows to facilitate the trade flows, \( F^* = 0 \).

Now we can summarize the findings in the following proposition.

---

33The socially optimal allocation requires \( F_t = 0 \), \( e_t P^f_x C_{x,t} = e_t P^f_y C_{y,t} \), and \( a P^f_y C_{y,t} = (1 - a)P^f_x C_{x,t} \). The optimal value of \( \gamma \) depends positively on the working capital requirements, \( \psi_x \), \( \psi_{EX} \), and \( \psi_{IM} \) and the domestic interest rate \( i \).
Proposition 2: The socially optimal allocation \((h^*, C^*, U^*)\) can be delivered by a wide range of first-best policy combinations, \((i, \tau_x^c, \tau_y^c, \gamma)\). Setting \(i\) independently implies that the optimal values of \(\tau_x^c, \tau_y^c, \text{and } \gamma\) are pinned down by the following conditions,

\[
\tau_x^c = 1 - \frac{B(1+i_w)}{1+\psi_x i}, \quad \tau_y^c = 1 - \frac{B(1+i_w)}{(1+\psi_{IM}i)(1+\psi_{EX}i)}, \quad \text{and } \gamma = 1 - \frac{1}{a\psi_x + (1-a)[(1+\psi_{EX}i)\psi_{IM} + \psi_{EX}]],}
\]

and the domestic money growth rate \(\mu\) is determined endogenously, \(1+i = (1+\mu)(1+i_w)\). The economy can achieve its optimal imports \(z^*\) without relying on capital inflows, \(F^* = 0\). Sufficiently high working capital requirements, \(a\psi_x + (1-a)(\psi_{IM} + \psi_{EX}) > 1\), implies \(\gamma > 0\).

Once the values of these policy variables are set optimally, the representative household attains the first-best utility level \(U^*\). For any given \(i\), the optimal choices of \(\tau_x^c, \tau_y^c, \text{and } \gamma\) are increasing in the working capital requirements.\(^{34}\) Changes in the exogenous values of \(\psi_x, \psi_{EX}, \text{and } \psi_{IM}\) result in adjustment in \(\tau_x^c, \tau_y^c, \text{and } \gamma\), while leaving the optimal utility level unaffected.

Proposition 2 implies that policy considerations such as adopting a fixed exchange rate regime or implementing the Friedman rule would simply impose an additional constraint on \(\mu\), while leaving the real allocation unchanged. Hence, we have the following two corollaries of Proposition 2.

Corollary 1: Adopting a fixed exchange rate regime by the small open economy implies that \(\varepsilon = \mu = 0\) and \(i = i_w\), and the first-best policy combination that can support the socially optimal allocation \((h^*, C^*, U^*)\) in a monetary competitive equilibrium has

\[
\tau_x^c = 1 - \frac{B(1+i_w)}{1+\psi_x i_w}, \quad \tau_y^c = 1 - \frac{B(1+i_w)}{(1+\psi_{IM}i_w)(1+\psi_{EX}i_w)}, \quad \text{and } \gamma = 1 - \frac{1}{a\psi_x + (1-a)[(1+\psi_{EX}i_w)\psi_{IM} + \psi_{EX}]].
\]

Corollary 2: In order to support the allocation \((h^*, C^*, U^*)\) as a monetary competitive equilibrium outcome, the first-best policy combination featuring the Friedman rule \((i = 0)\) must have uniform taxation of all consumption goods, \(\tau_x^c = \tau_y^c = \tau^c\). The optimal values of \(\tau^c, \mu, \text{and } \gamma\) are given by

\[
\tau_x^c = \tau_y^c = \tau^c = 1 - B(1+i_w), \quad \mu = \frac{i_w}{1+i_w}, \quad \text{and } \gamma = 1 - \frac{1}{a\psi_x + (1-a)[\psi_{IM} + \psi_{EX}]].
\]

The case with \(i = 0\) described in Corollary 2 highlights the fact that the requirement of using \(\gamma\) to direct the liquidity flows in the domestic financial market in order to address

\(^{34}\)The optimal choice of \(\tau_x^c\) is increasing in \(\psi_x\), and that of \(\tau_y^c\) is increasing in \(\psi_{EX}\) and \(\psi_{IM}\). Similarly, \(\gamma\) is an increasing function of \(\psi_x, \psi_{EX}, \text{and } \psi_{IM}\). The higher the working capital requirements, the stronger the policy actions on discouraging the consumption of the goods.
the intratemporal and intertemporal distortions is because of the quantitative constraints on exports and imports resulting from the liquidity constraints; and not because of the presence of a positive domestic nominal interest rate, \(i\).

### 4.3 Constrained Optimal Policy with \(\gamma = 0\) When \(B = 1\)

Using the first-best allocation described in Section 4.2 as a benchmark, we can now examine the more interesting and relevant second-best results when there is no wealth redistribution between the young and old via the tax rebate, \(T_t^Y = \gamma_t = 0\), for all \(t \geq 1\). The government can only make lump-sum transfers of newly printed money to the young, and all its revenue from various taxes is rebated as lump-sum transfers to the old only.

As shown in section 4.2, a non-zero \(\gamma\) is a necessary first-best policy for achieving the social optimum.\(^{35}\) With \(\gamma = 0\), \(F\) may be non-zero in the constrained, second-best allocation, we can use equations (33)-(35) to describe the transitional dynamics of the economy towards its stationary equilibrium with the international debt to imports ratio, \(F/z\), given by

\[
F/z = \frac{1}{1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}}) i_w} \left[ (1 + \psi_{\text{EX}} i) \psi_{\text{IM}} - (1 - \psi_{\text{EX}}) - \frac{a(1 - \psi_x)}{(1 - a)} \Omega_{x,y} \right].
\]

This ratio depends on the policy variables \(i\), \(\tau_x^c\), and \(\tau_y^c\). For any given \(i\), an increase in the ratio \((1 + \tau_y^c)/(1 + \tau_x^c)\) widens the wedge \(\Omega_{x,y}\) but reduces the ratio \(F/z\).

For analytical convenience, from now on, we will focus on the economic welfare of the representative household in the stationary equilibrium by setting \(B = 1\). The government chooses \(i\), \(\tau_x^c\), and \(\tau_y^c\) to maximize the stationary equilibrium welfare level of the representative household in the small open economy, \(U\). The first-order conditions give

\[
1 + \tau_x^c = \frac{(1 + i_w)}{1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}}) i_w} \left[ \frac{(1 + \psi_{\text{EX}} i)(1 - \psi_{\text{EX}}) - a(1 - \psi_x)}{(1 + \psi_{\text{EX}} i)} \right],
\]

\[
1 + \tau_y^c = \frac{(1 + i_w)}{1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}}) i_w} \left[ \frac{1 + \psi_{\text{IM}} i_w}{1 + \psi_{\text{IM}} i} \right],
\]

and

\[
i \left[ (1 - a)(\psi_{\text{EX}} - \psi_x) - \frac{(1 + \psi_{\text{EX}} i)(1 - \psi_x)}{1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}}) i_w} \right] = 0 \Rightarrow \begin{cases} i \geq 0, & \text{if } \psi_{\text{EX}} = \psi_x = 1, \\ i = 0, & \text{otherwise.} \end{cases}
\]

\(^{35}\)The first-best allocation requires \(\gamma \neq 0\), except in the case in which \((1 - a) \psi_{\text{IM}} + (1 - \psi_{\text{EX}}) + a \psi_x = 1\).
Only in the special case in which \( \psi_{\text{EX}} = \psi_x = 1 \) can the optimal \( i \) take on any non-negative value.\(^{36}\) In all other cases, the second-best optimal allocation requires the implementation of the Friedman rule. Using \( \hat{i} = 0 \), we obtain the second-best optimal tax rates \( \hat{\tau}_x \) and \( \hat{\tau}_y \),

\[
1 + \hat{\tau}_x = \frac{(1 + i_w)}{1 + (1 - \psi_{\text{EX}})i_w} [1 - (\psi_{\text{EX}} - \psi_x)i_w] \quad \text{and} \quad 1 + \hat{\tau}_y = \frac{(1 + i_w)}{1 + (1 - \psi_{\text{EX}})i_w} [1 + \psi_{\text{IM}}i_w],
\]

the corresponding equilibrium values of imports and capital flows to imports ratio,

\[
\hat{\tau} = z^* \left[ \frac{1 + \psi_{\text{IM}}i_w}{1 - (\psi_{\text{EX}} - \psi_x)i_w} \right] \frac{\hat{\tau}_x^{1+\eta}}{1 + \psi_{\text{IM}}i_w} \frac{\hat{\tau}_y^{1+\eta}}{1 + \psi_{\text{IM}}i_w},
\]

\[
\hat{F} = \frac{1}{1 + (1 - \psi_{\text{EX}})i_w} \left[ \psi_{\text{IM}} + \psi_{\text{EX}} - 1 - \left( \frac{a}{1-a} \right) \frac{(1 - \psi_x)(1 + \psi_{\text{IM}}i_w)}{1 - (\psi_{\text{EX}} - \psi_x)i_w} \right],
\]

and the optimal utility level

\[
\hat{U} = U^* \left[ \frac{1 + (1 - \psi_{\text{EX}})i_w}{1 - (\psi_{\text{EX}} - \psi_x)i_w}^{a} \left[ 1 + \psi_{\text{IM}}i_w \right]^{1-a} \right] < U^*.
\]

Recall that the first-best optimal level of international borrowing is zero, \( F^* = 0 \). With the assumption that \( i_w > 0 \), \( \hat{F} > 0 \) implies that there are distortions from international borrowing, the second-best policy fails to achieve the first-best outcome. The sufficient condition for \( \gamma > 0 \) discussed in Section 4.2, \( a\psi_x + (1 - a)(\psi_{\text{IM}} + \psi_{\text{EX}}) > 1 \), implies \([1 + (1 - \psi_{\text{EX}})i_w] < [1 - (\psi_{\text{EX}} - \psi_x)i_w]^a [1 + \psi_{\text{IM}}i_w]^{1-a} \) and therefore \( \hat{U} < U^* \). In contrast to the first-best environment in which \( U^* \) is independent of the working capital requirements, the second-best utility level \( \hat{U} \) is a decreasing function of \( \psi_x \), \( \psi_{\text{EX}} \), and \( \psi_{\text{IM}} \). The higher the working capital requirements in the production and international trade are, the higher the distortions and thus the lower the value of \( \hat{U} \) will be.

To gain some insight into the optimality of the Friedman rule in the second-best environment, we substitute equations (36) and (37) into the expressions for the two wedges, \( \Omega_{c,h} \) and \( \Omega_{x,y} \), and the ratio \( \frac{F}{z} \) to investigate how the three kinds of inefficiencies discussed in Section 3.3 may depend on the choice of \( i \).

\[
\Omega_{c,h} = \frac{1 + \psi_{\text{EX}}i + (1 - \psi_{\text{EX}})i_w}{1 + \psi_{\text{EX}}i - (\psi_{\text{EX}} - \psi_x)i_w}, \quad \Omega_{x,y} = \frac{(1 + \psi_{\text{EX}}i)(1 + \psi_{\text{IM}}i_w)}{(1 + \psi_{\text{EX}}i) - (\psi_{\text{EX}} - \psi_x)i_w}, \quad \text{and} \quad \frac{F}{z} = \frac{1}{1 + \psi_{\text{EX}}i + (1 - \psi_{\text{EX}})i_w} \left[ (1 + \psi_{\text{EX}}i)\psi_{\text{IM}} - (1 - \psi_{\text{EX}}) - \frac{a(1 - \psi_x)(1 + \psi_{\text{IM}}i_w)}{(1 - a)(1 + \psi_{\text{EX}}i) - (\psi_{\text{EX}} - \psi_x)i_w} \right].
\]

\(^{36}\)With \( \psi_{\text{EX}} = \psi_x = 1 \), the optimal value of \( \frac{F}{z} \) is equal to \( \psi_{\text{IM}} \).
In the case with \(i_w = 0\), \(\Omega_{c,h} = \Omega_{x,y} = 1\), the intratemporal distortions disappear regardless of the value of \(i\), while the severity of the intertemporal distortion depends negatively on the choice of \(i\). Given that the ratio \(F/z\) is increasing in \(i\), the second-best policy combination 
\(\hat{i} = \hat{\tau}_C^* = \hat{\tau}_y^* = 0\) can achieve the first-best optimal allocation by having \(\hat{z} = z^*\) and borrowing 
\(\hat{F} = [\psi_{IM} - (1 - \psi_{EX}) - \frac{\psi_{EX}}{1 - \psi_{EX}}(1 - \psi_x)]z^*\), resulting in \(\hat{h} = h^*, \hat{C} = C^*,\) and \(\hat{U} = U^*\).

When \(i_w > 0\), we have \(\Omega_{c,h} > 1, \Omega_{x,y} > 1, \frac{d\Omega_{c,h}}{di} < 0, \frac{d\Omega_{x,y}}{di} < 0,\) and \(\frac{d(F/z)}{di} > 0\), reflecting a tradeoff between the intratemporal and intertemporal distortions. A lower \(i\) can improve the intertemporal distortion (reduce \(F/z\)), while worsening the intratemporal distortions (increasing \(\Omega_{c,h}\) and \(\Omega_{x,y}\)). As the intertemporal distortion resulting from the international borrowing needed for importing \(z = z^*\) becomes too severe, the government finds it optimal to reduce \(z\) so that \(\hat{z} < z^*\). Setting \(\hat{i} = 0\), the second-best policy combination \((\hat{i}^*, \hat{\tau}_y^*, \hat{\tau}_C^*)\) minimizes the ratio \(F/z\) and thus the intertemporal distortion, in exchange for some intratemporal distortions as shown by the increases in the wedges \(\Omega_{c,h}\) and \(\Omega_{x,y}\). The economy achieves the second-best optimal allocation, \((\hat{h}^*, \hat{C}^*, \hat{U}^*)\). A higher \(i_w\) implies a higher welfare cost of the intertemporal distortion from external debt, with \(\hat{i} = 0\), the optimal consumption tax rates adjust, \((1 + \tau_y^*)/(1 + \tau_x^*)\) rises to induce the economy to economize on its use of international borrowing used per unit of imports, \(\frac{d(F/z)}{diw} < 0\), while worsening the intratemporal distortions, \(\frac{d\Omega_{c,h}}{diw} > 0\) and \(\frac{d\Omega_{x,y}}{diw} > 0\). These findings are summarized in Proposition 3.

**Proposition 3:** If the government is not allowed to distribute its tax revenue to the young, 
\((\gamma = 0)\), then the first-best optimal allocation \((h^*, C^*, U^*)\) will not be attainable. It will be optimal to implement the Friedman rule, \(\hat{i} = 0\), and to set the consumption tax rates as follows,
\[
1 + \hat{\tau}_C^* = \frac{(1 + \hat{i}_w)}{1 + (1 - \psi_{EX})\hat{i}_w}[1 - (\psi_{EX} - \psi_x)i_w] \quad \text{and} \quad 1 + \hat{\tau}_y^* = \frac{(1 + \hat{i}_w)}{1 + (1 - \psi_{EX})\hat{i}_w}[1 + \psi_{IM}\hat{i}_w],
\]
to achieve the second-best optimal allocation \((\hat{h}, \hat{C}, \hat{U})\) and its corresponding trade and capital flows \((\hat{z}, \hat{F})\), where \(\hat{z} < z^*\) and \(\hat{F} > 0\). Although an increase in the world interest rate has ambiguous effects on the trade and capital flows, \(z\) and \(F\), it leads the economy to economize on its use of international borrowing per unit of imports, \(\frac{d(F/z)}{diw} < 0\).

An increase in \(\psi_{IM}\) indicates a higher working capital requirement for imports. The higher financing cost of imports leads the optimal tax rate \(\hat{\tau}_y^*\) to rise to discourage consumption of good \(y\), while having no effects on the optimal tax rate \(\hat{\tau}_C^*\). In contrast, an increase in \(\psi_{EX}\) reduces the optimal level of \(\hat{\tau}_C^*\) but increases the optimal level of \(\hat{\tau}_y^*\). As the economy
faces a higher working capital requirement in its exports, it is optimal to respond in two ways. First, it devotes more resource to the production of good \( x \) for domestic consumption and encourages domestic households to increase its consumption of good \( x \) by lowering \( \hat{\tau}_c^c \). Second, as it is more costly to earn revenue from exports to pay for its imports, the economy should discourage its imports of good \( y \) by raising the tax rate \( \tau_y^c \) on the households. The intuition underlying these results is the use of differentiated consumption tax rates in directing the liquidity flows in the domestic financial market (discouraging the consumption of the good that its production/acquisition is subject to a high working capital requirement).

Contrary to the trade literature abstracting from intertemporal considerations, this result highlights the importance of the connection between intratemporal and intertemporal allocations and its implications on the effects of taxes on imports. When there are financial frictions in trade, the working capital requirements and the world interest rate play important roles in the determination of the optimal taxes on imports. The following discussion examines further the importance of the feasibility of differentiated consumption taxes.

The case with uniform consumption taxation

As presented in Proposition 3, the second-best policy is generally characterized by differentiated consumption tax rates, \( \tau_x^c < \tau_y^c \). Taxing the consumption of both goods uniformly prevents the domestic economy from achieving the second-best optimal allocation. In the case with a uniform consumption tax, \( \tau_x^c = \tau_y^c = \tau^c \), the government chooses \( i \) and \( \tau^c \) to maximize the stationary equilibrium welfare level \( U \). Appendix C shows that the optimal policy combination \( (\tilde{i}, \tilde{\tau}^c) \) is unique. The optimal interest rate \( \tilde{i} \) tends to be positive but lower than the world nominal interest rate \( i_w \). The optimal uniform consumption tax rate is given by

\[
1 + \tilde{\tau}^c = \frac{1 + i_w}{1 + \psi_{EX} i + (1 - \psi_{EX})i_w} \cdot \left[ a \left( (1 + \psi_{EX} \tilde{i}) - (\psi_{EX} - \psi_x) i_w \right) \frac{(1 - a)(1 + \psi_{IM} i_w)}{(1 + \psi_x i)} + (1 - a)(1 + \psi_{IM} i_w) \right],
\]

while the optimal \( i \) is characterized by the following equation,

\[
\left[ \frac{a \psi_x}{1 + \psi_x i} + \frac{(1 - a)\psi_{EX}}{1 + \psi_{EX} i} \right] + \left[ \frac{(1 - a)\psi_{EX}}{1 + \psi_{EX} i} \frac{a}{1 + \psi_{IM} i} \right] = \left\{ \psi_{EX} + \frac{(1 - a) \psi_{IM} (1 + \psi_{IM} i_w)}{(1 + \psi_{EX} i)^2} \left( \frac{a \psi_{EX} - \psi_x (1 - a)}{(1 + \psi_x i)^2} \right) \right\},
\]

Examining the effects of the choice of \( i \) on the three inefficiencies by applying the expression of \( \tilde{\tau}^c \) shows that the use of uniform consumption taxation alters the tradeoff between
intertemporal and intratemporal distortions faced by the government. The wedges \( \Omega_{c,h} \) and \( \Omega_{x,y} \) are now positively related to \( i, d\Omega_{c,h} / di > 0 \) and \( d\Omega_{x,y} / di > 0 \), and an increase in \( i \) tends to reduce the ratio \( F/z \) even though its effect is generally ambiguous. Hence, in order to address the intertemporal distortion, the government abandons the Friedman rule and increases the domestic interest rate, while worsening the intratemporal distortions. The optimal domestic interest rate will be positive, \( \tilde{i} > 0 \). With the optimal values of \( i \) and \( \tau^c \), the optimal value of imports, \( \tilde{z} \), international borrowing, \( \tilde{F} \), and real allocation \( \tilde{h}, \tilde{C}, \tilde{U} \) can be derived.

\[
\tilde{z} = z^* \left\{ \frac{[1 + \psi_\text{EX}\tilde{i} + (1 - \psi_\text{EX})i_w)(1 + \psi_\text{IM}\tilde{i})}{a[(1 + \psi_\text{EX}\tilde{i}) - (\psi_\text{EX} - \psi_x)i_w)](1 + \psi_\text{IM}\tilde{i}) + (1 - a)(1 + \psi_\text{IM}i_w)(1 + \psi_x\tilde{i})} \right\}^{\frac{1 + \eta}{\eta}} \cdot \left[ \frac{(1 + \psi_\text{EX}\tilde{i})(1 + \psi_\text{IM}\tilde{i})}{1 + \psi_x\tilde{i}} \right]^{\frac{\alpha(1 + \eta)}{\eta}},
\]

\[
\tilde{F}/\tilde{z} = \frac{(1 + \psi_\text{EX}\tilde{i})}{1 + \psi_\text{EX}\tilde{i} + (1 - \psi_\text{EX})i_w} \left[ \psi_\text{IM} - \frac{1 - \psi_\text{EX}}{1 + \psi_\text{EX}}\left( \frac{a}{1 - a} \right)(1 - \psi_x)(1 + \psi_\text{IM}\tilde{i}) \right].
\]

The optimal value of imports is lower than its first- and second-best optimal levels, \( \tilde{z} < \tilde{\tilde{z}} < z^* \), and the optimal utilization of international capital inflows has the following ranking, \( 0 = \frac{F^*}{z^*} < \frac{\tilde{F}}{\tilde{z}} < \frac{\tilde{F}}{\tilde{z}} \).

As illustrated in Section 4.2, the lump-sum transfer \( \gamma \) is the necessary tool for eliminating the intertemporal distortion. The domestic interest rate and consumption tax rates are the policy instruments for correcting the intratemporal distortions, and they are imperfect substitutes for \( \gamma \) in dealing with the intertemporal distortions. When \( \gamma = 0 \), the government can raise the domestic interest rate to reduce the use of international capital inflows only partially as it has to avoid worsening the intratemporal distortions too much.

For any given \( i_w > 0 \), the higher the value of \( F/z \), the less efficient the domestic economy’s utilization of capital inflows to facilitate its trade flows, and the severer the problem of excessive international borrowing. Although \( \tilde{z} < \tilde{\tilde{z}} < z^* \), we find that \( 0 = \frac{F}{z^*} < \frac{\tilde{F}}{\tilde{z}} < \frac{\tilde{F}}{\tilde{z}} \), indicating that with uniform consumption taxation, the domestic economy fails to utilize its international borrowing as efficiently as in the second-best environment, leading to a lower level of economic welfare. We can derive the following rankings \( \tilde{C} < \tilde{\tilde{C}} < C^* \), \( \tilde{h} < \tilde{h} < h^* \), and \( \tilde{U} < \tilde{\tilde{U}} < U^* \). The maximized steady-state utility level \( \tilde{U} \) is lower than the second-best level \( \tilde{U} \). This welfare ranking can be explained by the ratio of international borrowing to imports, \( F/z \).
Proposition 4: If the government has only $i$ and $\tau^c$ as its policy instruments ($\gamma = 0$ and $\tau^c = \tau^c_x = \tau^c_y$), then the optimal domestic interest rate will be positive and lower than the world interest rate. This policy mix will deliver the maximized steady-state utility level $\bar{U}$, where $\bar{U} < \bar{\bar{U}} < U^*$. The presence of uniform taxation of consumption not only reduces the economy’s imports but also worsens its efficiency in using international capital flows to facilitate its trade flows, $\tilde{z} < \tilde{\tilde{z}} < z^*$ and $0 = \tilde{F}^* < \tilde{F} < \tilde{\bar{F}}^*$.

Schmitt-Grohé and Uribe (2003) show that the Friedman rule will be optimal if the policymaker taxes all the consumption goods uniformly. In this paper, when the lump-sum transfer $\gamma$ is not feasible, the availability of consumption taxes is sufficient for the optimality of the Friedman rule only if differentiated consumption tax rates are allowed. Imposing a uniform tax rate on the consumption goods not only prevents the Friedman rule from being optimal, but also hinders the economy’s efficiency in channeling capital inflows to facilitate its trade flows.

As illustrated in Appendix C, if the government has to deal with various distortions using only the domestic nominal interest rate, there will be further changes in relationships between the domestic interest rate and the tradeoff between the intertemporal and intratemporal distortions. When $\gamma = \tau^c_x = \tau^c_y = 0$, an increase in $i$ will reduce $\Omega_{c,h}$ and increase $\Omega_{x,y}$, while having an ambiguous effect on $F/z$. As a result, it will be optimal to raise $i$ to improve the intertemporal consumption allocation and the intratemporal leisure-consumption allocation, while worsening the intratemporal consumption allocation between goods $x$ and $y$. The result is summarized in Proposition 5.

Proposition 5: If the domestic nominal interest rate is the only policy instrument available to the domestic government ($\gamma = \tau^c = 0$), the optimal domestic interest rate, $i^{**}$, will be higher than the optimal level when $\tau^c$ is feasible, $\tilde{i}$, but lower than the world interest rate, $i_w$. The constrained maximized steady state utility level will be lower than that in the case with both $i$ and $\tau^c$ used as policy instruments.

4.4 Discussions

4.4.A. The distortions from the liquidity constraints on production and international trade

In the case with cash-in-advance constraints imposed on consumption only, the young
households must save by holding domestic currency so as to finance their consumption when old, and the domestic financial intermediaries do not play any role in facilitating transactions. By simply distributing the initial stock of domestic currency to the initial old and keeping a zero money supply growth rate, the domestic government can effectively conduct an inter-generational transfer from the current or future generations to the initial old and achieve the socially optimal allocation described in Section 3.2.

When there are liquidity constraints on not only the consumers but also the firms and importers, the financial intermediaries intermediate between the savers and the borrowers, providing working capital to facilitate production, international trade, and consumption. The liquidity constraints effectively generate a linkage between the gross exports and imports flows to international borrowing that is in addition to the standard balance of payments equation. As the economy is required to make a fractional advance payment on its imports, the financial friction precludes the use of the economy’s proceeds from exports in settling the payment for its imports in full in the same period, resulting in distortions in the intratemporal consumption allocation. International borrowing can help easing the constraints, while introducing distortions to the intertemporal (intergenerational) consumption allocation. Hence, there is an essential role of the distribution of some of the government’s tax revenue to the young as lump-sum transfers in coordinating the liquidity flows in the domestic economy.

4.4.B. The crucial role of the lump-sum transfer of tax revenue to the young

In the existing literature of the optimality of the Friedman rule in overlapping generations frameworks, the generation-specific lump-sum taxes/transfers take a passive role, serving either to neutralize the wealth redistribution effect of monetary policy or to replace inflation tax as an instrument to implement the intergenerational transfers required for achieving the optimal allocation. See, for example, Gahvari (1988, 2007), Bhattacharya and Haslag (2001), Bhattacharya, Haslag, and Russell (2005), and Haslag and Martin (2007). In contrast, the lump-sum transfer, captured by $\gamma$, has an active role in the present paper. Recall that $\gamma_t \equiv T^Y_t / M_t$. As shown in Section 4.2, in order to achieve the socially optimal allocation, it is necessary to make a lump-sum transfer of tax revenue to the young, $\gamma > 0$. A positive $\gamma$ helps to fill the gap between the liquidity flows in the financial markets and allows the economy to attain the optimal intratemporal consumption allocation, while eliminating the use of international borrowing ($F = 0$) and therefore avoiding the distortions to the intergenerational consumption allocation. The government withdraws the excess liquidity from
the old via taxation and redistributes it to the young as lump-sum transfers. The liquidity withdrawn will be channeled back to the domestic credit market only at the end of the period when the young deposit their saving to the domestic financial intermediaries.

The above discussion also highlights the reason why the distortions to the intertemporal consumption allocation cannot be eliminated by using the lump-sum transfers of newly issued domestic money to the young $T_t^m = \mu_{t+1} M_t$. As the young will deposit the lump-sum transfers of newly issued money into the financial intermediaries, an increase in $\mu$ increases the supplies of liquidity to both the borrowers and savers and thus fails to address the gap between their liquidity flows.

In the absence of the lump-sum transfer of tax revenue to the young, the domestic interest rate and consumption taxation serve as its imperfect substitutes in addressing the distortion in the intertemporal consumption allocation. Given that the availability of the consumption taxes affects the impacts of the domestic interest rate on the three kinds of distortions, it also determines whether the government should increase or decrease the interest rate so as to help reducing the intertemporal distortions.

4.4.C. The heterogeneity of working capital needs among production and trading activities

In the model, although the pattern of specialization in production is assumed exogenously, the volumes of exports and imports and the trade balance are determined endogenously. By modeling explicitly the heterogeneity in the working capital requirements on domestic production, exports, and imports, we identify their individual impacts on the domestic economy’s equilibrium trade and capital flows and illustrate their distinctive roles in the determination of the optimal values of the policy variables. In contrast to the more obvious roles of $\psi_x$ and $\psi_{IM}$, the working capital requirement on the production of domestic goods for exports, $\psi_{EX}$, may seem irrelevant to the domestic consumers. However, as described in Section 4.3, our general equilibrium analysis highlights the channel in which the value of $\psi_{EX}$ determines the optimal values of the consumption tax rates $\tau_x^c$ and $\tau_y^c$ imposed on the domestic consumers and affects their optimal utility level.

4.4.D. Testable predictions of the model

Even though the model is highly stylized, it does provide some useful testable predictions. Some novel ones are as follows. First, a country’s short-term external debt to imports ratio tends to be positively related to the working capital requirements on its international
trading activities (either in the forms of cash in advance or letters of credit). Second, for countries facing similarly high working capital requirements on trade, those with higher taxes on imports tend to have lower short-term external debt to imports ratios. Third, whether a country’s short-term external debt to imports ratio is positively or negatively related to its domestic interest rate will depend on its relative level of different taxes on imports and domestic products. The empirical testing of these predictions will be left for future research.

5 Conclusions

This paper examines the effects of international capital inflows on the availability and allocation of liquidity in the domestic credit market and highlights the special role of financial intermediaries in meeting the liquidity needs of borrowers and savers by making loans and creating inside money in a small open economy inhabited by an infinite sequence of two-period-lived, overlapping generations. Using a “generation” to capture the entry and exit of firms from credit markets, this set up allows for the distributional effects on policies that are absent in a representative agent economy. The liquidity constraints generate a tradeoff between the intratemporal and intertemporal distortions, resulting in a positive relation between international trade flows and international borrowing that is in addition to the standard balance-of-payments equation. As this relation depends on policy variables, the domestic government chooses optimal policy to address these distortions, affecting not only the need but also the efficiency of the economy’s use of international borrowing in facilitating its trade flows. This specific international dimension of policy making has not been studied in the literature.

In a second-best environment in which the lump-sum transfers of tax revenue to the young are not allowed, the optimal policy combination requires the implementation of the Friedman rule and differentiated consumption taxes, taxing the imported goods at a rate higher than the one on the domestically produced goods. In comparison to the case with uniform consumption tax, despite of a higher tax rate on the imported goods, the optimal policy combination not only promotes the economy’s trade flows (both exports and imports) but also reduces the ratio of international borrowing to imports. The results shed new light on the connection between intertemporal and intratemporal trade in determining the optimality of the Friedman rule and its association with taxes on imports.
Figure 1: The Timing of Events in the Small Open Economy in Period $t$.

\textbf{Period $t$}

\begin{itemize}
\item Financial intermediaries enter the period with funds carried over from last period.
\end{itemize}

\begin{itemize}
\item $D_t + e_{t-1}F_t = M_t$.
\end{itemize}

\begin{itemize}
\item The young households (workers and entrepreneurs) are born.
\end{itemize}

\begin{itemize}
\item Members of each young household separate.
\end{itemize}

\begin{itemize}
\item The current state of the world is given by $(A_{xt}, P_{xt}^f, P_{yt}^f, i_{yt+1})$.
\end{itemize}

\begin{itemize}
\item The government announces its policies with commitment, $(T^m_t, T^{O}_t, \tau^c_{xt}, \tau^c_{yt})$.
\end{itemize}

\begin{itemize}
\item Intermediaries allocate loans across domestic borrowers (entrepreneurs and importers).
\end{itemize}

\begin{itemize}
\item $M_t = b_{xt} + b_{yt}$.
\end{itemize}

\begin{itemize}
\item Entrepreneurs operate domestic firms and hire workers to produce good $x$.
\end{itemize}

\begin{itemize}
\item Domestic firms export good $x$ to and importers import good $y$ from the world goods markets.
\end{itemize}

\begin{itemize}
\item The old buy goods in domestic goods markets using cheques drawn on their deposit accounts.
\end{itemize}

\begin{itemize}
\item $D_t(1 + i_{dt}) + T^{O}_{t} = (1 + \tau^c_{x})P_{xt}C_{xt} + (1 + \tau^c_{y})P_{yt}C_{yt}$.
\end{itemize}

\begin{itemize}
\item The young save all their income by making deposits to the intermediaries.
\end{itemize}

\begin{itemize}
\item $W_t h_t + T^m_t + T^Y_t = D_{t+1}$.
\end{itemize}

\begin{itemize}
\item Intermediaries determine their new international borrowing, $F_{t+1}$.
\end{itemize}

\begin{itemize}
\item The foreign exchange market is open.
\end{itemize}

\begin{itemize}
\item $e_t F_{t+1} + e_t P_{xt}^f EX_t = e_t F_t(1 + i_{wt}) + e_t P_{yt}^f IM_t$.
\end{itemize}

\begin{itemize}
\item Cheques are cleared; and loan repayments are made.
\end{itemize}

\begin{itemize}
\item $(b_{xt} + b_{yt})(1 + i_t) = D_t(1 + i_{dt}) + e_t F_t(1 + i_{wt})$.
\end{itemize}

\begin{itemize}
\item Intermediaries have funds collected from domestic depositors and borrowed from abroad.
\end{itemize}

\begin{itemize}
\item $D_{t+1} + e_t F_{t+1} = M_{t+1}$.
\end{itemize}

\textbf{Period $t+1$}

\begin{itemize}
\item The same sequence of events described above recurs.
\end{itemize}
Appendix

A. The Socially Optimal Allocation in a Centralized, Non-monetary Economy

We can rewrite the optimization problem of the benevolent government as follows.

$$\max_{\{L_xt,C_xt,C_yt,F_t+1\}} (1 - B) \left\{ - \beta C_xt^a C_yt^{1-a} + \sum_{t=1}^{\infty} B^t \left[ - \frac{(L_xt)^{1+\eta}}{1+\eta} \right. \right.$$  
$$+ \left. \zeta_{it} \left( A_xt L_xt + F_{t+1} - C_xt - p_{yt} C_yt - F_t (1 + r_{wt}) \right) + \zeta_{2t} F_{t+1} \right\}$$

where $$\zeta_{it}, i = 1, 2$$ are the multipliers associated with the constraints. Using the first-order conditions derived from this decision problem, we can obtain the optimal conditions (22), (23), and (24).

When $$A_xt$$, $$p_{yt}$$, and $$r_{wt}$$ are time invariant, the optimal allocation ($$L^*_x, C^*_x, C^*_y$$) is given by

$$F^* = 0, \quad h^* = L^*_x = \left[ A_x a^a \left( \frac{1-a}{p_y} \right) \frac{1}{B} \right]^{\frac{1}{\theta}}, \quad C^*_x = a A_x h^*, \quad C^*_y = \left( \frac{1-a}{p_y} \right) A_x h^*,$$

$$C^* = a^a \left( \frac{1-a}{p_y} \right)^{1-a} A_x h^*, \quad \text{and} \quad U^* = \left[ A_x a^a \left( \frac{1-a}{p_y} \right) \frac{1}{B} \right]^{\frac{1+a}{\eta}} \left[ - \frac{1}{1+\eta} + B \right].$$

It can be shown that $$h^*|_{B=1} < h^*|_{B=\beta}, C^*|_{B=1} < C^*|_{B=\beta}, U^*|_{B=1} < U^*|_{B=\beta} \quad \text{and} U^*|_{B=1} > U^*|_{B=\beta}.$$

B. The Monetary Competitive Equilibrium

With the definitions of $$m_t, z_t, \gamma_t$$, we derive equations (30)-(35) as follows. Equation (30) is derived from equation (15). Equation (31) is obtained by substituting equations (3), (4), (8), (11), and (12) into equation (19). Using $$C_t = C_xt^a C_yt^{1-a}$$ and equations (4), (8), and (12), we can get equation (32). Substituting equations (14'), (19), and (28) into equation (29') yields equation (33). We derive equation (34) from equations (9), (13), and (19). Finally, we get equation (35) by substituting equations (4), (8), (11), and (31) into equation (7').

The Stationary Monetary Competitive Equilibrium with Liquidity Constraints

Applying the stationary assumption, we have $$\Omega_{c,h} = \frac{B \psi (1+i_{m})}{(1+\psi_{z})(1+i_{z})}$$ and $$\Omega_{x,y} = \frac{B \psi (1+i_{m})(1+\psi_{m})(1+\psi_{x})(1+\psi_{z})(1+i_{z})}{(1+\psi_{z})(1+\psi_{x})}$$, obtain $$(1+i) = (1+\mu)(1+i_{w})$$ from equation (30), and can solve for the stationary equilibrium value of $$z$$ by substituting equations (33) and (34) into equation (35). We can then derive the equilibrium values of $$h, C, F,$$ and $$m$$ by using (31)-(34).

C. The Benevolent Government’s Maximization of the Representative Household’s Lifetime Utility ($$B = 1$$) in a Decentralized, Monetary Economy with Liquidity Constraints without Lump-sum Transfers of Tax Revenue ($$\gamma = 0$$)

$$U = \Lambda_1 \frac{1+a}{\eta} \left[ - \frac{1}{1+\eta} + \frac{\beta a}{1-a} \frac{1}{\Lambda_3} \right] = \Lambda_1 \frac{1+a}{\eta} \left[ \frac{\beta a (1+\eta)}{(1-a) \Lambda_3} - 1 \right],$$

$$dU = \Lambda_1 \frac{1+a}{(1-a) \Lambda_3} \left[ - \frac{1}{\eta} \left( 1 - \frac{(1-a) \Lambda_3}{\beta a} \right) \frac{d \Lambda_1}{\Lambda_1} + \frac{d \Lambda_1}{\Lambda_1} - \frac{d \Lambda_3}{\Lambda_3} \right].$$
where

$$\frac{d\Lambda_1}{\Lambda_1} = -\frac{a d\tau_x^c}{(1 + \tau_x^c)} - \frac{(1 - a)d\tau_y^c}{(1 + \tau_y^c)} - (1 - a) \left[ \frac{\psi_{\text{IM}}}{(1 + \psi_{\text{IM}}i)} + \frac{\psi_{\text{EX}}}{(1 + \psi_{\text{EX}}i)} \right] - \frac{a \psi_x}{(1 + \psi_x i)},$$

$$\frac{d\Lambda_3}{\Lambda_3} = \frac{\psi_{\text{EX}}}{(1 + \psi_{\text{EX}} i)} - \frac{\psi_{\text{EX}}}{(1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}})i_w)} - \frac{\psi_x}{(1 + \psi_x i)}$$

$$+ \frac{a}{1-a} \left[ \frac{(\psi_{\text{EX}} - \psi_x)i_w}{(1 + \psi_{\text{EX}} i)} \right] \frac{d\tau_x^c}{(1 + \tau_x^c)} + \frac{a}{1-a} \left[ 1 - \frac{(\psi_{\text{EX}} - \psi_x)i_w}{(1 + \psi_{\text{EX}} i)} \right] + \frac{a}{1-a} \left[ \frac{(\psi_{\text{EX}} - \psi_x)i_w}{(1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}})i_w)} \right].$$

$$\left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] |_{i=0} > 0, \quad \text{and} \quad \left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] |_{i=i_w} < 0.$$

With $\gamma = 0$, when the government can choose $\tau_x^c$, $\tau_y^c$ and $i$, we have

$$\frac{dU}{d\tau_x^c} = 0 \quad \Rightarrow \quad [\beta a(1 + \eta) - (1 - a)\Lambda_3] = \eta \beta a \left[ 1 - \frac{1}{1-a} \left[ 1 - \frac{(\psi_{\text{EX}} - \psi_x)i_w}{(1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}})i_w)} \right] \right].$$

$$\frac{dU}{d\tau_y^c} = 0 \quad \Rightarrow \quad [\beta a(1 + \eta) - (1 - a)\Lambda_3] = \eta \beta a \left[ 1 - \frac{1}{1-a} \left[ 1 - \frac{(\psi_{\text{EX}} - \psi_x)i_w}{(1 + \psi_{\text{EX}} i + (1 - \psi_{\text{EX}})i_w)} \right] \right].$$

$$\frac{dU}{d\tau_x^c} = \frac{dU}{d\tau_y^c} = 0 = \left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] = \left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right]$$

$$\left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] |_{i=0} > 0, \quad \text{and} \quad \left. \frac{dU}{d\Lambda_1} \right|_{i=0} > 0 \quad \text{and} \quad \left. \frac{dU}{d\Lambda_3} \right|_{i=0} < 0 \quad \Rightarrow \quad 0 < \hat{i} < i_w.$$

Using $\frac{dU}{d\tau_x^c} = \frac{dU}{d\tau_y^c} = 0$, and $\psi_{\text{EX}} \neq \psi_x$, we have the optimal interest rate, $\hat{i}$,

$$\frac{dU}{d\tau_x^c} = \frac{dU}{d\tau_y^c} = 0 = \left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] = \left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right]$$

$$\left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] |_{i=0} > 0, \quad \text{and} \quad \left. \frac{dU}{d\Lambda_1} \right|_{i=0} > 0 \quad \text{and} \quad \left. \frac{dU}{d\Lambda_3} \right|_{i=0} < 0 \quad \Rightarrow \quad 0 < \hat{i} < i_w.$$

With $\gamma = 0$, the government chooses $\tau_x^c = \tau_y^c = \tau^c$ and $i$. We get the optimal values, $\hat{\tau}^c$ and $\hat{i}$, from

$$\frac{dU}{d\tau^c} = 0 \quad \Rightarrow \quad \beta a = (1 - a)\Lambda_3,$$

$$\frac{dU}{di} = \frac{\beta a\Lambda_1}{(1 - a)\Lambda_3} \left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] > 0, \quad \text{and} \quad \left. \frac{dU}{d\Lambda_1} \right|_{i=i_w} < 0 \quad \Rightarrow \quad 0 < \hat{i} < i_w.$$

With $\gamma = \tau^c = 0$, the government can choose $i$ only. We obtain the optimal interest rate, $i^{**}$, from

$$\frac{dU}{di} = \frac{\beta a\Lambda_1}{(1 - a)\Lambda_3} \left[ \frac{1}{\eta} \frac{1 - (1 - a)\Lambda_3}{\beta a} \right] \frac{d\Lambda_1}{di} + \left[ \frac{1}{\Lambda_1} \frac{d\Lambda_1}{di} - \frac{1}{\Lambda_3} \frac{d\Lambda_3}{di} \right] = 0,$$

$$1 < \frac{(1 - a)\Lambda_3}{\beta a}, \quad \left. \frac{dU}{di} \right|_{i=0, \tau^c=0} > \left. \frac{dU}{di} \right|_{i=0} > 0 \quad \text{and} \quad \left. \frac{dU}{d\Lambda_1} \right|_{i=i_w} < 0 \quad \Rightarrow \quad 0 < \hat{i} < i^{**} < i_w.$$
References

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