International Outsourcing, Exchange Rates, and Monetary Policy

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Abstract

Firms’ decisions to outsource the production of intermediate inputs abroad depend on the macroeconomic environment set by governments’ monetary and foreign exchange policies, while the relocations of production have important implications on the liquidity demands in the financial markets, which in turn affect the policy effectiveness. This paper constructs a two-country, monetary model with segmented financial markets to incorporate the microeconomic foundations of firms’ make-or-buy decisions and highlight the working capital needs of both of the buyers and suppliers of intermediate inputs. The interdependence of firms’ sourcing decisions and governments’ conducts of policies are examined by identifying the endogenous adjustments of international outsourcing at both the extensive and intensive margins. It shows that the adjustments at the extensive margin can alter qualitatively the impacts of a currency revaluation and help explaining the perverse effect on the trade balance. The adjustments at the intensive margin demonstrate how firms’ sourcing decisions and payment arrangements act to dampen the effects of monetary shocks.

Keywords: International Outsourcing, Liquidity Constraints, Monetary Policy, Currency Revaluation.

JEL Classification: E44, F41

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1 Introduction

International outsourcing has become an important phenomenon in globalization. Domestic firms outsource to unaffiliated foreign suppliers to take advantage of lower costs of labor and intermediate inputs abroad. The effects of these international transactions on the flows of goods and labor have been studied extensively in the theoretical literatures on international trade and labor economics. However, the financial dimension of the transactions has been largely neglected; and the implications of international outsourcing for the macroeconomy and for the conducts of monetary and foreign exchange policies have received relatively little attention. The relocation of production of intermediate inputs affects the liquidity demands in the domestic and foreign loan markets. Monetary and foreign exchange policies influence the availability of liquidity in the financial markets. It is important for the firms to understand how these policies affect their tradeoffs between in-house production and sourcing abroad. It is also crucial for the policymakers to recognize the impacts of the presence of international outsourcing on the transmission channels of their policies. The investigation of this interdependence would provide new insights into the effectiveness of monetary policy. It also helps understanding the perverse effect of currency revaluations on the trade balance.

This paper incorporates the microeconomic foundations of firms’ make-or-buy decisions into a two-country, monetary model with segmented financial markets to highlight two key features of international outsourcing. First, a firm’s decision to use an imported intermediate input is optional and sensitive to the economic environment. Second, firms’ make-or-buy decisions determine not only the locations of production of the intermediate inputs but also the loan markets to which the intermediate good producers seek external financing for their working capital needs.

To emphasize that outsourcing is not necessary but optional, the intermediate inputs produced domestically and abroad are assumed to be perfect substitutes in the model. By incurring a fixed

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1 As reported in the World Bank’s International Trade Statistics 2013, the share of intermediate goods in world non-fuel exports was equal to 55% in 2011. Although the measures of international intermediate trade do not allow for a distinction between arm’s-length and intra-firm trade, Lanz and Miroudot (2011) analyze the intra-firm trade statistics of the United States in 2009 and report the shares of arm’s length transactions in US exports and imports of intermediate goods to be 71.3% and 51.8%, respectively.

2 Spencer (2005) and Helpman (2006) survey the theoretical literature that combines trade and the organizational choices of firms to provide insights into the forces driving international outsourcing. A recent review of the international trade literature on multinational firms has been presented in Antrás and Yeaple (2013). Feenstra and Hanson (2001) provide a detailed discussion of the impacts of trade in intermediate inputs on wages and employment. Some examples of recent theoretical studies are Antrás, Garicano, and Rossi-Hansberg (2006), Baldwin and Robert-Nicoud (2007), and Holmes and Thornton Snider (2011) analyzing the wage effects, and Keuschnigg and Ribi (2009) and Koskela and Stenbacka (2009, 2010) examining the employment effects of outsourcing.
cost of international outsourcing, a domestic firm can import the intermediate input at a lower unit cost. Depending on their productivity levels in producing the final good, some domestic firms prefer international outsourcing to producing their intermediate inputs in-house. Hence, firms’ reliance on the imported intermediate inputs is endogenously determined, the economy can adjust its use of the imported intermediate inputs not only at the intensive margin (the changes in the quantities demanded for imports of the firms that have already been sourcing from abroad) but also at the extensive margin (the changes in the number of firms entering in outsourcing arrangements). Focusing on the adjustment in the intensive margin, the model shows how the effects of temporary monetary policy shocks are weakened by the presence of international outsourcing. Understanding the adjustments at the extensive margin in response to permanent policy changes such as exchange rate revaluations provides new insights into the relationship between firms’ endogenous sourcing decisions and a country’s trade balance.

In order to highlight the impacts of international outsourcing on the demands for liquidity in financial markets, we assume cash-in-advance constraints and segmented financial markets to model the role of financial flows in facilitating the flows in goods and labor. The cash-in-advance assumption highlights the liquidity services provided by money. Financial market segmentation implies asymmetric access to liquidity by different market participants; financial intermediaries channel funds collected from depositors to provide working capital for production and international trade. The financial frictions are important in affecting the relative unit cost of intermediate inputs between the two locations. Our general equilibrium framework demonstrates how the domestic country’s output of final good is affected by the interactions of the supplies and demands of loanable funds in both the domestic and foreign loan markets.

To keep the model simple, we assume that labor is the only primary factor of production in the world economy, and that production fragmentation occurs in the final good sector of the domestic country only. Domestic firms can choose between domestic in-house production or international outsourcing. We rule out domestic outsourcing and foreign integration by construction so as to focus on the implications of firms’ make-or-buy decisions on the allocations in the labor and financial markets of both countries. The outsourcing relationship involves the domestic firms outsourcing some tasks to arm’s length firms in the foreign country, referred to as the production of the intermediate good for convenience. The value added by foreign labor to the domestic production is

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3The assumptions that support this construction will be discussed in Section 3. The choice between in-house production and domestic outsourcing has no impact on the foreign labor market. Foreign integration can affect the domestic and foreign labor markets, but the intra-firm trade does not give rise to changes in the external financing of the buyers and suppliers of intermediate goods in the domestic and foreign financial markets.
therefore captured by the value of the domestic economy’s imports of intermediates.\footnote{The trade in intermediates in this model is defined in a broader sense to capture not only the trade in intermediate goods and services, but also the trade in tasks introduced by Grossman and Rossi-Hansberg (2008) to describe the value added contributed by the factors of production in different locations.}

The results are summarized as follows. First, under a fixed nominal exchange rate regime, the effects of monetary shocks depend not only on the presence of international outsourcing activities but also on the contractual upfront payment arrangements between the domestic (source) firms and their foreign suppliers. The domestic firms’ decisions to outsource shift the financing of the working capital required for the production of the intermediate input from the domestic loan market to the foreign loan market. The upfront payment arrangement determines the foreign suppliers’ reliance on the foreign loan market in meeting their working capital needs, affecting the responsiveness of liquidity demands to interest rates. With a low upfront payment for the intermediate input, domestic production will be less sensitive to the liquidity shocks in the domestic loan market, so that the effectiveness of the domestic country’s monetary policy will be dampened.

Second, a foreign currency revaluation leads more domestic firms to outsource to foreign firms and results in an improvement in the foreign country’s trade balance. The general equilibrium adjustment mechanism is the reason behind these counter-intuitive results. When there is no international outsourcing, an increase in the value of foreign currency results in reductions (increases) in the domestic (foreign) households’ demands for imported consumption goods, deteriorating the foreign country’s trade balance. With international outsourcing, there are additional effects via the adjustments of trade in intermediates. A foreign currency revaluation leads the domestic firms that have been outsourcing abroad to adjust at the intensive margin by reducing their intermediate imports. As their production of the domestic consumption good decreases, the domestic price of the domestic consumption good rises substantially, leading some domestic firms to switch from producing their intermediate inputs in-house to outsourcing abroad. The adjustment of the imports of intermediates at the extensive margin plays a dominant role in determining the trade flows, resulting in a perverse effect on the trade balance.

Third, a reduction in the fixed cost associated with outsourcing makes the foreign country better off and the domestic country worse off under both flexible and fixed exchange rate regimes. Given that a revaluation of the foreign currency benefits the domestic country and makes the foreign country worse off, it can be used as a policy tool to redistribute some of the welfare gain of the foreign country to the domestic country so that both countries can benefit from a reduction in the fixed cost of international outsourcing and attain a higher aggregate welfare level. This result
highlights the asymmetry in the welfare effects of international outsourcing and illustrates the welfare-redistributive role of a fixed exchange rate regime when there is international outsourcing.

The remainder of the paper is organized as follows. In Section 2, we discuss the related literature and contributions of this paper. The model is presented in Section 3. Section 4 analyzes the effects of changes in some exogenous variables on the world economy. Some welfare analyses are presented in Section 5. Section 6 concludes the paper.

2 Related Literature

The international trade of intermediates and vertical specialization have been modeled in the literature of open-economy macroeconomics. Kose and Yi (2001, 2006), Ambler, Cardia, and Zimmerman (2002), Head (2002), Huang and Liu (2007), Burstein, Kurz, and Tesar (2008), and Arkolakis and Ramanarayanan (2009) have examined the roles of intermediate input trade and vertical structure of production in the propagating mechanism of international business cycles using dynamic stochastic general equilibrium models. Kollmann (2002), Huang and Liu (2006), and Shi and Xu (2007) examine the optimal monetary policy in the presence of intermediate input trade. Devereux and Engel (2007) study the desirability of flexible exchange rate in a two-country model with intermediate goods. This paper contributes to this macroeconomic literature by focusing on trade in intermediates via international outsourcing and emphasizing the role of financial flows in facilitating production and trade. First, the models in the literature make trade in intermediates necessary by assuming that each type of intermediate input is produced exclusively by the firms of one country, and that both the domestic and imported intermediates must be used in the production of final goods. In contrast, this paper allows the use of the intermediate inputs to be determined endogenously and adjusted in both the intensive and extensive margins, providing a better understanding of the macroeconomic implications of individual firms’ intermediate input sourcing decisions. Second, the financial aspects of the transactions are often omitted in this literature. Our study demonstrates how firms’ sourcing decisions affect the demands for liquidity in financial markets for financing the production and trade of intermediate inputs and therefore play an important role in the transmission of the effects of monetary and foreign exchange policies.

\[5\text{For example, Kose and Yi (2001, 2006) assume that the elasticity of substitution between domestic and foreign intermediate goods is equal to 1.5. Ambler, Cardia, and Zimmerman (2002), Devereux and Engel (2007), Huang and Liu (2007) and Shi and Xu (2007 and 2010) assume Cobb-Douglas production functions so that the domestic and foreign intermediate inputs have a unitary elasticity of substitution. Devereux and Genberg (2007) assume a Leontief technology that imported intermediate inputs must be used in fixed proportion with domestic inputs.}\]

\[6\text{Some recent theoretical studies of gain from multinational production focusing intra-firm trade are presented by Bauer and Langenmayr (2013), Garetto (2013), Irarrazabal, Moxnes, and Opremolla (2013), and Ramondo and}\]
Our emphasis on the financial aspects of international outsourcing also contributes to two strands of the trade literature. First, as common in the literature on the organizational choices of firms, following Antràs and Helpman (2004), this paper models firms’ make-or-buy decisions as a tradeoff between the fixed sourcing cost and the variable cost of intermediate inputs. The investigation of the financial flows required to facilitate the flows of goods and labor illustrates how the tradeoff is affected by the financing costs. Our examination of the financing of the contractual upfront payments in affecting intermediate good trade in a general equilibrium framework complements the discussions of the effects of financial frictions on firms’ sourcing decisions and choices of trade modes by Antràs, Desai, and Foley (2009), Feenstra, Li, and Yu (2009), and Manova and Yu (2012). Second, the global trade collapse during 2008-2009 has raised attention to the negative impacts of domestic financial market frictions on firms’ ability to exports and on countries’ trade flows. See Manova (2010) for a detailed survey of the literature on trade and finance. This paper points out that the relative availability of liquidity in the domestic and foreign financial markets is also important in determining the international trade flows when production becomes increasingly fragmented across countries.

Some recent empirical studies find that Chinese trade flows do not respond to exchange rate movements as suggested by conventional wisdom. Marquez and Schindler (2007), Thorbecke and Smith (2010), and Cheung, Chinn, and Qian (2012) re-examine Chinese trade flows by using disaggregated data. They conclude that the rapid changing economic structure may have contributed to the unstable and perverse effects at the aggregate level. Some theoretical studies, for example, Devereux and Genberg (2007) and Dong (2012), analyze the role of intermediate input trade in the global imbalance adjustments and explain why a country’s trade surplus may not be responsive to its exchange rate. However, they cannot explain China’s accelerating increases in its trade surplus along with its currency revaluations. This study offers a theoretical framework to rationalize the puzzling positive correlation between China’s currency value and its trade surplus. Taking into account the rise of China as a major host country of international outsourcing and considering China as the foreign country in our model, we find that firms’ endogenous sourcing decisions are crucial in generating different impacts of currency revaluations on the trade balances of the consumer goods and intermediate goods and the overall trade balance. Our finding sheds light on the continuing

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7 See Spencer (2005) and Helpman (2006) for a detailed survey of this literature.
8 Lanz and Miroudot (2011) find that during the period of 2008-2009, the United States experienced a larger decline in its imports of intermediate goods than in its imports of final goods.
9 See Cheung, Chinn, and Qian (2012) for a detailed survey.
improvements in China’s trade balance in intermediate goods in spite of the substantial increases in its currency value in recent years. In addition, the prediction of having negative welfare impacts on its own economy, in spite of improvements in its trade balance, offers an explanation for China’s resistance to the external pressure to revalue its currency.

3 The Model

Consider a world economy consisting of two countries, home and foreign. All foreign variables and parameters will be indexed with asterisks (*). There are two final (consumer) goods, goods \( x \) and \( y \), one intermediate input, referred to as good \( I \), and one primary factor of production, labor. In each country, there are a monetary authority and a measure one of ex-ante identical, infinitely-lived, multi-member households. Each household consists of five members: a shopper, an entrepreneur, a worker, a financial intermediary, and an importer. Household members separate and conduct different tasks in different segmented markets during a period, while pooling their income and consumption at the end of the period.\(^{10}\) All markets are perfectly competitive so that everyone acts as a price taker. To introduce money to the world economy, all transactions are subject to cash-in-advance constraints, and payments must be made in terms of the sellers’ currency. Shoppers need cash to pay for their consumption purchases. Entrepreneurs and importers need working capital to facilitate their activities because of the timing frictions between the payments of operating costs and receipts from sales. All loans are intermediated through financial intermediaries.

3.1 Preferences

The preferences of the representative home household are given by the life-time utility function,

\[
U = \sum_{t=0}^{\infty} \beta^t u(C_{xt}, C_{yt}, h_t), \quad 0 < \beta < 1, \tag{1}
\]

where the instantaneous utility function \( u(C_{xt}, C_{yt}, h_t) = a \ln C_{xt} + (1 - a) \ln C_{yt} + \nu [1 - h_t] \), and \( \beta \) is the subjective discount factor. In period \( t \), the household consumes \( C_{jt} \) units of final good \( j \), \( j = x, y \), and supplies \( h_t \) units of its worker’s labor effort to the home labor market. The expenditure share on good \( x \) is given by the parameter \( a \), \( 0 < a < 1 \).\(^{11}\) The worker’s time endowment is normalized to one, and \( \nu \) is a positive parameter measuring the constant marginal utility of leisure.

\(^{10}\) The construction of multiple-member household follows from Lucas (1990). In spite of the distinction of the agents into shoppers, entrepreneurs, workers, importers, and financial intermediaries, they pool their resources at the end of each period, allowing for the tractability and retaining the simplicity of the representative household. As will be illustrated in Section 3.5, this allows the decision making of each individual member to be nested into the household’s optimization problem.

\(^{11}\) There is home bias in consumption in the case with \( \frac{1}{2} < a < 1 \).
The representative foreign household has a similar utility function, 
\[ U^* = \sum_{t=0}^{\infty} \beta^t u^*(C^*_x t, C^*_y t, h^*_t) \]
with \( 0 < \beta^* < 1 \), \( u^*(C^*_x t, C^*_y t, h^*_t) = (1 - a^*) \ln C^*_x t + a^* \ln C^*_y t + v^*[1 - h^*_t] \), \( 0 < a^* < 1 \), and \( v^* > 0 \).

### 3.2 Technology and Production

The entrepreneurs in the home country operate the home firms specializing in producing good \( x \) following the production function,
\[ Q_{ixt} = A_{xt} \theta_{it} I_{it}^\alpha, \quad A_{xt} > 0, \theta_{it} > 0, \text{ and } 0 < \alpha \leq 1. \tag{2} \]

Given the country-wide production parameters, \( A_{xt} \) and \( \alpha \), and the firm-specific productivity parameter, \( \theta_{it} \), the entrepreneur inputs \( I_{it} \) units of intermediate good \( I \) to produce \( Q_{ixt} \) units of good \( x \). In each period \( t \), the parameter \( \theta_{it} \) is uniformly distributed on the unit interval \([\bar{\theta}, \theta]\), where \( 0 \leq \bar{\theta} < \theta \) and \( \theta - \bar{\theta} = 1 \).

Every home firm can produce its intermediate input domestically in-house following an identical, linear production technology, using one unit of labor effort to produce one unit of good \( I \).
\[ I_{it} = l_{it}, \tag{3} \]

where \( l_{it} \) is the labor input hired from the home labor market. As all home firms face the same production technology (3), there will be no domestic outsourcing (trade in intermediates among the home firms). However, the home firms have the option to import good \( I \) from the foreign country by engaging in international outsourcing, facing the unit cost of \( q^*_t \) units of foreign currency plus a real fixed cost of international outsourcing, \( \kappa \) units of good \( x \).\(^{12}\)

Each home firm is subject to a cash-in-advance constraint, it has to borrow its working capital from a home financial intermediary to finance its operation (hiring domestic workers or importing good \( I \)) in order to maximize its profit. It is noted that the decreasing returns production technology given by (2) can be interpreted as inputting \( I_{it} \) units of the intermediate good and one unit of the entrepreneur’s effort, with the income shares denoted respectively by \( \alpha \) and \( 1 - \alpha \), so that the profit is the compensation to the entrepreneur. As the entrepreneur’s own effort is not subject to a cash-in-advance constraint, the working capital need of the home firm is increasing in \( \alpha \).

Following production functions (2) and (3), if a home firm with productivity parameter \( \theta_{it} \) in the production of good \( x \) chooses to produce its intermediate input in-house, it will maximize its nominal profit \( \pi_t(\theta_{it}) \) by borrowing \( b_{it} \) units of home currency from the home financial intermediary at the nominal interest rate \( i_t \) to finance its hiring of \( l_{it} \) units of labor from the home labor market.

\(^{12}\)Assuming the fixed cost to be measured in terms of good \( x \) helps simplifying the derivation. Although models in the literature on firms’ organizational choices and outsourcing usually denote the fixed cost in terms of labor input, their assumption of an exogenously fixed real wage implies that the fixed cost can indeed be denoted in terms of the good that is used as the measurement of the real wage.
at the wage rate of $w_t$ units of home currency. The output of good $x$ will be sold at the home goods market at the price of $P_{xt}$ units of home currency.

$$\pi_t(\theta_{it}) \equiv \max_{b_{it}, l_{it}} \left\{ P_{xt} A_{xt} \theta_{it} l_{it}^\alpha + (b_{it} - w_t l_{it}) - b_{it} (1 + i_t) \right\}$$

subject to the liquidity constraint,

$$w_t l_{it} \leq b_{it}.$$  \hspace{1cm} (4)

The first-order condition of $l_{it}$ is given by

$$\alpha P_{xt} A_{xt} \theta_{it} l_{it}^{\alpha - 1} = (1 + i_t) w_t.$$  \hspace{1cm} (5)

The home firm hires workers until the marginal revenue product of labor $\alpha P_{xt} A_{xt} \theta_{it} l_{it}^{\alpha - 1}$ is equal to the effective marginal cost of labor $(1 + i_t) w_t$. Solving this condition yields the optimal level of $l_{it}$. We can then derive the firm's optimal profit, $\pi_t(\theta_{it})$.

$$l_{it} = \left( \frac{\alpha P_{xt} A_{xt} \theta_{it}}{(1 + i_t) w_t} \right)^{\frac{1}{\alpha - 1}}$$ and $\pi_t(\theta_{it}) = (1 - \alpha) P_{xt} A_{xt} \theta_{it} \left( \frac{\alpha P_{xt} A_{xt} \theta_{it}}{(1 + i_t) w_t} \right)^{\frac{\alpha}{\alpha - 1}}$.

If the home firm chooses to use the imported intermediate input to produce good $x$, it will have an outsourcing contract specifying the fraction $\xi$ of the contract payment to be paid upfront to its foreign supplier before the beginning of the production process. The remaining fraction $1 - \xi$ will be paid after the production and sale of good $x$. The parameter value of $\xi$, $0 \leq \xi < 1$, is assumed to be exogenously given.\textsuperscript{13} The smaller the value of $\xi$, the higher the liquidity burden on the foreign supplier, and the lesser the working capital that the home firm needs to borrow from the home loan market. As will be illustrated in Section 4.1, the value of $\xi$ is assumed to be small and plays a crucial role in determining the effects of various exogenous changes on the world economy.

Taking as given production function (2), the home-currency price of good $x$, $P_{xt}$, the foreign-currency price of good $I$, $q_{it}^*$, the nominal exchange rates (prices of foreign currency in terms of home currency) at the beginning of the period, $e_t$, and at the end of the period, $\hat{e}_t$, and the nominal interest rate, $i_t$, the home firm maximizes its nominal profit $\pi_{it}(\theta_{it})$ by borrowing $b_{it}^o$ units of home currency to finance the contractual upfront payment for importing $I_{it}$ units of good $I$ from a foreign supplier and paying the remaining balance $(1 - \xi) q_{it}^* I_{it}$ at the end of the period.

$$\pi_{it}(\theta_{it}) \equiv \max_{b_{it}^o, I_{it}} \left\{ P_{xt} (A_{xt} \theta_{it} I_{it}^\alpha - \kappa) + (b_{it}^o - \xi q_{it}^* I_{it}) - b_{it}^o (1 + i_t) - (1 - \xi) \hat{e}_t q_{it}^* I_{it} \right\}$$

subject to the liquidity constraint,

\textsuperscript{13}Some factors that determine the value of $\xi$ are the existence of established trading relationship and the legal protection of contract enforcement. See Section 4.1.d for a discussion on the plausible value of $\xi$ in China.
\[ \xi e_t q_t^* I_{lt} \leq b_{lt}^o. \] (6)

The presence of the real fixed cost of international outsourcing \( \kappa \) implies that the net output of good \( x \) produced by the home firm is \( A_{xt} \theta_{lt} I_{lt}^\alpha - \kappa \). The first-order condition of \( I_{lt} \) is given by

\[ \alpha P_{xt} A_{xt} \theta_{lt} I_{lt}^{\alpha-1} = (\xi (1 + i_t) e_t + (1 - \xi) \bar{e}_t) q_t^*. \] (7)

The home firm inputs \( I \) to the production of good \( x \) until the marginal benefit \( \alpha P_{xt} A_{xt} \theta_{lt} I_{lt}^{\alpha-1} \) is equal to the effective marginal cost \( (\xi (1 + i_t) e_t + (1 - \xi) \bar{e}_t) q_t^* \). Using this condition, we can derive the firm’s optimal demand for the intermediate input and profit level.

\[ I_{lt} = \left[ \frac{\alpha P_{xt} A_{xt} \theta_{lt}}{\xi (1 + i_t) e_t + (1 - \xi) \bar{e}_t} q_t^* \right]^{\frac{1}{\alpha - 1}}, \text{ and } \pi_t^o(\theta_{lt}) = (1 - \alpha) P_{xt} A_{xt} \theta_{lt} \left[ \frac{\alpha P_{xt} A_{xt} \theta_{lt}}{\xi (1 + i_t) e_t + (1 - \xi) \bar{e}_t} q_t^* \right]^{\frac{1}{\alpha - 1}} - P_{xt} \kappa. \]

The lower the fraction of upfront payment \( \xi \), the lower the firm’s working capital need \( \xi e_t q_t^* I_{lt} \), and the weaker the negative effect of an increase in \( i_t \) on its optimal demand \( I_{lt} \).

Both expressions of the optimal profits \( \pi_t(\theta_{lt}) \) and \( \pi_t^o(\theta_{lt}) \) are increasing in \( \theta_{lt} \). As will be described in Sections 3.4 and 3.5, given the timing of events, after observing its own \( \theta_{lt} \), each home firm will make its outsourcing decision before knowing the current state of the world \( s_t \). Taking into account all possible realizations of \( s_t \), the optimal cutoff level \( \theta_{lt}^o \) will be determined at where the expected values \( E[\pi_t(\theta_{lt}^o)] \) and \( E[\pi_t^o(\theta_{lt}^o)] \) are equal. Firms with \( \theta_{lt} \in [\underline{\theta}_t, \bar{\theta}_t] \) choose to produce input \( I \) in-house, while those with \( \theta_{lt} \in (\underline{\theta}_t, \bar{\theta}_t] \) prefer to source good \( I \) from the foreign country.

In the foreign country, the foreign entrepreneurs operate firms to produce final good \( y \) and/or intermediate good \( I \), using labor input following the production functions.

\[ Q_{yt}^* = A_{yt}^* l_{yt}^* \alpha^*, \quad A_{yt}^* > 0 \text{ and } 0 < \alpha^* < 1, \] (8)

and

\[ Q_{lt}^* = \phi_{lt}^* l_{lt}^* \phi_{lt}^*, \quad \phi_{lt}^* > 0. \] (9)

The output of good \( y \), \( Q_{yt}^* \), depends on labor input, \( l_{yt}^* \), and the country-wide productivity parameters, \( A_{yt}^* \) and \( \alpha^* \). The output of good \( I \), \( Q_{lt}^* \), depends on labor input, \( l_{lt}^* \), and the country-wide productivity parameter, \( \phi_{lt}^* \). A foreign firm will produce good \( I \) for a home firm if they have a contractual arrangement. The two goods are assumed to be produced by the same firm for convenience. The representative foreign firm can be considered as having two branches, one branch produces good \( y \), and another produces good \( I \). Using the production functions (8) and (9), taking as given the foreign prices of good \( y \) and good \( I \), \( P_{yt}^* \) and \( q_t^* \), the wage rate in the foreign labor

\[ \footnote{In the case with a fixed exchange rate regime, \( e_t = \bar{e}_t = \pi \), the expression \( \xi (1 + i_t) e_t + (1 - \xi) \bar{e}_t = (1 + \xi i_t) \pi \).} \]
market, $w_t^*$, the nominal interest rate on foreign-currency-denominated loans, $i_t^*$, and the fraction of upfront contract payment, $\xi_t$, the representative foreign entrepreneur optimizes by solving,

$$
\pi_t^* \equiv \max_{b_t^*, I_{It}^*} \left\{ P_{yt}^* A_{yt}^* I_{yt}^* \alpha^* + (1 - \xi_t) q_t^* \phi_t^* I_{It}^* + \left(b_t^* + \xi_t q_t^* \phi_t^* I_{It}^* - w_t^* \left(I_{yt}^* + I_{It}^*\right)\right) - (1 + i_t^*) b_t^* \right\},
$$

subject to the liquidity constraint,

$$
w_t^* \left(I_{yt}^* + I_{It}^*\right) \leq b_t^* + \xi_t q_t^* \phi_t^* I_{It}^*,
$$

(10)

where $\xi_t q_t^* \phi_t^* I_{It}^* = \xi_t Q_{It}^*$ is the upfront contract fee received from the buyers of good $I$ for supplying $Q_{It}^*$ following the production technology (9). The foreign firm’s working capital need is increasing in $\alpha^*$ but decreasing in $\xi$.

By borrowing $b_t^*$ units of foreign currency from the foreign financial intermediary and receiving $\xi_t q_t^* Q_{It}^*$ from the home firm to finance its hiring of workers, the firm will receive the sale revenue from good $y$ and the remaining contract payment from producing good $I$ and maximize its profit, $\pi_t^*$. The first-order condition for $I_{yt}^*$ is given by

$$
\alpha^* P_{yt}^* A_{yt}^* I_{yt}^* \alpha^{*-1} = (1 + i_t^*) w_t^*,
$$

(11)

the marginal revenue product and the effective marginal cost of labor are equalized, determining the optimal level of $I_{yt}^*$.

$$
I_{yt}^* = \left(\frac{\alpha^* P_{yt}^* A_{yt}^*}{(1 + i_t^*) w_t^*}\right)^{\frac{1}{\alpha^*}}.
$$

Using the first-order condition for $I_{It}^*$, we get

$$
q_t^* \phi_t^* = \frac{(1 + i_t^*) w_t^*}{(1 + \xi_t^*)}.
$$

(12)

The foreign firm will produce good $I$ only if the marginal benefit of hiring an additional unit of labor $q_t^* \phi_t^*$ is equal to the effective unit labor cost $(1 + i_t^*) w_t^*/(1 + \xi_t^*)$. The linear production technology (9) implies that $q_t^* Q_{It}^* = \left(\frac{1 + i_t^*}{1 + \xi_t^*}\right) w_t^* I_{It}^*$, the foreign firm always earns zero profit from producing good $I$, while the home firm has no incentive to engage in foreign integration.

The optimal profit of the foreign firm, $\pi_t^*$, derived from producing good $y$, is given by

$$
\pi_t^* = (1 - \alpha^*) P_{yt}^* A_{yt}^* \left(\frac{\alpha^* P_{yt}^* A_{yt}^*}{(1 + i_t^*) w_t^*}\right)^{\frac{\alpha^*}{1 - \alpha^*}}.
$$

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15 The smaller the value of the fraction $\xi$, the higher the liquidity burden the foreign supplier bears, and the more the working capital it needs to borrow from the foreign loan market.

16 The model does not preclude the domestic firms from hiring foreign labor to produce the intermediate good. Given international labor immobility, a domestic firm choosing to hire foreign labor will have to operate the production in the foreign country following the production function, $Q_{It}^* = \phi_t^* I_{It}$, to bear the working capital burden of financing the wage bill $w_t^* I_{It}^*$ solely, and to pay for the real fixed cost of importing, $\kappa$. However, as long as $\xi < 1$, this option will be dominated by simply purchasing the intermediate good from a foreign firm.
For simplicity, it is assumed that international trade in equity is prohibited.\footnote{This is not a restrictive assumption given the evidence of equity home bias.} In order to eliminate the wealth effects from the realizations of $\theta_{it}$ among the home households, every home household is assumed to own a share of every home firm so that there is complete risk-sharing within the home country. In other words, the representative household owns all firms of its country.

3.3 The Monetary Authorities

In order to study the effects of foreign currency devaluation/revaluation on the world economy, it is assumed that the monetary authority of the home country follows an exogenous monetary policy, characterized by an open market purchase $B_t$, and does not conduct any foreign exchange policy, while the monetary authority of the foreign country unilaterally maintains an exogenous, fixed nominal exchange rate. $e_t = \hat{e}_t = \tau$, by adjusting its sterilized foreign exchange sales. Under a fixed $\tau$, the foreign monetary authority can achieve an interest differential $i^*_t \neq i_t$ by restricting agents from trading assets denominated in a foreign currency to prevent arbitrage.\footnote{The modeling of the open market operations in the home loan market follows the literature on the liquidity effects of monetary policy (See Lucas (1990) and Fuerst (1992)). The modeling of the monetary policy of the foreign country tries to capture the characteristics of China’s managed fixed exchange rate regime and strong controls on capital flows. As discussed by Aizenman (2015), China’s growing trade surplus has been in tandem with its massive international reserve hoarding and sterilization. Chang, Liu, and Spiegel (2015) study China’s optimal monetary policy in a dynamic stochastic general equilibrium model that features a nominal exchange rate peg and sterilized central bank interventions.}

In order to keep the fixed rate $\tau$, the foreign monetary authority sells $Z^*_t$ units of foreign currency to the beginning-of-period foreign exchange market, and fully sterilizes its impact on the stock of foreign currency in circulation by selling $Z^*_t$ units of foreign-currency-denominated bonds, leaving the quantity of foreign currency in circulation unchanged. Similarly, it sells $\hat{Z}^*_t$ units of foreign currency to the end-of-period foreign exchange market to meet the market demand.\footnote{The derivation described here can be applied to the case with a flexible exchange rate simply by imposing $Z^*_t = \hat{Z}^*_t = 0$ and allowing $e_t$ and $\hat{e}_t$ to adjust endogenously.}

Let $M_t$ denote the aggregate money stock of the home country at the beginning of period $t$. The open market purchase of $B_t$ units of home-currency-denominated bonds increases the quantity of home currency in circulation during period $t$ to the level of $M_t + B_t$. As the foreign monetary authority uses the home currency purchased, $\tau Z^*_t$, to buy the home-currency-denominated bonds, it does not affect the home currency in circulation.\footnote{Under the fixed exchange rate regime, when the foreign country runs a trade surplus in period $t$, the foreign monetary authority has to conduct official sales of foreign currency, resulting in an increase in its end-of-period foreign exchange reserve holding by $Z^*_t (1 + i^*_t) + \hat{Z}^*_t$ units of home currency.} Given our focus on the allocations of liquidity in the financial markets, we normalize the beginning-of-period, aggregate money stock of each country to unity over time, $M_t = M_{t+1} = 1$, and $M^*_t = M^*_{t+1} = 1$, $\forall t$. At the end of period $t$, after all the bonds are redeemed, the aggregate stocks of money in circulation of the home and foreign countries...
are given by $M_t - B_t i_t - \bar{Z}_t^* (1 + i_t) - \bar{Z}_t^*$ and $M_t^* + Z_t^* (1 + i_t^*) + \hat{Z}_t^*$, respectively. Hence, the home monetary authority will distribute a lump-sum transfer of $T_t = B_t i_t + \bar{Z}_t^* (1 + i_t) + \bar{Z}_t^*$ units of home currency to each home household, and the foreign monetary authority will impose a lump-sum tax of $T_t^* = Z_t^* (1 + i_t^*) + \hat{Z}_t^*$ units of foreign currency on each foreign household.

3.4. The Timing of Information and Transactions

The timing of information and transactions are summarized in Figure 1.

<table>
<thead>
<tr>
<th>Period $t$</th>
<th>Period $t+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>allocate cash then separate</td>
<td>hold $(m_{ht+1}, m_{ft+1})$</td>
</tr>
<tr>
<td>observe $s_t$</td>
<td>observe $(\kappa, \tau)$, and choose $(n_{ht+1}, \theta_{o_t+1})$</td>
</tr>
<tr>
<td></td>
<td>finance working capital, produce and trade</td>
</tr>
<tr>
<td></td>
<td>repay loans, reunite, and consume</td>
</tr>
<tr>
<td></td>
<td>observe $\theta_{it}$, sign outsourcing contract if $\theta_{it} &gt; \theta_{it}^*$</td>
</tr>
<tr>
<td></td>
<td>hold $(m_{ht}, m_{ft})$, observe $(\kappa, \bar{\tau})$, and choose $(n_{ht}, \theta_{o_t})$</td>
</tr>
</tbody>
</table>

Figure 1: The timing of events of the representative home household

The values of $\kappa$ and $\bar{\tau}$ are assumed to be exogenously fixed and known to everyone.\(^{21}\) Entering period $t$ with cash balances $(m_{ht}, m_{ft})$, the representative home household decides on the optimal values of $n_{ht}$ and $\theta_{o_t}^*$. It deposits $n_{ht}$ units of home currency and $m_{ft}$ units of foreign currency in the home financial intermediary, allocates the remaining $m_{ht} - n_{ht}$ units of home currency to the shopper.\(^{22}\) After the household members separate and go to different markets, each home firm’s productivity parameter $\theta_{it}$ is revealed to everyone. As it takes time to make a contractual arrangement abroad, the home entrepreneur has to make the make-or-buy decision based on the comparison of $\theta_{it}$ and $\theta_{it}^*$ before observing the current state $s_t = (A_{xt}, A_{yt}^*, \phi_{It}^*, B_t)$, while knowing that $s_t$ is independently and identically distributed across time following the probability density function $G(s_t)$. The decision to outsource is irreversible and cannot be changed until the beginning of next period.\(^{23}\) However, the quantity of the intermediate input imported is assumed to be

\(^{21}\text{Changes in the fixed cost associated with international outsourcing would not occur very often. Similarly, devaluations/revaluations of a currency under a the fixed exchange rate regime are not supposed to happen frequently. Hence, any changes in these parameters will be treated as unanticipated so that the households would not take into account the possibilities of these changes when making their decisions.}

\(^{22}\text{To economize on the notations, each household is assumed to deposit to the intermediary of its own country only.}

\(^{23}\text{The assumption of the sluggish adjustment of firms’ outsourcing decision is to capture the evidence presented by}
determined by the home entrepreneur after the realization of  is revealed.

Because of the cash-in-advance constraints on transactions and financial market segmentation, financial intermediaries play an important role in allocating liquidity in the financial markets. Taking as given the nominal interest rate, , and the fixed exchange rate , the home intermediary collects the deposits from the home household, makes loans and to the home entrepreneurs and to the home importer, and purchases units of home-currency-denominated bonds so as to maximize the benefit to its depositors, facing the following liquidity constraint,

\[
n_{ht} + \bar{m}_{ft} \geq L_{xt} + L_{yt} + b_{ht}, \quad \text{where} \quad L_{xt} = \int_{\theta}^{\bar{\theta}} b_{lt} \, d\ell + \int_{\theta}^{\bar{\theta}} b_{lt}^{*} \, d\ell, \quad (13)
\]

is the total quantity of loans allocated to the home entrepreneurs as stated in equations (4) and (6). At the end of period , the intermediary receives the repayments and pays its depositors. As will be shown in Section 3.5 and Appendix A, the optimization problem of the financial intermediary can be nested into the optimization problem of the representative home household, and constraint (13) must be applied to capture the impacts of financial market segmentation.

The requirement of using the sellers’ currency for transactions implies that some economic agents have to trade for their desired currency. In the beginning-of-period foreign exchange market, the home firms outsourcing abroad sell units of home currency, the home importers sell units of home currency, and the foreign importers sell units of foreign currency. In order to pay for their remaining balances to the foreign suppliers at the end of period , the home firms outsourcing abroad arrange the purchases of units of foreign currency from the end-of-period foreign exchange market.

Labor is internationally immobile. The home worker supplies units of labor effort to the market at the nominal wage rate . For a home firm producing its intermediate input in-house, it hires units of labor from the home labor market using the home currency borrowed from the home intermediary . For a home firm engaging in international outsourcing, its liquidity needs are determined by the fraction . It borrows units of home currency and convert them into foreign currency in the foreign exchange market so as to make the upfront payment of units of foreign currency to its foreign supplier of good .

In the goods markets, the home firms sell good to the home shoppers and the foreign importers at the price of units of home currency, while the foreign firms sell good to the foreign shoppers.

\[\text{(Jabour (2013) that outsourcing is a persistent strategy.)}\]

\[\text{24 The loan market is perfectly competitive, there is no default on loans, and the financial intermediaries do not face reserve requirements. However, capital control imposed by the foreign monetary authority prevents home intermediary from holding foreign bonds, } b_{ft} = 0, \text{ and foreign intermediary from holding domestic bonds, } b_{ht}^{*} = 0.\]
and the home importers at the price of $P_{yt}^*$ units of foreign currency.

The representative home importer uses the cash balance of $L_{yt}/\bar{\epsilon}$ units of foreign currency to purchase $\text{IM}_{yt}$ units of good $y$ and then sells them to the home shopper at the price of $P_{yt}$ units of home currency so as to maximize the profit,

$$\max_{\text{IM}_{yt}} \left\{ P_{yt} \text{IM}_{yt} + \left( \frac{L_{yt}}{\bar{\epsilon}} - P_{yt}^* \text{IM}_{yt} \right) \bar{\epsilon} - L_{yt}(1 + i_t) \right\},$$

subject to the cash-in-advance constraint,

$$\frac{L_{yt}}{\bar{\epsilon}} \geq P_{yt}^* \text{IM}_{yt}. \quad (14)$$

The first-order condition indicates an equalization of the marginal cost and benefit of imports,

$$(1 + i_t)\bar{\epsilon} P_{yt}^* = P_{yt}. \quad (15)$$

Taking the home-currency prices of the final goods as given, the representative home shopper purchases both goods for consumption, facing the cash-in-advance constraint,

$$m_{ht} - n_t \geq P_{xt} C_{xt} + P_{yt} C_{yt}. \quad (16)$$

At the end of the period, repayments are made, bonds are redeemed, deposits are paid out, and profits of firms are distributed to their shareholders. After all transactions are completed, household members are reunited, pool their earnings, and consume the final goods purchased by the shopper. The home household then holds the cash balances $m_{ht+1}$ and $m_{ft+1}$ for next period,

$$m_{ht+1} = [n_{ht} + \bar{\epsilon} m_{ft} - L_{xt} - L_{yt} - b_{ht}] + [m_{ht} - n_{ht} - P_{xt} C_{xt} - P_{yt} C_{yt}] + [L_{xt} - w_t l_{xt} - \bar{\epsilon} \xi q_t^* Q_{It}] + P_{xt} Q_{xt} - \bar{\epsilon} (1 - \xi) q_t^* Q_{It} + P_{yt} \text{IM}_{yt} + b_{ht}(1 + i_t) + w_t h_t + T_t, \quad (17)$$

and

$$m_{ft+1} = \frac{L_{yt}}{\bar{\epsilon}} - P_{yt}^* \text{IM}_{yt}, \quad (18)$$

where $l_{xt} = \int q_t^* l_{it} \, di$ is the aggregate demand for labor and $Q_{xt}^\text{ih} = \int q_t^* Q_{ixt} \, di$ is the aggregate supply of good $x$ of the home firms producing their intermediate inputs in-house, $Q_{It} = \int q_t^\text{It} I_{it} \, di$ is the aggregate demand for good $I$ and $Q_{xt}^\text{os} = \int q_t^\text{os} (Q_{ixt} - \kappa) \, di$ is the aggregate supply of good $x$ (net of the total fixed cost of outsourcing) of the home firms outsourcing abroad, and $Q_{zt} = Q_{xt}^\text{ih} + Q_{zt}^\text{os}$ is the aggregate output of good $x$ of all home firms.

### 3.5 The Optimization Problem of the Representative Home Household

Given the money holdings of the representative home household at the beginning of the current period, $(m_h, m_f)$, and based on its knowledge of the probability density function $G(s)$, the household’s value function $V(m_h, m_f)$ is defined as follows.
\[ V(m_h, m_f) = \max_{n_h, \theta^o} \int C_x, C_y, h, b, L_x, L_y, L_y \max \left\{ u(C_x, C_y, h) + \beta V(m_h', m_f') \right\} G(s) ds, \]

subject to (4) – (7) and (13) – (18).

For convenience, the time subscripts of the current-period variables have been dropped, and the next-period values of the variables are denoted by primes. All of the optimal conditions are presented in Appendix A, and those for \( C_x, C_y, h, n_h \), and \( \theta^o \) are discussed below.

\[ P_x C_x = a(m_h - n_h), \quad \text{and} \quad P_y C_y = (1 - a)(m_h - n_h), \quad (19) \]

\[ \frac{\beta w}{m_h' - n_h'} = v, \quad (20) \]

\[ E \left[ \frac{u_x}{P_x} \right] = \beta E \left[ 1 + i \right] E \left[ \frac{u_x'}{P_x'} \right], \quad \text{where} \quad E \left[ \frac{u_x}{P_x} \right] = \frac{1}{m_h - n_h}, \quad (21) \]

\[ E \left[ \frac{1 - \alpha}{\alpha} \right] (1 + i) w \left[ \frac{\alpha P_x A_x \theta^o}{(1 + i) w} \right]^{\frac{1}{1 - \alpha}} = E \left[ \frac{1 - \alpha}{\alpha} \right] (1 + \xi i) q^* \left[ \frac{\alpha P_x A_x \theta^o}{\bar{P}(1 + \xi i) q^*} \right]^{\frac{1}{1 - \alpha}} - P_x \kappa. \quad (22) \]

Equation (19) gives the optimal consumption allocation of the shopper, a fraction \( a \) of the money holding is spent on good \( x \) and the remaining fraction is on good \( y \) because the two consumption goods have a unitary elasticity of substitution in the household’s preferences. Equation (20) states that the worker is willing to work if the household’s discounted expected future marginal utility gain from consuming the additional goods purchased by the nominal wage, \( \beta w \left[ \frac{1}{m_h - n_h} \right] \), is equal to the current marginal disutility of working, \( v \). The expectations in equations (21) and (22) are carried out over all possible states of the world using the probability density function \( G(s) \).

Equation (21) is the intertemporal Euler equation that gives the optimal deposit decision based on the first-order condition for \( n_h \). The household should deposit the amount \( n_h \) so as to equate the current marginal utility of consumption and the discounted expected future marginal utility from consumption purchased by using the gross return from deposits \( 1 + i \). In other words, \( n_h \) is chosen so that the shadow value of bringing an additional unit of money to the goods market for the purchase of consumption good and the shadow value of depositing an additional unit of money to the home loan market for future consumption is expected to be equal.

Equation (22) determines the optimal cutoff value of the firm-specific productivity parameter for international outsourcing, \( \theta^o \), by equating the expected profits, \( E[\pi(\theta^o)] = E[\pi^o(\theta^o)] \). International outsourcing involves a real fixed cost of \( \kappa \) units of good \( x \), while increasing the expected nominal profit by reducing the effective unit cost of the intermediate good from \( (1 + i) w \) to \( (1 + \xi i) \bar{q}^* \).

Each firm observes its \( \theta_i \) and considers either hiring \( l_i = \left[ \frac{\alpha P_x A_x \theta_i}{(1 + i) w} \right]^{\frac{1}{1 - \alpha}} \) units of labor for in-house
production or using \( I_i = \left[ \frac{\alpha P_x A_x \theta}{(1+\xi)\bar{\pi}q^*} \right]^{\frac{1}{\alpha}} \) units of imported intermediate good. Similar to the finding in the literature, as plotted in Figure 2, the existence of a fixed cost implies that only firms with higher productivity levels \( \theta_i > \theta^o \) engage in outsourcing. The number of home firms sourcing internationally is \( \bar{\theta} - \theta^o \). What is new here is the asymmetry introduced by the presence of \( \xi \) to this tradeoff; the details will be discussed below.

![Expected profits](image)

**Figure 2:** The optimal cutoff productivity level for international outsourcing, \( \theta^o \)

3.6 **Stationary Rational Expectations Equilibrium**

In the stationary rational expectations equilibrium of the world economy, the representative household of each country optimizes; and all markets clear. The detailed solution of the equilibrium is presented in Appendix B. At the beginning of each period, the money stock in each country is normalized to one. Based on the knowledge of the probability density function \( G(s) \) and the exogenous values of \( \bar{\pi} \) and \( \kappa \), the deposit decisions, \( n_h \) and \( n^*_f \), and the international outsourcing decision, \( \theta^o \), are made before the realization of \( s \). Define \( 1 - n \equiv m_h - n_h \) and \( 1 - n^* \equiv m^*_f - n^*_f \). Since \( s \) is independently and identically distributed over time, it is optimal for each home shopper to carry \( 1 - n \) units of home currency, and each foreign shopper to carry \( 1 - n^* \) units of foreign currency to the goods markets, regardless of the households’ initial cash holdings \( m_h \) and \( m^*_f \). Hence, the optimal condition for \( n \) given by equation (21) becomes \( \beta E[1 + i] = 1 \), and similarly, we have \( \beta^* E[1 + i^*] = 1 \) for the optimal \( n^* \). The equilibrium values of \( n \), \( n^* \) and \( \theta \) are jointly determined by their interactions. Households’ deposit decisions \( n \) and \( n^* \) affect the tightness of liquidity in the goods and loan markets and therefore influence the home entrepreneurs’ incentives to outsource abroad. Home firms’ productivity cutoff level for outsourcing \( \theta^o \) has opposing impacts
on the liquidity demands in the home and foreign loan markets and affects the relative market tightness, which, in turn, leads the households to adjust $n$ and $n^*$. As will be discussed in Section 4.2, permanent liquidity shocks caused by changes in $\tau$ or $\kappa$ will affect the interactions among $n$, $n^*$, and $\theta^o$ and thus have impacts on their equilibrium values.

Given $\theta^o$, $n$, and $n^*$, the realization of $s = (A_x, A^*_y, \phi^*_I, B)$ specifies the temporary liquidity shocks to the world economy and pins down the equilibrium liquidity allocation to each market participant. Table 1 summarizes the ultimate allocation of liquidity in various markets to give a clear picture of how the equilibrium real allocation depends on the liquidity allocation.

Table 1: The Equilibrium Ultimate Allocation of Liquidity in the World Economy

<table>
<thead>
<tr>
<th></th>
<th>Home Country</th>
<th>Foreign Country</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goods Markets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 - n = P_x C_x + P_y C_y$</td>
<td>Home household’s consumption spending</td>
<td>$1 - n^* = P_y^* C_y + P_y^* C_y^*$</td>
</tr>
<tr>
<td><strong>Loan Markets</strong></td>
<td>$n + B = w l_x + P_x C_y^*$</td>
<td>$n^* = w^* l_y^* + P_y^* C_y + w^* I^*$</td>
</tr>
<tr>
<td></td>
<td>producing good I in-house</td>
<td>producing good x</td>
</tr>
<tr>
<td><strong>Foreign Exchange Markets</strong></td>
<td>$P_y^* C_y - P_y C_y^* + w^* I^*$</td>
<td>$z^* \equiv Z^* + \frac{Z^<em>}{1 + i^</em>}$</td>
</tr>
<tr>
<td></td>
<td>Net private excess demand for foreign currency</td>
<td>present value sum of foreign official sales of foreign currency</td>
</tr>
</tbody>
</table>

Each shopper allocates its consumption expenditure between the two final goods. The total supply of liquidity in the home loan market $n + B$ is ultimately used for facilitating home firms’ in-house production of their intermediate inputs, $w l_x$, and foreign importers’ purchases of good $x$, $P_x C_x^*$. The supply of liquidity in the foreign loan market, $n^*$, is allocated for facilitating the foreign firms’ production of good $y$, $w^* l_y^*$, the home importers’ purchases of good $y$, $P_y^* C_y$, and the home firms’ international outsourcing activities (having the foreign firms producing good $I$ for the home country), $w^* I^* = \left(\frac{1+\xi I^*}{1+\tau^*}\right) q^* Q I^*$. The interest rates $i$ and $i^*$ and good prices $P_x$ and $P_y^*$ adjust endogenously to clear all markets and determine the equilibrium real activities.

Transactions in the foreign exchange markets under a fixed exchange rate $\tau$ result in the foreign monetary’s official intervention meeting the net private excess demand for foreign currency. The equilibrium expression of $w^* I^*$ highlights the role of the upfront contractual payment in transmitting the impacts of temporary liquidity shocks. The fraction $\xi$ indicates how the burden of working capital financing is shared between the home firm and its foreign supplier of the intermediate good,
determining the responsiveness of \( w^*l_I^* \) to changes in \( i \) and \( i^* \), respectively.

\[
w^*l_I^* = \left( \frac{1 + \xi i^*}{1 + i^*} \right)^{\frac{1}{\beta}} \left[ \frac{v^*(1 - n^*)}{\phi_i^* \beta \zeta} \right]^{\frac{1}{\beta}} \left[ \frac{\alpha P_x A_y}{z + \tilde{\phi} - \tilde{\phi}^*} \right]^{\frac{1}{\beta}} \left[ \frac{1 - \alpha}{2 - \alpha} \right]^{\frac{1}{\beta}} \left[ \frac{\tilde{\phi}^*}{\beta} - \theta_i^* \right]^{\frac{1}{\beta}} - \frac{P_y C^*_y}{\beta} = \frac{P_y C^*_y}{\beta} + P_y C_y + z^*.
\]

It also affects the present value sum of foreign official sales of foreign currency, \( z^* \equiv Z^* + \tilde{Z}^* \), and plays a key role in determining the foreign trade balance, \( TB^* \equiv P_y C_y - \frac{P_y C^*_y}{\beta} + q^*Q_I^* = Z^* + \tilde{Z}^* \).

We will show in Section 4.1 that, the lower the value of \( \xi \), the less sensitive the liquidity demand \( w^*l_I^* \) and the equilibrium response of \( TB^* \) to the home monetary and productivity shocks.

### 4 Analysis

Our analysis will proceed in three parts. First, we will study the effects of monetary and productivity shocks of the home country on the world economy, taking as given the presence of international outsourcing. Second, we will analyze the effects of devaluations/revaluations of the foreign currency on the home entrepreneurs’ outsourcing decisions, the liquidity allocations in financial markets, and the trade balances. Third, the effects of a reduction in the fixed cost associated with international outsourcing on the world equilibrium allocation of production will be examined.

#### 4.1 Effects of Different Realizations of the Current State of the World, \( s \)

Given the predetermined values of \( n, n^*, \) and \( \theta^* \), we can use two equations, \( ES_x = 0 \) and \( ES^*_{loan} = 0 \), in two unknowns, \( P_x \) and \( i^* \), to summarize the various market forces in determining the general equilibrium of the world economy for different realizations of \( s \). The derivation of the functions for the excess supply of good \( x, ES_x = Q^x_{y+x} + Q^x_{os} - C_x - C^*_x \), and the excess supply of liquidity in the foreign loan market, \( ES^*_{loan} = n^* - w^*l_I^* - w^*l_y^* - P_y C_y \) is presented in Appendix C.

\[
ES_x(P_x, i^*, B, A_x, A_y, \phi_i^*, \theta^*, n, n^*) \quad \text{and} \quad ES^*_{loan}(P_x, i^*, B, A_x, A_y, \phi_i^*, \theta^*, n, n^*)
\]

A plus or minus sign underneath an argument in each of the functions denotes the sign of its respective partial derivative. Furthermore, it is shown that the responsiveness of the excess supplies to adjustments in \( P_x \) and \( i^* \) depends crucially on the fraction of the required upfront payments, \( \xi \).

With outsourcing, \( \theta^* < \tilde{\phi} \), we have \( \frac{\partial^2 ES_x}{\partial \xi \partial P_x} < 0 \), \( \frac{\partial^2 ES^*_{loan}}{\partial \xi \partial P_x} > 0 \), \( \frac{\partial^2 ES_x}{\partial \xi \partial i^*} > 0 \), and \( \frac{\partial^2 ES^*_{loan}}{\partial \xi \partial i^*} < 0 \).

An increase in \( P_x \) lowers the demands \( C_x \) and \( C^*_x \) but increases the supplies \( Q^x_{y+x} \) and \( Q^x_{os} \) in the market for good \( x \), leading to an increase in \( ES_x \) and liquidity demands \( wIx \) and \( w^*l_I^* \). The increase in liquidity demand \( wIx \) in the home loan market results in an increase in \( i \), generating negative effects on the home country’s import \( C_y \) and the supplies \( Q^x_{y+x} \) and \( Q^x_{os} \). In the foreign loan market, the liquidity demand \( w^*l_y^* \) decreases as \( C_y \) decreases, and the liquidity demand \( w^*l_I^* \) increases as \( Q_I \).
increases. As the former effect dominates, there will be a net increase in \( ES_{loan}^* \). With outsourcing, as only a fraction \( \xi \) of the working capital is financed in the home loan market, the negative effects of an increase in \( i \) on \( Q_{os}^* \) and \( w^*l_y^* \) will be weak if \( \xi \) is small. Hence, the smaller the value of \( \xi \), the larger the increase in \( ES_x \) and the smaller the increase in \( ES_{loan}^* \) will respond to an increase in \( P_x \).

An increase in \( i^* \) reduces the demand \( C_x^* \) and the supply \( Q_{os}^* \) in the market for good \( x \). The first effect would dominate the second effect and result in a net increase in \( ES_x \). In the market for foreign loan, an increase in \( i^* \) leads to decreases in liquidity demands, \( w^*l_y^* \) and \( w^*l_y^* \), causing an increase in \( ES_{loan}^* \). The smaller the value of \( \xi \), the higher the reliance of the working capital financing for production of good \( I \) is on the foreign loan market, the smaller the increase in \( ES_x \) and the larger the increase in \( ES_{loan}^* \) will result from an increase in \( i^* \).

Different realizations of the current state \( s \) may result in \( ES_x \neq 0 \) and/or \( ES_{loan}^* \neq 0 \). As shown in Appendix C, with no international outsourcing, there is trade in final goods only, the equilibrium adjustments of \( P_x \) and \( i^* \) to different realizations of \( s \) depend on the values of \( a, a^*, \alpha, \) and \( \alpha^* \). The parameters \( a \) and \( a^* \) measure the degree of home bias in households’ preferences, determining the strengths of the liquidity flows required for facilitating international final good trade, \( P_xC_x \) and \( P_y^*C_y \). The parameters \( \alpha \) and \( \alpha^* \) represent the shares of inputs that are subject to the liquidity constraints, measuring the liquidity needs of the final good producers, \( w^*l_x \) and \( w^*l_y^* \). In the case with outsourcing, in addition to final-good trade, there is intermediate-good trade, and the equilibrium adjustments of \( P_x \) and \( i^* \) depend on not only the values of \( a, a^*, \alpha, \) and \( \alpha^* \) but also the value of \( \xi \). The fraction \( \xi \) determines the foreign firms’ reliance on the foreign loan market in meeting their working capital needs for the production of good \( I \).

**Result 1.** Under a fixed exchange rate, holding \( n, n^* \) and \( \theta^o \) constant, the transmission of the effects of temporary shocks (different realizations of \( s \)) is through not only the channel of final good trade but also the channel of intermediate good trade. International outsourcing introduces adjustments via the liquidity demand \( w^*l_y^* \) and output supply \( Q_{os}^* \), while the fraction of upfront contractual payment \( \xi \) determines the strength of these additional adjustments. For a small value of \( \xi \), the presence of international outsourcing dampens the effects of the home monetary shocks \( B \), and alters qualitatively the impacts of the home productivity shocks \( A \).

To demonstrate the role of \( \xi \), Figure 3 plots the two linearized relations between \( P_x \) and \( i^* \) that satisfy respectively equations \( ES_x = 0 \) and \( ES_{loan}^* = 0 \) to present a graphical illustration of the effects of the monetary and productivity shocks of the home country.\(^{25}\) The two linearized

\(^{25}\) As these two excess supply functions are nonlinear, we plot the linearized relations between \( P_x \) and \( i^* \) using the first-order Taylor approximations of \( ES_x = 0 \) and \( ES_{loan}^* = 0 \) around the equilibrium. The derivation is presented in Appendix C. A realization of \( s \) determines all the values of \( B, A_x, A_y^*, \) and \( \phi^*_I \). To identify the effects, our analysis
relations are both negatively sloped; the value of $\xi$ determines their relative slope and the extent they respond to the shocks. As shown in Appendix C, the linearized relation that satisfies $ES_x = 0$ will be flatter (steeper) than the one satisfying $ES^*_{loan} = 0$ if $\xi$ is small (large). How $\xi$ affects the responses of the two relations to the shocks is illustrated below.

Figure 3: The effects of temporary shocks in the home country
The solid lines show the original linearized relations between $P_x$ and $i^*$ around the general equilibrium and the dotted lines reflect the effects of the shocks on the relations. Under a fixed $\tau$ regime, the linearized relation of $ES^*_{loan} = 0$ is steeper than that of $ES_x = 0$, except when $\xi$ is sufficiently high.

(a) Monetary shock: an increase in $B$ raises $ES_x$ and reduces $ES^*_{loan}$.
$\Rightarrow$ shifting the $ES_x = 0$ relation downward and the $ES^*_{loan} = 0$ relation rightward.

(b) Productivity shock: an increase in $A_x$ raises both $ES_x$ and $ES^*_{loan}$.
$\Rightarrow$ shifting the $ES_x = 0$ relation downward and the $ES^*_{loan} = 0$ relation leftward.

4.1.a The Effects of a Larger Realization of $B$
An increase in the open market purchase of home-currency-denominated bonds, $B$, increases the liquidity supply in the home loan market and has a downward force on $i$. Holding $P_x$ and $i^*$ constant, a lower $i$ encourages the supplies $Q_{x}^{ih}$ and $Q_{x}^{os}$ and the demands $C_y$ and $I^*_l$, resulting in an excess supply of good $x$, $ES_x > 0$, and an excess demand for foreign loans, $ES^*_{loan} < 0$.

In the case with no international outsourcing, $\theta^o = \overline{\theta}$, $i$ and $P_x$ will fall, and $i^*$ will rise to clear allows only one of these exogenous shocks to vary at a time. We will discuss the effects of changes in $B$ and $A_x$, and omit the discussion on $A'_y$ and $\phi_I^*$, while their effects are summarized in Table 2.
the markets if there is home bias in consumption in each household’s preferences and the liquidity needs of the final good producers are high \((a, a^*, \alpha, \text{and } \alpha^* \text{ are sufficiently high})\). Both the direct effect of the increase in \(B\) and the indirect effects via the decrease in \(P_x\) and the increase in \(i^*\) have negative impacts on the home interest rate, \(i\), but positive impacts on the foreign price of good \(y\), \(P_y^*\), and the foreign trade balance, \(TB^*\).

In the case with international outsourcing, \(\theta^o < \overline{\theta}\), the equilibrium changes in \(P_x\) and \(i^*\) also depends on the fraction \(\xi\). When \(\xi\) is small, a decrease in \(P_x\) has a strong negative effects on \(ES_x\) and a weak negative effect on \(ES_{\text{loan}}^*\); while an increase in \(i^*\) has a weak positive effects on \(ES_x\) and a strong positive effect on \(ES_{\text{loan}}^*\). Hence, a small decrease in \(P_x\) and a small increase in \(i^*\) will be sufficient to achieve a new general equilibrium. Similarly, the effects on \(i\), \(P_y^*\), \(TB^*\) will be qualitatively similar to but quantitatively much smaller than in the case with no outsourcing.\(^{26}\)

4.1.b The Effects of a Higher Realization of \(A_x\)

If \(P_x\) and \(i^*\) remain unchanged, a higher realization of \(A_x\) will induce the home firms to expand their production and increase their demands for home-currency-denominated loans, leading to \(ES_x > 0\) and an increase \(i\). A higher value of \(A_x\) affects the foreign loan market directly by increasing \(l^*_f\), and indirectly via its upward force on \(i\) that leads to decreases in the demands \(C_y\) and \(l^*_f\). The decrease in \(w^*l^*_y + P_y^*C_y\) dominates the increase in \(w^*l^*_f\) and leads to \(ES_{\text{loan}}^* > 0\).

With no outsourcing, \(\theta^o = \overline{\theta}\), reaching a new general equilibrium requires only an equal proportional decrease in \(P_x\) to respond to an increase in \(A_x\), leaving the marginal revenue product schedules \(\alpha P_x A_x \theta_i l^*_i \alpha^{-1}\) unchanged. The home firms have no incentive to change their inputs. There are no effects on the allocation in the liquidity in each loan market and the interest rates \(i\) and \(i^*\). Given that each shopper has a constant expenditure on each good, an increase in \(A_x\) and an equal proportional decrease in \(P_x\) simply cause \(Q_x\), \(C_x\) and \(C^*_x\) to increase proportionally, while leaving the production and allocation of good \(y\) and thus \(P_y^*\), \(Q_y^*\), \(C_y\), \(C^*_y\), and \(TB^*\) unchanged.

When there is international outsourcing, \(\theta^o < \overline{\theta}\), an equal proportional decrease in \(P_x\) will restore \(ES_{\text{loan}}^* = 0\), it remains to have \(ES_x > 0\) because of the presence of the total real fixed cost of outsourcing, \(\kappa(\overline{\theta} - \theta^o)\). When \(\xi\) is small, the combination of a more than proportional decrease in \(P_x\) and an increase in \(i^*\) can help to achieve a general equilibrium. As there is a net decrease in \(P_x A_x\), the home firms reduce their liquidity demands and cause the equilibrium value of \(i\) to fall. There will be increases in \(P_y^*\) and \(TB^*\) because of the decrease in \(P_x\) and the increase in \(i^*\).

\(^{26}\)When \(\xi\) is large, clearing the markets requires a combination of an increase in \(P_x\) and a decrease in \(i^*\), while the equilibrium effects on \(i\), \(P_y^*\), and \(TB^*\) will be ambiguous.

\(^{27}\)If \(\xi\) is large, a combination of a less than proportional decrease in \(P_x\) and a decrease in \(i^*\) will help achieving the general equilibrium, implying an increase in \(i\) and decreases in \(P_y^*\) and \(TB^*\).
4.1.c The Case with a Flexible Exchange Rate Regime

It would be of interest to investigate whether the dependence of the effects of the monetary and productivity shocks on the contractual upfront payment arrangement, $\xi$, would hold under a flexible exchange rate regime ($Z^* = \tilde{Z}^* = 0$). The derivation of the $ES_X$ and $ES_{loan}$ equations in the case with a flexible exchange rate are presented in Appendix C, we have

$$ES_X(P_x, i^*, B, A, A_x^*, A_y^*, \phi_I^*, \theta^o, n, n^*) \quad \text{and} \quad ES_{loan}^*(P_x, i^*, B, A, A_x^*, A_y^*, \phi_I^*, \theta^o, n, n^*).$$

There are two interesting findings. Firstly, the presence of the endogenous adjustment of $e$ alters the relation of $ES_{loan}^*$ with $P_x$, $i^*$, and the monetary and productivity shocks qualitatively. Contrary to the quantity adjustments (official sales of foreign currency, $Z^*$ and $\tilde{Z}^*$) in the foreign exchange market under a fixed $e$ that respond to accommodate the changes in the demand for foreign liquidity, the price adjustments (relative price of foreign currency, $e$) in the foreign exchange market under a flexible $e$ tend to counteract the impacts on the foreign liquidity, resulting in a $ES_{loan}^*$ relation that is qualitatively different from the one in the case with a fixed $\tau$. Secondly, the quantitative and qualitative differences in the effects of the shocks for different values of $\xi$ do not prevail in the case of a flexible exchange rate regime. It highlights how the value of $\xi$ determines the strength of the direct effects of the home country’s shocks on the liquidity allocation in the foreign loan market, influences the directions and magnitudes of the official intervention required to accommodate the fixed nominal exchange rate $\tau$, and therefore matters for the transmission mechanism.

4.1.d Discussion

As summarized in Table 2, under a fixed exchange rate regime, the signs of these effects depend on the presence of international outsourcing activities and the contractual upfront payment arrangements between the home firms and their foreign suppliers. The numerical example in Table 3 confirms these analytical findings, the values of $\theta^o$ and $\xi$ are crucial in determining the directions and magnitudes of the effects of the shocks in cases with a fixed $\tau$.

Table 2: The Effects of Monetary and Productive Shocks under a Fixed Exchange Rate

<table>
<thead>
<tr>
<th>$\Delta B$</th>
<th>$\Delta A_x$</th>
<th>$\Delta A_y^*$</th>
<th>$\Delta \phi_I^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^o = \overline{\theta}$</td>
<td>$\theta^o &lt; \overline{\theta}$</td>
<td>$\theta^o &lt; \overline{\theta}$</td>
<td>$\theta^o = \overline{\theta}$</td>
</tr>
<tr>
<td>small $\xi$</td>
<td>large $\xi$</td>
<td>small $\xi$</td>
<td>large $\xi$</td>
</tr>
</tbody>
</table>
| $\Delta P_x$ | $-$ | $-$ | $+$ | $-$ | $-$ | $-$ | $0$ | $0$ | $0$ | $-$ | $+$
| $\Delta i^*$ | $+$ | $+$ | $-$ | $0$ | $+$ | $-$ | $0$ | $0$ | $0$ | $+$ | $-$
| $\Delta i$ | $-$ | $-$ | $-$ | $0$ | $-$ | $-$ | $0$ | $0$ | $0$ | $-$ | $-$
| $\Delta P_y^*$ | $+$ | $+$ | $?$ | $0$ | $+$ | $-$ | $-$ | $-$ | $-$ | $-$ | $+$
| $\Delta TB^*$ | $+$ | $+$ | $?$ | $0$ | $+$ | $-$ | $0$ | $0$ | $0$ | $+$ | $?$
It would be useful to have some idea what would the plausible value of \( \xi \) be. As reported by the World Bank Enterprise Survey on China (2012), among the 2600 firms surveyed, the median response to the question about the percentage of the firm’s total annual sales of goods or services that were paid for after delivery in the fiscal year of 2011 was 70%. We will conjecture that the value of \( \xi \) will be small and may plausibly be around the value of 0.3. Table 3 shows that the signs of the effects switch only when \( \xi \) is higher than 0.8, giving support to focusing on the analytical results for the case with small \( \xi \).

Table 3 : Numerical Exercises of Shocks in \( B \) and \( A_x \) for Sections 4.1 and 5.1

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>Fixed, ( \bar{e} = 1.05 )</th>
<th>Flexible ( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( % \Delta(M+B)=0.01, B_{\alpha}=0.0001 )</td>
<td>( % \Delta(M+B)=0.01, B_{\alpha}=0.0001 )</td>
<td>( % \Delta A_{\alpha}=10, A_{\alpha}=4.4 )</td>
</tr>
<tr>
<td>( % \Delta A_{\alpha}=10, A_{\alpha}=4.4 )</td>
<td>( % \Delta A_{\alpha}=10, A_{\alpha}=4.4 )</td>
<td></td>
</tr>
</tbody>
</table>

| \( \% \Delta C_a \) | 1.000 0.993 0.993 0.993 | 1.000 0.987 0.987 0.987 | 1.000 0.993 0.993 0.993 |
| \( \% \Delta C_y \) | 0.524 0.513 0.513 0.513 | 0.503 0.509 0.509 0.509 | 0.524 0.513 0.513 0.513 |
| \( \% \Delta I \) | 0.484 0.495 0.495 0.495 | 0.503 0.498 0.498 0.498 | 0.484 0.494 0.494 0.494 |
| \( \% \Delta I^s \) | -0.164 -0.230 -1.750 1.028 | -0.017 -0.018 -0.017 -0.017 | 0.000 -0.023 -0.172 0.102 |
| \( \% \Delta J \) | -0.526 0.199 1.637 -0.992 | 0.000 0.001 0.000 0.000 | 0.000 0.020 0.161 -0.098 |
| \( \% \Delta J^s \) | -0.653 -0.168 -1.315 0.791 | -0.009 -0.009 -0.008 -0.008 | -0.091 -0.107 -0.210 -0.021 |
| \( \% \Delta TB \) | 5.929 0.155 1.263 -0.753 | 0.000 0.001 0.000 0.000 | 0.000 0.015 0.124 -0.075 |
| \( \% \Delta A^s \) | -1.231 -9.405 -57.532 64.376 | 0.000 -0.031 -0.017 0.000 | 0.000 -0.926 -7.339 4.739 |
| \( \% A^s \) | 1.016 0.013 0.014 0.015 | 0.000 0.022 0.003 0.003 |

4.2 Effects of Permanent Changes in \( \bar{e} \) and \( \kappa \)

In order to study the effects of changes in the fixed exchange rate \( \bar{e} \) or the fixed cost associated with international outsourcing \( \kappa \) on the world economy, we need to examine not only the \( ES_x = 0 \) and \( ES_{\text{loan}} = 0 \) equations but also the optimal conditions for \( \theta^o \), \( n \), and \( n^* \) because the households determine the outsourcing cutoff productivity level and deposit decisions based on the given values of \( (\bar{e}, \kappa) \). As presented in Appendix D, since our focus is now on the effects on \( \theta^o \), \( n \), and \( n^* \), we could simplify the derivation and allow for the analytical results by assuming no uncertainty in the realization of \( s \). This assumption implies that the households will choose their deposit decisions so that the optimal conditions \( \beta E[1+i] = \beta(1+i) = 1 \) and \( \beta^* E[1+i^*] = \beta^*(1+i^*) = 1 \) are always satisfied. As expected, the value of \( \xi \) does not matter much in the cases with permanent shocks.
The intuition is that the value of $\xi$ is crucial in determining the strengths of the effects on liquidity demands generated by temporary shocks when the outsourcing and deposit decisions are sluggish. However, in the case with permanent changes in $\sigma$ and $\kappa$, the decisions $\theta^0$, $n$, and $n^*$ are allowed to respond so that the liquidity supplies can adjust to accommodate the changes in liquidity demands, making $\xi$ less important in the transmissions of the effects of permanent shocks.

4.2.a Effects of a Revaluation of the Foreign Currency

Consider now that at the beginning of the current period, before any decision is made, the foreign monetary authority announces a permanent revaluation of its currency. In the case without international outsourcing, the increase in $e$ affects the relative price of import and induces adjustments in final good trade. As the foreign country increases $P_x C_x^*$ and the home country reduces $P_y^* C_y$, $TB^* = P_y^* C_y - P_x C_x^*/e$ deteriorates. In the case with outsourcing, there will be additional adjustments via the trade in intermediate good, $q^* Q_I$, which may alter quantitatively or even qualitatively the equilibrium effect on $TB^* = q^* Q_I + P_y^* C_y - P_x C_x^*/e$.

**Result 2.** In the case with outsourcing, a revaluation of the foreign currency induces not only adjustments in final good trade but also adjustments in outsourcing activities at both the intensive and extensive margins. The adjustment at the extensive margin (in the number of home firms sourcing abroad) could be the dominant force in determining the equilibrium effects, resulting in an improvement in the foreign trade balance when the foreign currency value increases.

In the case without international outsourcing, an increase in $e$ will reduce the home importers’ demands for good $y$, $C_y$, leading the foreign firms to reduce their loan $w^* l_y^*$ from the foreign loan market. The foreign interest rate $i^*$ will fall if $n^*$ does not adjust. Expecting a decrease in the demand for liquidity in the foreign loan market, the foreign households reduce their deposits, $n^*$ falls to satisfy the optimal condition $\beta^* E[1 + i^*] = 1$. Similarly, if $n$ remains unchanged, an increase in $e$ will increase the demands for good $x$ from the foreign importers, $C_x^*$, inducing the home firms to increase their loans $w_l x$ from the home loan market and generating an upward force on $i$. Expecting an increase in the demand for liquidity in the home loan market, the home households increase their deposits, $n$ rises until the optimal condition $\beta E[1 + i] = 1$ is satisfied. Although these changes in $n$ and $n^*$ will induce further adjustments in the loan markets that would offset some of the initial changes, it can be shown that $\frac{dn^*}{de} < 0$, $\frac{dn}{de} > 0$, $\frac{dP_y^* C_y}{de} < 0$, $\frac{d(P_y C_y^*)}{de} > 0$, and $\frac{dTB^*}{de} < 0$, the deterioration of the trade balance $TB^*$ requires the foreign monetary authority to conduct a smaller official sale of foreign currency in the foreign exchange market, $Z^* = TB^*$ falls.\(^{28}\)

\(^{28}\)As discussed above, the values of $a$, $a^*$, $\alpha$ and $\alpha^*$ are assumed to be sufficiently high.
In the case with outsourcing, in addition to the effects on trade in final goods described above, there are adjustments in outsourcing activities at both the intensive and extensive margins. If \( \theta^o \) and \( n \) remain unchanged, an increase in \( \pi \) will increase the foreign importers’ demands for good \( x \) but will reduce the home firms’ demands for input \( I \) (at the intensive margin) and their production of good \( x \), resulting in a large excess demand for good \( x \) and therefore a substantial increase in \( P_x \). Using equation (22), we can identify two effects of an increase in \( \pi \) on \( \theta^o \). The first one is a demand effect reflected by an increase in \( P_x \) due to the higher foreign demand for good \( x \), increasing both \( E[\pi(\theta_i)] \) and \( E[\pi^o(\theta_i)] \). The second one is a cost effect as a higher \( \pi \) increases the unit cost and a higher \( P_x \) increases the nominal fixed cost of outsourcing, reducing \( E[\pi^o(\theta_i)] \). Although the two effects have opposing effects on the home firms’ incentives to outsource, the demand effect dominates and results in an increase in the number of home firms sourcing abroad (a decrease in \( \theta^o \)). In Figure 4, we indicate explicitly the fixed exchange rate and real cost of outsourcing that the expectations are conditional on. Holding the real fixed cost constant at \( \hat{\kappa} \), when \( \pi \) is increased to \( \hat{\pi} \), there will be a shift from \( E[\pi(\theta_i)|\pi, \hat{\kappa}] \) up to \( E[\pi(\theta_i)|\hat{\pi}, \hat{\kappa}] \) and a twist from \( E[\pi^o(\theta_i)|\pi, \hat{\kappa}] \) to \( E[\pi^o(\theta_i)|\hat{\pi}, \hat{\kappa}] \), resulting in a decrease in the equilibrium cutoff level from \( \overline{\theta^o} \) to \( \hat{\theta} \).

Figure 4: Effects of changes in \( \pi \) and \( \kappa \) on \( \theta^o \), (\( \hat{\kappa} < \kappa \) and \( \overline{\pi} < \hat{\kappa} \))
The adjustment at the extensive margin results in a decrease in the liquidity demand in the home loan market $wl_x$ and an increase in the liquidity demand in the foreign loan market $w^*l_y^*$, leading the households to adjust their deposit decisions in the directions different from those in the case without outsourcing. In the home loan market, given that the home firms’ liquidity demand $wl_x$ is more sensitive to an increase in $\tau$ than the foreign importers’ liquidity demand $P_xC_x^*$ does, there will be a net decrease in liquidity demand in the home loan market. To eliminate the downward pressure on $i$, the home households will reduce their deposits $n$ to satisfy the optimal condition $\beta E[1 + i] = 1$. In the foreign loan market, the increase in liquidity demand for the production of the intermediate good $w^*l_y^*$ dominates the decreases in the liquidity demands for the production and trade of final good $y$, $P_y^*C_y$ and $w^*l_y^*$. Following the optimal condition $\beta^* E[1 + i^*] = 1$, the foreign households will increase their deposits $n^*$ to ease the upward pressure on $i^*$.

The decrease in $n$ implies an increase in the home real wage $w$; and the increase in $n^*$ implies decreases in the foreign real wage, $w^*$, and price of good $I$, $q^*$. Although the increase in $\tau$ would raise the home-currency price of good $I$, $\tau q^*$, the relative unit cost of producing to importing good $I$, $w/\tau q^*$, increases and induces more home firms to switch to international outsourcing, leading $\theta^\circ$ to fall further. The interactions of the adjustment at the extensive margin in outsourcing (the number of firms sourcing abroad) with the households’ deposit decisions are the driving forces of the equilibrium effects. It is shown that a revaluation of the foreign currency (an increase in $\tau$) has positive equilibrium effects on $n^*$ and $w/\tau q^*$, and a negative equilibrium effect on $\theta^\circ$.

When $\theta^\circ \in (\underline{\theta}, \overline{\theta})$, $\frac{dn^*}{d\tau} > 0$, $\frac{d(w/\tau q^*)}{d\tau} > 0$, $\frac{d\theta^\circ}{d\tau} < 0$, $\frac{d(P_xC_x^*/\tau)}{d\tau} < 0$, and $\frac{dT^*}{d\tau} > 0$.

If $\frac{d((1-n)/\tau)}{d\tau} < 0$, then $\frac{dP_y^*C_y}{d\tau} < 0$, $\frac{dq^*Q_I}{d\tau} > 0$, and $\frac{d(q^*Q_I + P_y^*C_y)}{d\tau} > 0$.

A positive effect on $n^*$ indicates a negative effect on $P_xC_x^*/\tau$. The equilibrium effects on $n$ and $(1-n)/\tau$ are ambiguous. If the direct effect of an increase in $\tau$ dominates, we will get $\frac{d((1-n)/\tau)}{d\tau} < 0$, implying a negative effect on $P_y^*C_y$ and positive effects on $q^*Q_I$ and $q^*Q_I + P_y^*C_y$. Although the net trade in consumer goods, $P_y^*C_y - P_yC_y/\tau$, is ambiguous, we show that the foreign trade balance, $TB^* = q^*Q_I + P_y^*C_y - P_yC_y/\tau$, improves, requiring the foreign monetary authority to conduct larger official sales of the foreign currency in the foreign exchange market, $z^* \equiv Z^* + \tilde{Z}^*_{1+i}$ rises.
Discussions:

The presence of international outsourcing activities is shown to be an important determinant of the effect of a revaluation on the trade balance $TB^*$. By looking into the liquidity flows in the foreign exchange market, we can gain some insights into the interactions at work. The demand for foreign currency is derived from the home country’s demand for imports of final good $y$ and intermediate input $I$, and the supply of foreign currency is derived from the foreign country’s import of final good $x$. When $\theta^o = \overline{\theta}$, there is trade in final goods only, and changes in $\pi$ induce adjustments of $n$ and $n^*$. As $P_y^*C_y$ is negatively related to $\pi$, and $P_x^*C_x^*/\pi$ is positively related to $\pi$, we have the usual downward sloping demand and upward sloping supply schedules of foreign currency in the foreign exchange market. An increase in $\pi$ reduces $TB^*$, the foreign monetary authority reduces its official sale of foreign currency $Z^*$ to meet the decrease in the net private demand.

The incorporation of international outsourcing activities alters the responses of the demand and supply of foreign currency to changes in $\pi$. When $\theta^o < \overline{\theta}$, with the adjustments of $\theta^o$, $n$, and $n^*$, an increase in $\pi$ effectively increases the demand $w^*l_I^* + P_y^*C_y$ and reduces the supply $P_x^*C_x^*/\pi$ of foreign currency, resulting in an upward sloping demand and a downward sloping supply schedules in the foreign exchange market.\(^{29}\) The foreign monetary authority has to conduct larger official sales $Z^*$ and $\hat{Z}^*$ to meet the increase in the net private demand due to a larger $TB^*$. As a result, a revaluation of foreign currency will make the home country’s trade deficit deteriorate further.

This finding provides a rationale for the puzzling evidence of a positive relationship between China’s currency value and its trade surplus during the period of 1994-2008. Figure 5 presents the official exchange rate of China’s currency, the Renminbi (RMB), in units of the US dollar and China’s trade balance in goods and services (TB) as a percentage of its gross domestic product (GDP) from 1985 to 2016. It also includes the real effective exchange rate (REER) and the decomposition of the trade balance.\(^{30}\) The correlation coefficient between the official USD/RMB rate and China’s TB/GDP ratio was $-0.72$ during the period of 1985-1993 as China’s overall trade balance tended to improve when RMB was devalued. In contrast, the correlation coefficient was positive and equal to 0.74 during the period of 1994-2008 and 0.26 for the period of 1994-2016.\(^{31}\) Similar patterns of the correlation coefficients between the REER the TB/GDP ratio are obtained.

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\(^{29}\)Recall that $w^*l_I^* = \left(1+\frac{\epsilon^*}{\mu_I^*}\right)q^*Q_I$ and $TB^* = q^*Q_I + P_y^*C_y - P_x^*C_x^*/\pi$.

\(^{30}\)The data are obtained from the World Integrated Trade Solution and the World Development Indicators Databases of the World Bank. The real effective exchange rate is the nominal effective exchange rate (a measure of the value of RMB against a weighted average of several foreign currencies) divided by a price deflator. Data on the decomposition of the trade balance into the four major product categories (capital goods, consumer goods, intermediate goods, and raw materials) are available from 1992 only.

\(^{31}\)The global trade collapse in 2008-2009 and the slow economic recovery may have contributed to this significant drop in the positive correlation.
### Figure 5: China’s Exchange Rates and Net Exports to GDP Ratios

<table>
<thead>
<tr>
<th>Year</th>
<th>Official USD/RMB Rate</th>
<th>Adjusted Official USD/RMB Rate</th>
<th>TB/GDP (All)</th>
<th>TB/GDP (Intermediate)</th>
<th>TB/GDP (Consumer)</th>
<th>TB/GDP (Materials)</th>
<th>TB/GDP (Products)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>10.43</td>
<td>-</td>
<td>2.78</td>
<td>0.37</td>
<td>3.28</td>
<td>-</td>
<td>3.28</td>
</tr>
<tr>
<td>1986</td>
<td>10.26</td>
<td>-</td>
<td>2.78</td>
<td>0.37</td>
<td>3.28</td>
<td>-</td>
<td>3.28</td>
</tr>
<tr>
<td>1987</td>
<td>10.25</td>
<td>-</td>
<td>2.78</td>
<td>0.37</td>
<td>3.28</td>
<td>-</td>
<td>3.28</td>
</tr>
<tr>
<td>1988</td>
<td>10.25</td>
<td>-</td>
<td>2.78</td>
<td>0.37</td>
<td>3.28</td>
<td>-</td>
<td>3.28</td>
</tr>
<tr>
<td>1989</td>
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<td>-</td>
<td>2.78</td>
<td>0.37</td>
<td>3.28</td>
<td>-</td>
<td>3.28</td>
</tr>
<tr>
<td>1990</td>
<td>10.25</td>
<td>-</td>
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<td>3.28</td>
<td>-</td>
<td>3.28</td>
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<tr>
<td>1991</td>
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<td>-</td>
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<td>-</td>
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<td>-</td>
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<tr>
<td>1993</td>
<td>10.26</td>
<td>-</td>
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<td>0.37</td>
<td>3.28</td>
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</tr>
<tr>
<td>1994</td>
<td>10.26</td>
<td>-</td>
<td>2.78</td>
<td>0.37</td>
<td>3.28</td>
<td>-</td>
<td>3.28</td>
</tr>
</tbody>
</table>

**Source:** The World Development Indicators and the World Integrated Trade Solution

**Figure Notes:**
- The official USD/RMB rate is the rate at which the Chinese currency is exchanged for the US dollar.
- The adjusted official USD/RMB rate adjusts for inflation and other factors to provide a more accurate comparison.
- TB/GDP ratios indicate the proportion of total exports as a percentage of GDP.
After a substantial devaluation of the RMB in 1994, the official USD/RMB exchange rate had been kept relatively stable, and it remained steady at 12.08 U.S. cents per RMB from 1997 to 2005. The surge of China’s TB/GDP ratio to 4.46% in 2005 ignited the political pressure from policymakers of other countries, including those of the U.S., on RMB to revalue. China introduced a new regime in 2005 to allow for graduate revaluations of the RMB. Its trade surplus not only did not fall but rose substantially to 6.48% of its GDP in 2008 despite of a 20% cumulative appreciation of the RMB against the US dollar. Although many studies in the literature have explained why the trade flows might not be responsive to the currency revaluations, they could not explain the accelerating increase in China’s trade surplus. The decomposition of the trade balance reveals that the net trade flows of different categories of products responded quite differently to the changes in the currency value. For the period of 1994-2008, the correlation coefficient of the official USD/RMB rate and the TB/GDP ratio was equal to 0.91 for the intermediate goods and −0.02 for the consumer goods, while for the period of 1994-2016, they were equal to 0.84 and −0.80, respectively. These correlations are in line with our model predictions that a revaluation of the foreign currency would increase the foreign country’s exports of intermediate goods, while having an ambiguous effect on its trade balance of the consumer goods.

The increases in the differences in the labor costs of China and the U.S. are consistent with the increases in China’s outsourcing activities and exports of intermediate goods. According to the Bureau of Labor Statistics of the United States Department of Labor, the hourly labor compensation in China’s manufacturing sector increased from USD0.60 in 2002 to USD1.74 in 2009, while those of the U.S. increased from USD27.36 to USD34.19. Although the hourly labor compensation in China increased from about 2% of the U.S. level in 2002 to 5% in 2009, the level differences were widened and the labor cost in China remained very low.

Our model provides a theoretical framework to illustrate how the rise of China as a major host country of international outsourcing may have altered the response of the overall trade balance to its currency revaluation, helping to rationalize the relation between its currency revaluation and trade surplus.

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32Although its trade flows and trade surplus have been declining since global trade collapsed in 2008, China has become the world’s largest exporter since 2009. According to the World Trade Report 2013, its share of world export was 11% in 2011. The upward trend in the trade balance has been observed in the recent years.

33The hourly labor costs in China and the U.S. are from the website of the Bureau of Labor Statistics of the United States Department of Labor, http://www.bls.gov/fls/home.htm, which provides data from 2002 to 2009 only. Some reasons for the increases in China’s labor costs were rising labor productivity, inflation, and the increased requirements for social insurance payments by firms.
4.2. Effects of Reductions in the Costs Associated with International Outsourcing

We now consider the effects of a reduction in the fixed cost of international outsourcing $\kappa$. In order to disentangle the direct effects of a reduction in $\kappa$ and the indirect effects of the induced official interventions under a fixed nominal exchange rate regime, our analysis will proceed in two steps. Firstly, we investigate the effects on the households’ decisions and the nominal exchange rate by assuming that $z^\ast$ is held constant and $e$ is allowed to adjust endogenously to clear the foreign exchange market. Secondly, maintaining the nominal exchange rate at $\bar{e}$ requires the foreign monetary authority to conduct official sales/purchases of its currency in the foreign exchange market, we identify the required endogenous adjustment of $z^\ast$ and the effects on the world economy. We can then combine the effects to obtain the equilibrium impacts of an increase in $\kappa$ under a fixed exchange rate regime and have a better understanding of the driving forces behind the findings.

**Result 3.** A reduction in the fixed cost of international outsourcing encourages more home firms to switch to sourcing abroad under a flexible exchange rate, while the endogenous adjustment in the official sale of foreign currency for maintaining a fixed exchange rate will discourage outsourcing.

Holding $z^\ast$ constant, a reduction in $\kappa$ encourages some home firms to switch from producing their intermediate inputs domestically to importing from aboard, resulting in a fall in $\theta^o$, a decrease in liquidity demand $wl_x$ in the home loan market, and an increase in the liquidity demand $w^*l^*_I$ in the foreign loan market. In the foreign exchange market, the increase in the home country’s demand for foreign currency $q^*Q_I$ generates an upward force on $e$, it will discourage the foreign country’s export of good $y$, $P_y^*C_y$, but leave the home country’s export of good $x$, $P_x^*C_x/e$, unchanged if $n$ and $n^*$ do not adjust. In the foreign loan market, a decrease in $P^*_yC_y$ also reduce the liquidity demand $w^*l^*_y$, leading the foreign households to reduce their deposits $n^*$ until the optimal condition $\beta^*E[1 + i^*] = 1$ is satisfied. In the home loan market, if the decrease in $wl_x$ is dominated by the increase in $P_x^*C_x$, there will be a net increase in liquidity demand, and the home households will increase their deposits $n$ until the optimal condition $\beta E[1 + i] = 1$ is met. It can be shown that

\[
\frac{d\epsilon}{d\kappa} \bigg|_{dz^\ast = 0} < 0, \quad \left. \frac{d\theta^o}{d\kappa} \right|_{dz^\ast = 0} > 0, \quad \left. \frac{d((1 - n)/e)}{d\kappa} \right|_{dz^\ast = 0} > 0, \quad \text{and} \quad \left. \frac{dn^*}{d\kappa} \right|_{dz^\ast = 0} > 0.
\]

As shown in Figure 4, a reduction from $\kappa$ to $\hat{\kappa}$, while letting $e$ adjust freely, causes the equilibrium exchange rate to increase from $\bar{e}$ to $\hat{e}$ and cutoff productivity level to decrease from $\theta^o$ to $\hat{\theta^o}$, encouraging more home firms to outsource abroad.

Keeping the exchange rate at $\bar{e}$ in face of a lower $\kappa$ is like conducting a devaluation of the foreign currency to bring the exchange rate from the new market equilibrium level $\hat{e}$ down to its original
fixed level $\bar{\pi}$, requiring the a reduction in the foreign official sale of foreign currency $z^*$. As discussed in Section 4.2.a, a reduction in $\bar{\pi}$ has a positive effect on $\theta^o$ and a negative effect on $TB^*$. As the direct effect from a lower $\kappa$ and the indirect effect from a lower $z^*$ affect $\theta^o$ in opposing directions, the equilibrium effect is ambiguous. Figure 4 illustrates how keeping the nominal exchange rate at $\bar{\pi}$ means further shifts in the expected profits schedules which may result in a new equilibrium level at $\bar{\theta}^o$, having a net increase in the cutoff level, $\hat{\bar{\theta}}^o < \theta^o < \bar{\theta}^o$. The numerical example presented in Table 4 shows that the indirect effect could dominate and result in an increase in $\theta^o$.

5 Welfare Analysis

The equilibrium effects of the exogenous changes on the welfare of each household are generally ambiguous as the direct and indirect effects are in opposing directions. Tables 3 and 4 construct some numerical examples to illustrate the welfare impacts of various exogenous changes. It should be noted that the equilibrium welfare effects depend on the specifications of the utility functions, and that the aim of these exercises is to illustrate the theoretical properties of the model rather than to match the observed data.

5.1 Welfare Effects of Different Realizations of $s$

Given $\bar{\pi}$ and $\kappa$, holding $\theta^o$, $n$, and $n^*$ constant, we can use our findings of changes in $i$, $i^*$, $P_x$, and $P_y^*$ to determine the impacts on households’ consumption and leisure levels and pin down the welfare effects of different realizations of $s$. The results of the numerical exercises are presented in Table 3.

First, the presence of international outsourcing weakens the welfare effects of domestic monetary policy on the world economy. An increase in the open market purchase of home-currency-denominated bonds $B$ reduces $P_x$ and $i$ but increases $P_y^*$ and $i^*$, leading to increases in $C_x$, $C_y$, and $h$ but decreases in $C_x^*$, $C_y^*$, and $h^*$. Because of home bias in consumption, households experience substantial welfare gains (losses) from the increases (decreases) in the consumption of the domestically produced final good when its price falls (rises). In addition, a decrease (an increase) in the domestic interest rate lowers (raises) the domestic-currency price of the imported final good, leading to an increase (a decrease) in the consumption of the imported final good. As the welfare changes from the adjustments in consumption dominate the impacts from the adjustments in work effort, $u$ increases and $u^*$ decreases. The smaller the value of $\xi$, the less responsive the adjustments in $P_x$, $i$, $P_y^*$ and $i^*$, and the weaker the equilibrium effects on $u$ and $u^*$ will be.

Second, no matter whether there are international outsourcing activities, an improvement in
the production technology of a final good (an increase in $A_x$ or $A_{y}^x$) benefits the households of both countries. The increases in consumption of the final good that experiences a positive productivity shock play dominant roles in improving the welfare levels of the households.\footnote{When $\xi$ is small, an increase in $\phi_{x}^y$ reduces $P_x$ and $i$ but increases $P_{y}^x$ and $i^*$, leading to increases in $C_x$, $C_y$, and $h$ but decreases in $C_{x}^*$, $C_{y}^*$, and $h^*$, resulting in an increase in $u$ and a decrease in $u^*$.}

5.2 Welfare Effects of Changes in $\bar{\epsilon}$ or $\kappa$

The adjustments of $\theta^o$, $n$, and $n^*$ in response to the changes in $\bar{\epsilon}$ or $\kappa$ determine the allocation of liquidity in the world economy. As shown in Table 4, given that transactions are facilitated by liquidity, the adjustments of deposit decisions $n$ and $n^*$ are good indicators of the welfare effects.\footnote{The numerical example assumes $\xi = 0.3$. However, the qualitative effects on the welfare levels of an increase in $\bar{\epsilon}$ or a reduction in $\kappa$ are not sensitive to the value of $\xi$.}

Table 4 : Numerical Exercises of Permanent Changes in $\bar{\epsilon}$ and $\kappa$ for Sections 4.1 and 5.1\footnote{a These numerical exercises assume no uncertainty. The parameter values are $\beta = \beta^* = 0.96$, $a = a^* = 0.6$, $\alpha = \alpha^* = \frac{2}{3}$. $\nu = 1$, $\nu^* = 0.9$, $\beta = 0$, $A_x = A_{y}^x = 4$, $\phi_{x}^y = 1$, $\theta = 0$, $\bar{\epsilon} = 1$, and $\xi = 0.3$. In the cases with $\theta^o = \bar{\epsilon}$, there is no international outsourcing as the value of $\kappa$ is prohibitively high. $b$ The equilibrium under this fixed exchange rate $\bar{\epsilon}$ is also the equilibrium under a flexible exchange rate regime. $c$ If $\bar{\epsilon} = 1$ and $\kappa$ is prohibitively high originally, these are examples of fixed exchange rate levels that will benefit both countries when $\kappa$ becomes lower. $d$ Comparing to the equilibrium with $\kappa = 0.2$ and $\bar{\epsilon} = 1.07$, this fixed exchange rate level can make both countries better off when $\kappa$ falls to equal to 0.1.}

<table>
<thead>
<tr>
<th>Case I: $\theta^o = \bar{\epsilon}$</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$ is prohibitively high</td>
<td>Outsourcing with $\kappa = 0.2$, $\theta^o = \bar{\epsilon}$</td>
<td>Outsourcing with $\kappa = 0.1$, $\theta^o = \bar{\epsilon}$</td>
</tr>
<tr>
<td>$\bar{\epsilon}$</td>
<td>$\bar{\epsilon}$</td>
<td>$\bar{\epsilon}$</td>
</tr>
<tr>
<td>$n$</td>
<td>$n$</td>
<td>$n$</td>
</tr>
<tr>
<td>$n^*$</td>
<td>$n^*$</td>
<td>$n^*$</td>
</tr>
<tr>
<td>$\bar{\epsilon}$</td>
<td>$\bar{\epsilon}$</td>
<td>$\bar{\epsilon}$</td>
</tr>
<tr>
<td>$P_x$</td>
<td>$P_x$</td>
<td>$P_x$</td>
</tr>
<tr>
<td>$P_{y}^x$</td>
<td>$P_{y}^x$</td>
<td>$P_{y}^x$</td>
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<tr>
<td>$P_{y}^*$</td>
<td>$P_{y}^*$</td>
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</tr>
<tr>
<td>$P_{x}^*$</td>
<td>$P_{x}^*$</td>
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<tr>
<td>$C_x$</td>
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<td>$C_y$</td>
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<tr>
<td>$h$</td>
<td>$h$</td>
<td>$h$</td>
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<tr>
<td>$C_{x}^*$</td>
<td>$C_{x}^*$</td>
<td>$C_{x}^*$</td>
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<td>$C_{y}^*$</td>
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<td>$h^*$</td>
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<tr>
<td>$u^*$</td>
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<td>$u^*$</td>
</tr>
<tr>
<td>$u + u^*$</td>
<td>$u + u^*$</td>
<td>$u + u^*$</td>
</tr>
<tr>
<td>$P_x C_x/\bar{\epsilon}$</td>
<td>$P_x C_x/\bar{\epsilon}$</td>
<td>$P_x C_x/\bar{\epsilon}$</td>
</tr>
<tr>
<td>$P_{y}^x C_y/\bar{\epsilon}$</td>
<td>$P_{y}^x C_y/\bar{\epsilon}$</td>
<td>$P_{y}^x C_y/\bar{\epsilon}$</td>
</tr>
<tr>
<td>$q \phi_{x}^y$</td>
<td>$q \phi_{x}^y$</td>
<td>$q \phi_{x}^y$</td>
</tr>
<tr>
<td>$T D^*$</td>
<td>$T D^*$</td>
<td>$T D^*$</td>
</tr>
<tr>
<td>$z^*$</td>
<td>$z^*$</td>
<td>$z^*$</td>
</tr>
</tbody>
</table>

As shown respectively in Cases II and III of Table 4, holding $\kappa$ constant, an increase in $\bar{\epsilon}$ leads some home firms to switch to international sourcing ($\theta^o$ falls); and the reallocations of liquidity in
the loan markets induce a decrease in $n$ and an increase in $n^*$. A decrease in $n$ implies that higher consumption expenditures are available for the home shoppers while fewer loans are available for the hiring of workers in the home country, $C_x$ and $C_y$ increase and $h$ falls. An increase in $n^*$ indicates that the foreign shoppers’ consumption expenditures are lower and that the foreign firms obtain more loans to hire workers in the foreign country, $C^*_x$ and $C^*_y$ decrease and $h^*$ rises. Hence, a revaluation of the foreign currency leads to an increase in $u$ and a decrease in $u^*$. It is found that $u$ is increasing and $u^*$ is decreasing in $\bar{\tau}$, and the total utility level of the two counties, $u + u^*$, can be increasing in $\bar{\tau}$ when the utility function is linear in leisure.\(^{36}\) In contrast, with no outsourcing, a foreign currency revaluation increases $n$ and reduces $n^*$, improving the welfare of the foreign households, while making the home households worse off. Case I of Table 4 illustrates that when $\theta^o = \bar{\theta}$, $u$ is decreasing and $u^*$ is increasing in $\bar{\tau}$, and the total utility $u + u^*$ displays an inverted U relationship as $\bar{\tau}$ increases, reaching its maximum at the flexible exchange rate equilibrium. These results highlight the important role of international outsourcing in the determination of the welfare effects of currency revaluations on the world economy.

5.2.b Welfare Effects of a Reduction in $\kappa$

A reduction in the fixed cost of outsourcing improves the welfare of the foreign households but makes the home households worse off under a flexible exchange rate regime. As the comparison of columns I.2, II.1, and III.2 in Table 4 illustrates, when the exchange rate is flexible ($z^* = 0$), a decrease in $\kappa$ increases $e$ and $n$ but reduces $\theta^o$ and $n^*$. The changes in households’ consumption expenditures result in decreases in $C_x$ and $C_y$ and increases in $C^*_x$ and $C^*_y$. Although $h$ falls and $h^*$ rises when more production activities are relocated to the foreign country, the welfare changes from the adjustments in consumption dominate, leading to a decrease in $u$ and an increase in $u^*$ and resulting in a small net increase in the total utility level $u + u^*$.

The presence of a fixed exchange rate regime reinforces the welfare effects. The foreign exchange intervention that prevents the exchange rate from increasing and keeps it at the level $\bar{\tau}$ works like a policy to devalue the foreign currency, reducing the home country’s welfare and improving the foreign country’s welfare further. Consider the original equilibrium given by column II.1. Under a

\(^{36}\)It is noted that the increase in $u + u^*$ cannot go unbounded as the foreign labor effort, $h^*$, will reach its upper bound eventually. When there is diminishing marginal utility of leisure, the total utility level of the two counties, $u + u^*$, may not be monotonic in $\bar{\tau}$. For example, in the case with $u(C_x, C_y, h) = a \ln C_x + (1-a) \ln C_y + \frac{1}{1+\sigma} v [1 - h]^{1+\sigma}$, and $u^*(C^*_x, C^*_y, h^*) = (1 - a^*) \ln C^*_x + a^* \ln C^*_y + \frac{1}{1+\sigma^*} v^* [1 - h^*]^{1+\sigma^*}$, where $\sigma = \sigma^* = -0.2$, it is found that $u + u^*$ increases as $\bar{\tau}$ rises, reaches its maximum when $\bar{\tau}$ is somewhere above its flexible exchange rate equilibrium, and becomes decreasing in $\bar{\tau}$ thereafter.

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flexible exchange rate, a reduction in the value of $\kappa$ from 0.2 to 0.1 results in the new equilibrium presented in column III.2. Keeping the exchange rate at the original level would lead to the new equilibrium given by column III.1. Overall, the reallocations of liquidity in the loan markets result in an increase in $n$ and a decrease in $n^*$. The home households are worse off as their consumption levels $C_x$ and $C_y$ decrease and work effort $h$ increases. The foreign households are better off as $C_x^*$ and $C_y^*$ increase and $h^*$ decreases. Holding $\bar{\epsilon}$ constant, the increase in $u^*$ is dominated by the decrease in $u$, resulting in a net decrease in their total utility $u + u^*$.

5.2.c Discussion
As demonstrated above, a decrease in $\kappa$ can have substantial distributional welfare effects between countries, while the total utility gain of the world economy is insignificant or may even be negative under a fixed $\bar{\epsilon}$. Using the results from the changes in $\bar{\epsilon}$ and $\kappa$, we can conjecture that when a reduction in $\kappa$ is accompanied by a revaluation of the foreign currency (an increase in $e$), it is possible to result in an increase in outsourcing activities (a decrease in $\theta_o$) and improvements in the welfare levels of both countries (increases in both $u$ and $u^*$). However, due to the welfare conflict between the two countries generated by the foreign exchange policy, the foreign country would resist the revaluation of its currency. The numerical exercise confirms the validity of this conjecture. Suppose that the world is in the initial equilibrium with $\kappa = 0.2$ and $\bar{\epsilon} = 1.0700$ (column II.3). There exists some fixed exchange rate levels that can benefit both countries and allow for larger total welfare gains when the value of $\kappa$ is decreased to 0.1 (see for example when $\bar{\epsilon} = 1.0930$, column III.4). As the foreign country would be better off in the equilibrium with $\kappa = 0.1$ and $\bar{\epsilon} = 1.0700$, (given by column III.3), it would have no incentive to revalue its currency.\footnote{Instead, it would want to devalue its currency so as to capture an even larger welfare gain.}

6 Conclusions
The paper complements the existing macroeconomic literature on international trade in intermediates by modeling explicitly firms’ make-or-buy decisions and highlighting the role of financial flows required to facilitate the flows of goods and services related to international outsourcing activities. A two-country, monetary model with segmented financial markets is constructed to examine the interdependence of firms’ international outsourcing decisions and governments’ conducts of monetary and foreign exchange policies.

Our results show that international outsourcing and its associated contractual upfront payment

\footnote{Instead, it would want to devalue its currency so as to capture an even larger welfare gain.}
arrangements between the domestic firms and their foreign suppliers determine the responsiveness of the liquidity allocation in the foreign loan market to the monetary and productivity shocks, and have an important role in the international transmission of the effects of the shocks when the nominal exchange rate is fixed. The paper also examines how the presence of international outsourcing alters the responses of the demand and supply of foreign currency in the foreign exchange market to currency revaluations. As firms can choose between producing their intermediate inputs in-house or buying them from foreign suppliers, the perfect substitution between the intermediate goods produced domestically and abroad implies that the adjustment at the extensive margin of international outsourcing would be significant and can have a dominant role in determining the effects on production and trade. This finding helps explain qualitatively why a foreign currency revaluation may increase the foreign country’s exports of intermediate goods and worsen the trade deficit of the home country even in the time frame in which firms are able to adjust their outsourcing decisions. Finally, the model demonstrates the distributional welfare effects of international outsourcing and highlights the role of a fixed exchange rate regime in welfare redistribution when there is international outsourcing, suggesting the needs to further explore the gains from international monetary policy coordination by modeling explicitly firms’ sourcing decisions of their intermediate inputs.
Appendix

A. The Optimization Problem of the Representative Home Household in Section 3.5

\[ V(m_h, m_f) = \max_{m_h, \theta^o} \int C_x, C_y, h, m_h, b_f, L_x, L_y, \mathrm{IM}_y \{ u(C_{xt}, C_{yt}, h_t) + \beta V(m_h', m_f') \} \mathcal{G}(s) ds, \]

subject to the liquidity constraints (A1) - (A4) and the evolution equations for \( m_h' \) and \( m_f' \),

\[ n_h + e m_f \geq L_x + L_y + b_h + e b_f, \]  
\[ L_x \geq \left( \frac{1-\alpha}{2-\alpha} \right) \left[ \frac{\alpha P_x A_x}{(1+i)w} \right]^\frac{1}{1-\alpha} \left[ \theta^o \frac{\alpha P_x A_x}{(1+i)w} \right] + \xi q^* \left[ \left( \frac{\alpha P_x A_x}{(1+i)w} \right)^\frac{1}{1-\alpha} \left[ \theta^o \frac{\alpha P_x A_x}{(1+i)w} \right] - \frac{\theta^o}{\theta^o - \theta^{o*}} \right] \]

\[ \frac{L_y}{\epsilon} \geq P_y^* \mathrm{IM}_y, \]  
\[ m_h - n_h \geq P_x C_x + P_y C_y, \]  
\[ m_h' + e m_f' = m_h + e m_f - P_x C_x - P_y C_y - L_y - e b_f + P_y \mathrm{IM}_y + b_h + W h + T \]

Let \( \rho_1, \rho_2, \rho_3, \) and \( \rho_4 \) be the multipliers associated with the liquidity constraints (A1) - (A4), respectively. \( \lambda \) denotes the multiplier associated with the no-arbitrage condition, \( |e(1+i) - \hat{e}(1+i^*)|b_f = 0 \), stating that \( b_f \) will be chosen freely if \( i = i^* \) under a fixed exchange rate regime or under a flexible exchange rate regime in which the covered interest parity condition \( e(1+i) = \hat{e}(1+i^*) \) always holds, and that capital control from the foreign monetary authority \( (i - i^*)b_f = 0 \) will be imposed whenever \( i \neq i^* \) under a fixed exchange rate regime.

We can then derive the first-order conditions for the home household’s optimization problem,

\[ n : \int \rho_1 \mathcal{G}(s) ds = \int \rho_4 \mathcal{G}(s) ds, \]

\[ \theta^o : \int \rho_2 \left\{ -w \left[ \frac{\alpha P_x A_x \theta^o}{(1+i)w} \right]^\frac{1}{1-\alpha} + \xi q^* \left[ \frac{\alpha P_x A_x \theta^o}{\xi(1+i) + (1-\xi)q^*} \right]^\frac{1}{1-\alpha} \right\} \mathcal{G}(s) ds \]

\[ + \int \beta V_{m_h} \left\{ \left[ \frac{1+i}{\alpha} - 1 \right] \left[ \frac{\alpha P_x A_x \theta^o}{(1+i)w} \right]^\frac{1}{1-\alpha} - q^* \left[ \frac{\xi(1+i) + (1-\xi)q^*}{\alpha} \right]^\frac{1}{1-\alpha} \right\} \mathcal{G}(s) ds = 0, \]

\[ C_x, C_y : \frac{u_x}{P_x} = \frac{u_y}{P_y} = \rho_4 + \beta V_{m_h}, \quad u_j = \frac{\partial u(C_x, C_y, l)}{\partial C_j}, \quad j = x, y, \]

\[ h : v = \beta V_{m_h} w, \]

\[ b_h : \rho_1 = \beta V_{m_h} i, \]
Solving the first-order conditions, we get the optimal conditions (19)

\[ b_f : \quad -\epsilon (\rho_1 + \beta V_{m'} + \beta V_{m''}) \widehat{c}(1 + i^*) + \lambda [\epsilon(1 + i) - \widehat{c}(1 + i^*)] = 0, \]

\[ L_x : \quad \rho_1 = \rho_2, \]

\[ L_y : \quad \rho_1 + \beta V_{m'} = \frac{\rho_2}{\epsilon} + \beta V_{m''} \widehat{c}. \]

\[ \text{IM}_y : \quad \rho_3 P_y^* = \beta V_{m''} [P_y - \widehat{c} P_y^*] \]

where \[ V_{m'} \equiv \frac{\partial V(m', m_f)}{\partial m_h} = \int [\rho_1 + \beta V_{m'}] G(s) ds = \int \frac{u_x}{P_x} G(s) ds = E \left[ \frac{u_x}{P_x} \right] = \frac{1}{m_h - n_h}, \]

and \[ V_{m_f} = \frac{\partial V(m_h, m_f)}{\partial m_f} = \int [\epsilon(\rho_1 + \beta V_{m'})] G(s) ds = \beta E \left[ V_{m'} \widehat{c}(1 + i^*) \right]. \]

Solving the first-order conditions, we get the optimal conditions (19) - (22),

\[ P_x C_x = a(m_h - n_h), \quad P_y C_y = (1 - a)(m_h - n_h), \quad \frac{\beta w}{m_h' - n_h'} = v, \quad \text{and} \quad E \left[ \frac{u_x}{P_x} \right] = \beta E \left[ 1 + i \right] E \left[ \frac{u_x}{P_x} \right], \]

\[ E \left[ \frac{1 - \alpha}{\alpha} (1 + i) w \frac{\alpha P_x A_x \theta^o w^o}{(1 + i)w} \right] = E \left[ \frac{1 - \alpha}{\alpha} \left[ \xi(1 + i) + (1 - \xi)\widehat{c} \right] q^x \frac{\alpha P_x A_x \theta^o}{[\xi(1 + i) + (1 - \xi)\widehat{c}] w^o} \right] - P_x \kappa. \]

Similarly, solving the foreign representative household’s problem gives the optimal conditions for \[ C_x^*, C_y^*, h^*, \text{and} n_f^*. \]

\[ P_x^* C_x^* = (1 - a^*) (m_f^* - n_f^*), \quad P_y^* C_y^* = a^* (m_f^* - n_f^*), \quad \frac{\beta^* w^*}{m_f^* - n_f^*} = v^*, \quad \text{and} \quad E \left[ \frac{u_x}{P_x} \right] = \beta E \left[ 1 + i^* \right] E \left[ \frac{u_x}{P_x} \right], \quad \text{where} \quad E \left[ \frac{u_x}{P_x} \right] = \frac{1}{m_f^* - n_f^*}. \quad (A5) \]

Given the s is i.i.d. across time, we have \[ 1 - n = m_h - n_h \] and \[ 1 - n^* = m_f^* - n_f^* \] regardless of \[ m_h \] and \[ m_f^*. \]

**B. Solving for the Stationary Competitive Equilibrium in Section 3.6**

In equilibrium, each household optimizes and all markets clear. The market-clearing conditions are as follows,

- labor markets: \[ h = l_x, \quad h^* = l_y^* + l_f^*; \]
- goods markets: \[ Q_I = Q_I^*, \quad C_x + \text{IM}_x = Q_x, \quad \text{IM}_x^* = C_x^*, \quad \text{IM}_y + C_y^* = Q_y^*, \quad \text{IM}_y = C_y; \]
- money markets: \[ m_h + m_h^* = 1, \quad m_f + m_f^* = 1, \]
- loan markets: \[ n_h + \bar{v} m_f = w l_x + \xi \bar{v} q^* Q_I + \bar{v} P_y^* C_y + b_h + \bar{v} b_f, \]
  \[ n_f^* + \frac{m_f^*}{\bar{v}} = w^* l_y^* + (w^* l_f^* - \xi q^* Q_I) + \frac{P_x C_x^*}{\bar{v}} + b_f^* + \frac{b_f^*}{\bar{v}}; \]
- bond markets: \[ B + b_h + b_h^* + \bar{v} Z^* = 0, \quad b_f + b_f^* = Z^*; \]
- foreign exchange market: beginning-of-period: \[ P_y^* C_y^* + \xi q^* Q_I + (b_f - m_f) = \frac{P_x C_x^* (b_h^* - m_h^*)}{\bar{v}} + Z^*; \]
  \[ \text{end-of-period:} \quad \frac{b_h^*(1 + i)}{\bar{v}} + (1 - \xi) q^* Q_I = \bar{Z}^* + b_f(1 + i^*). \]

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Financial intermediaries are allowed to exchange for the currency they need in the foreign exchange market when allocating their loans, we simplify the analysis by focusing on the equilibrium in which \( m_h = m_f = 1 \).

Under a fixed exchange rate regime, \( \pi = \hat{e} \) and capital controls is imposed, \( b_h^* = b_f = 0 \). Combining the foreign exchange market clearing conditions, we get the trade balance \( TB^* \equiv P_y^* C_y - \frac{P_f^* C_f^*}{\pi} + q^* Q_I = Z^* + \hat{Z}^* \), where the balance-of-payment equilibrium, \( TB^* + iZ^* = (1 + i)Z^* + \hat{Z}^* \), states that the foreign country’s current account surplus, consisting of its trade balance and monetary authority’s interest income from holding \( \pi Z^* \) units of home bonds, equals its capital account deficit, reflecting the increase in the monetary authority’s holding of home currency resulting from its official sales of foreign currency. The official sale \( z^* \equiv Z^* + \frac{\hat{Z}^*}{1+i} = P_y^* C_y + w^* l_I^* = \frac{P_f^* C_f^*}{\pi} \) serves to meet the private excess demand for foreign currency in the foreign exchange market. It is noted that \( w^* l_I^* = \xi q^* Q_I + \left( \frac{(1-\xi)q^* Q_I}{1+i} \right) \). Using equations (2), (3), (5), (7), (8), (11), (12), and (19), we get the equilibrium quantities, of good \( I \).

The foreign exchange market clearing conditions give \( P_y^* C_y + w^* l_I^* = \frac{P_f^* C_f^*}{\pi} \). The balance-of-payment equilibrium, \( TB^* + iZ^* = (1 + i)Z^* + \hat{Z}^* \), states that the foreign trade balance \( TB^* \equiv P_y^* C_y - \frac{P_f^* C_f^*}{\pi} + q^* Q_I \) equals its interest payment to the home intermediaries, \( \frac{(1-\xi)q^* Q_I}{1+i} \), while both its current and capital accounts are zero.

From households’ optimal conditions, we get the equilibrium prices under a fixed \( \pi \),

\[
P_x^* = \frac{P_x(1+i^*)}{\pi}, \quad P_y = P_y^* \pi (1+i), \quad w = \frac{v(1-n)}{\beta}, \quad w^* = \frac{v^*(1-n^*)}{\beta^*}, \quad \text{and} \quad q^* = \frac{(1+i^*)u^*(1-n^*)}{\psi^*_I (1 + \xi i^*)^{\pi^*}}.
\]

Using equations (2), (3), (5), (7), (8), (11), (12), and (19), we get the equilibrium quantities,

\[
l_y^* = \left( \frac{\alpha^* P_y^* A_y^*}{1+i^*} \right)^{\frac{\pi n}{\alpha}} \left( \frac{v^* (1-n^*)}{\beta^*} \right)^{-\frac{\pi n}{\alpha^*}}, \quad Q_y^* = \frac{A_y^*}{\psi^*_I (1+i^*)^{\pi^*}} \left( \frac{n^* (1-n^*)}{\beta^*} \right)^{-\frac{\pi n}{\alpha^*}},
\]

\[
l_x^* = \left( \frac{v(1-n)}{\beta} \right)^{-\frac{\pi n}{\alpha^*}} \left( \frac{\alpha P_x A_x}{1+i} \right)^{\frac{\pi n}{\alpha^*}} \left( \frac{1-\alpha}{2-\alpha} \right) \left[ \theta^0 \tilde{\theta}^* - \tilde{\theta}^* \right],
\]

\[
Q_I = \frac{\pi (1+i^*) v^* (1-n^*)}{\psi^*_I (1+\xi i^*)^{\beta^*}} \left( \frac{\alpha P_y A_y}{1+i} \right)^{\frac{\pi n}{\alpha^*}} \left( \frac{1-\alpha}{2-\alpha} \right) \left[ \theta^0 \tilde{\theta}^* - \tilde{\theta}^* \right],
\]

\[
l_I^* = \frac{1}{\psi^*_I} \left[ \frac{\pi (1+i^*) v^* (1-n^*)}{\psi^*_I (1+\xi i^*)^{\beta^*}} \right]^{\frac{\pi n}{\alpha^*}} \left( \frac{\alpha P_y A_y}{1+i} \right)^{\frac{\pi n}{\alpha^*}} \left( \frac{1-\alpha}{2-\alpha} \right) \left[ \theta^0 \tilde{\theta}^* - \tilde{\theta}^* \right], \quad \text{and} \quad w^* l_I^* = \left( \frac{1+\xi i^*}{1+i^*} \right) q^* Q_I,
\]

\[
Q_x = Q_x^{th} + Q_x^{os}, \quad Q_x^{th} = \frac{A_x}{\alpha^*} \left( \frac{\alpha P_x A_x}{1+i} \right)^{\frac{\pi n}{\alpha^*}} \left[ \frac{v(1-n)}{\beta} \right]^{\frac{\pi n}{\alpha^*}} \left[ \theta^0 \tilde{\theta}^* - \tilde{\theta}^* \right],
\]

\[
Q_x^{os} = A_x \left( \frac{1-\alpha}{2-\alpha} \right) \left( \frac{\alpha P_x A_x}{1+i} \right)^{\frac{\pi n}{\alpha^*}} \left[ \frac{\pi (1+i^*) v^* (1-n^*)}{\psi^*_I (1+\xi i^*)^{\beta^*}} \right]^{\frac{\pi n}{\alpha^*}} \left[ \theta^0 \tilde{\theta}^* - \tilde{\theta}^* \right] - \kappa \left( \tilde{\theta}^* - \theta^0 \right),
\]

\[
C_x = \frac{a(1-n)}{P_x}, \quad C_y = \frac{(1-a)(1-n)}{P_y^*} \pi (1+i), \quad C_x^* = \frac{\alpha^*(1-n^*)}{P_y^*}, \quad \text{and} \quad C_x^* = \frac{\pi (1-a^*)(1-n^*)}{P_x(1+i^*)}.
\]
Using the equilibrium values of \( w, w^*, q, l_x, Q_I, Q_{xh}, Q_{os}, l_y, Q_y, C_x, C_y, C_x^*, \) and \( C_y^* \), we rewrite the market-clearing conditions for good \( x \), good \( y \), home loans, and foreign loans, and foreign exchange respectively as the ones presented in Table 1 or as equations (A6) – (A10).

**Good x:**

\[
A_x \left( \frac{1-a}{2-a} \right) \left( \frac{\alpha P_x A_x}{1 + \xi i} \right) \frac{v^*(1-n^*)}{\beta^*} \left[ \alpha P_x A_x \right] \left( \frac{v(1-n)}{\beta} \right) \left[ \theta^0 \frac{\beta}{\theta - \theta^0} - \frac{\beta}{\theta - \theta^0} \right] = \frac{1}{P_x} \left[ a(1-n) + \frac{\theta^0(1-a^*)(1-n^*)}{\beta(1+i^*)} \right], \quad (A6)
\]

**Good y:**

\[
A_y \left( \frac{\alpha P_y A_y}{1 + i^*} \right) \frac{v^*(1-n^*)}{\beta^*} \left[ \alpha P_y A_y \right] \left( \frac{v(1-n)}{\beta} \right) \left[ \theta^0 \frac{\beta}{\theta - \theta^0} - \frac{\beta}{\theta - \theta^0} \right] = \frac{1}{P_y} \left[ \frac{(1-a)(1-n)}{\theta(1+i)} + a^*(1-n^*) \right], \quad (A7)
\]

**Home loan:**

\[
n + B = \left( \frac{1-a}{2-a} \right) \left( \frac{\alpha P_x A_x}{1 + \xi i} \right) \left( \frac{v(1-n)}{\beta} \right) \left[ \theta^0 \frac{\beta}{\theta - \theta^0} - \frac{\beta}{\theta - \theta^0} \right] + \frac{\theta^0(1-a^*)(1-n^*)}{1+i^*}, \quad (A8)
\]

**Foreign loan:**

\[
n^* = \left( \frac{1-a}{2-a} \right) \left( \frac{\alpha P_x A_x}{1 + i^*} \right) \left( \frac{v(1-n)}{\beta} \right) \left[ \theta^0 \frac{\beta}{\theta - \theta^0} - \frac{\beta}{\theta - \theta^0} \right] + \frac{\theta^0(1-a^*)(1-n^*)}{1+i^*}
\]

\[
+ \frac{1}{\beta} \left( \frac{1 + \xi i^*}{1 + \xi i} \right) \left( \frac{\alpha P_x A_x}{1 + \xi i} \right) \left( \frac{v(1-n^*)}{\beta^*} \right) \left[ \theta^0 \frac{\beta}{\theta - \theta^0} - \frac{\beta}{\theta - \theta^0} \right] = z^*. \quad (A9)
\]

Equations (A6) – (A10) characterize the equilibrium values of \( P_x, P_y, i, i^* \), and \( z^* \) as implicit functions of \( \theta^0, n, n^* \), and the current state \( s = (A_x, A_y, \phi_I, B) \). Hence, we can use equations (21), (22), and (A5) and take the expectations over \( s \) to determine the equilibrium values of \( \theta^0, n \), and \( n^* \).

**C. The Comparative Static Exercises in Section 4.1**

Using equations (A7), (A8), and (A10) to rewrite equations (A6) and (A9), the interactions of various market forces can be summarized by using two equations, \( ES_x \equiv Q_{xh} + Q_{os} - C_x - C_x^* = 0 \) and \( ES_{loan} \equiv n^* - w^*l^*_x - w^*l^*_y - P_y^*C_y = 0 \), in two unknowns, \( P_x \) and \( i^* \), so as to characterize the general equilibrium of the world economy.

\[
ES_x(P_x, i^*, B, A_x, A_y, \phi_I, \theta^0, n, n^*) \equiv A_x \left( \frac{\alpha P_x A_x}{1 + i} \right) \Omega + A_y \left( \frac{\alpha P_x A_x}{1 + i} \right) \Omega^* \left( \frac{\phi_I}{1 + i^*} \right) \Omega^* - \frac{1}{\theta - \theta^0} \left[ a(1-n) + \frac{\theta^0(1-a^*)(1-n^*)}{\beta(1+i^*)} \right] = 0, \quad (A6')
\]

\[
ES_{loan}^*(P_x, i^*, B, A_x, A_y, \phi_I, \theta^0, n, n^*) \equiv n^* - \frac{1}{\beta} \left( \frac{1 + \xi i^*}{1 + i^*} \right) \left( \frac{\alpha P_x A_x}{1 + \xi i} \right) \left( \frac{\phi_I}{1 + i^*} \right) \Omega^*, \quad (A9')
\]
where \( \Omega = \left( \frac{1-\alpha}{2-\alpha} \right) \left( \frac{\eta(1-n^*)}{\beta} \right)^{1-\alpha} \left[ \theta \eta^{-\frac{1-\alpha}{2-\alpha}} - \xi^{-\frac{1-\alpha}{2-\alpha}} \right] \), \( \Omega^* = \left( \frac{1-\alpha}{2-\alpha} \right) \left( \frac{\eta(1-n^*)}{\beta^*} \right)^{1-\alpha} \left[ \theta \eta^{-\frac{1-\alpha}{2-\alpha}} - \theta^* \eta^{-\frac{1-\alpha}{2-\alpha}} \right] \),

\[
1 + i = \frac{\alpha P_x A_x \Omega^{-\alpha}}{n + B - \bar{\eta}(1-\alpha)(1-n^*)}^{1-\alpha} \quad \text{and} \quad 1 + \xi = 1 - \xi + \frac{\xi \alpha P_x A_x \Omega^{-\alpha}}{n + B - \bar{\eta}(1-\alpha)(1-n^*)}^{1-\alpha}.
\]

Given the predetermined values of \( \theta^* \), \( n \), and \( n^* \), and the exogenously fixed exchange rate \( \bar{\eta} \), we use equations (A6') and (A9') to derive the responses of \( P_x \) and \( i^* \) to different realizations of \( s \), and then use equations (A7), (A8), and (A10) to obtain the effects on \( P_y^* \), \( i^* \), and \( TB^* \).

Totally differentiating equations (A6') and (A9') gives the following two differential equations,

\[
dES_x = \frac{dES_x}{dx} = \frac{\left( P_x C_x^* - \frac{(1 + \xi)q Q_t}{1 - \alpha} \left( 1 - \xi/(1 + \xi i) \right) \right) di^*}{1 + i} + \left( \frac{1 + i \lambda (1 + \xi)q Q_t}{1 - \alpha} \right) \frac{dP_x}{P_x} - \frac{(1 + \xi)(1 + \xi i)q Q_t}{1 - \alpha} \phi_i^* \phi_i^* + \frac{\left( (1 + \xi)(1 + \xi i)q Q_t \right)}{1 - \alpha} \phi_i^* \phi_i^*,
\]

\[
dES_{\text{loan}} = \frac{dES_{\text{loan}}}{dx} = \left[ \frac{\alpha^*}{1 + i} \left( P_y C_y + P_y C_y^* \right) + \frac{w l^*_y}{1 - \alpha} \left( 1 - \xi/(1 + \xi i) \right) \right] \frac{dP_x}{P_x} + \frac{dA_x}{A_x} - \frac{(1 + \xi)(1 + \xi i)q Q_t}{1 - \alpha} \phi_i^* \phi_i^* + \left( \frac{1 + \alpha^*}{1 + i^*} \right) \frac{dP_x}{P_x} + \frac{dA_x}{A_x} - \frac{(1 + \xi)(1 + \xi i)q Q_t}{1 - \alpha} \phi_i^* \phi_i^*.
\]

The case with a fixed exchange rate regime

With \( \epsilon = \bar{\eta} \), we have \( \frac{di}{1 + i} = \frac{dP_x}{P_x} + \frac{dA_x}{A_x} \), and get

\[
dES_x = \lambda_{1P_x} dP_x + \lambda_{1i^*} di^* + \lambda_{1B} dB + \lambda_{1A_x} dA_x + \lambda_{1A_y} dA_y + \lambda_{1\phi_i} d\phi_i^* \quad \text{and} \quad
\]

\[
dES_{\text{loan}} = \lambda_{2P_x} dP_x + \lambda_{2i^*} di^* + \lambda_{2B} dB + \lambda_{2A_x} dA_x + \lambda_{2A_y} dA_y + \lambda_{2\phi_i} d\phi_i^* \quad \text{and} \quad
\]

where

\[
\lambda_{1P_x} = \frac{\partial ES_x}{\partial P_x} = \frac{1}{1 - \alpha} \left[ P_x C_x + P_x C_y + \alpha P_x \kappa(\bar{\eta} - \theta^*) - \left[ w l_x + \xi q Q_t \right] (1 + i) \right] \frac{1}{P_x} > 0,
\]

\[
\lambda_{1i^*} = \frac{\partial ES_x}{\partial i^*} = \left[ P_x C_x^* \left( 1 + \alpha^* + (1 + \xi)(1 + \xi^*) \right) \left( 1 - \xi/(1 + \xi^*) \right) \right] \frac{1}{1 + i^*} > 0,
\]

\[
\lambda_{1B} = \frac{\partial ES_x}{\partial B} = \left( \frac{w l_x + \xi q Q_t}{w l_x} \right) (1 + i) > 0,
\]

\[
\lambda_{1A_x} = \frac{\partial ES_x}{\partial A_x} = \frac{1}{1 - \alpha} \left[ \left( P_x C_x + P_x C_y + P_x \kappa(\bar{\eta} - \theta^*) \right) - \left[ w l_x + \xi q Q_t \right] (1 + i) \right] \frac{1}{A_x} > 0,
\]

\[
\lambda_{1A_y} = \frac{\partial ES_x}{\partial A_y} = 0,
\]

\[
\lambda_{1\phi_i} = \frac{\partial ES_x}{\partial \phi_i^*} = \left( \frac{1 + \xi^*}{1 - \alpha} \right) q Q_t \phi_i^* > 0.
\]
\[ \lambda_{2P*} \equiv \frac{\partial ES_{\text{loan}}}{\partial P_x} = \left\{ \begin{array}{l} P_y C_y \left[ 1 + \frac{\alpha^*}{1 + i^*} \right] - \frac{w^* l_1^*}{1 - \alpha} \left[ \frac{1 - \xi}{1 + \xi i} \right] \end{array} \right\} \frac{1}{P_x} > 0, \]

\[ \lambda_{2i^*} \equiv \frac{\partial ES_{\text{loan}}}{\partial i^*} = \left\{ \begin{array}{l} \frac{w^* l_1^*}{1 - \alpha} \left[ \frac{1 - \xi}{1 + \xi i} \right] + \left[ \frac{P_y C_y + P_y C_y}{\alpha^*} \right] \frac{1}{1 + i^*} \\

- \left[ \frac{w^* l_1^*}{1 - \alpha} \left[ \frac{\xi(1 + i)}{1 + \xi i} \right] + P_y C_y \left[ 1 + \frac{\alpha^*}{1 + i^*} \right] \right] \frac{(1 - \alpha) P_x C_y}{\omega l x} \end{array} \right\} \frac{1}{1 + i^*} > 0, \]

\[ \lambda_{2B} \equiv \frac{\partial ES_{\text{loan}}}{\partial B} = \left\{ \begin{array}{l} \frac{w^* l_1^*}{1 - \alpha} \left[ \frac{\xi(1 + i)}{1 + \xi i} \right] + P_y C_y \left[ 1 + \frac{\alpha^*}{1 + i^*} \right] \end{array} \right\} \frac{(1 - \alpha)}{\omega l x} < 0, \]

\[ \lambda_{2A_x} \equiv \frac{\partial ES_{\text{loan}}}{\partial A_x} = \left\{ P_y C_y \left[ 1 + \frac{\alpha^*}{1 + i^*} \right] - \frac{w^* l_1^*}{1 - \alpha} \left[ \frac{1 - \xi}{1 + \xi i} \right] \frac{1}{A_x} > 0, \]

\[ \lambda_{2A_y}^* \equiv \frac{\partial ES_{\text{loan}}}{\partial A_y^*} = 0, \]

\[ \lambda_{2\phi_i^*} \equiv \frac{\partial ES_{\text{loan}}}{\partial \phi_i^*} = \frac{\alpha w^* l_1^*}{(1 - \alpha) \phi_i^*} < 0, \]

where we have assumed that \(a, \alpha^*, a\) and \(\alpha\) are sufficiently high such that \(\lambda_{1P_x} \lambda_{2i^*} - \lambda_{2P_x} \lambda_{1i^*} > 0\) when \(\theta^o = \overline{\theta}\), and \(\lambda_{1i^*} > 0\) and \(\lambda_{2i^*} > 0\) when \(\theta^o < \overline{\theta}\).

When \(\theta^o < \overline{\theta}\), as \(\frac{\partial^2 ES_x}{\partial \xi \partial P_x} = \frac{\partial \lambda_{1P_x}}{\partial \xi} < 0, \frac{\partial^2 ES_x}{\partial \xi \partial i^*} = \frac{\partial \lambda_{1i^*}}{\partial \xi} > 0, \frac{\partial^2 ES_{\text{loan}}}{\partial \xi \partial P_x} = \frac{\partial \lambda_{2P_x}}{\partial \xi} > 0, \) and \(\frac{\partial^2 ES_{\text{loan}}}{\partial \xi \partial i^*} = \frac{\partial \lambda_{2i^*}}{\partial \xi} < 0, \)

we find that \(\lambda_{1P_x} \lambda_{2i^*} - \lambda_{2P_x} \lambda_{1i^*} > 0\) when \(\xi\) is small, and \(\lambda_{1P_x} \lambda_{2i^*} - \lambda_{2P_x} \lambda_{1i^*} < 0\) when \(\xi\) is large.

Let \((\overline{i}, \overline{P})\) be the general equilibrium values of \(i^*\) and \(P_x\) that solve \(ES_x = 0\) and \(ES_{\text{loan}}^* = 0\) simultaneously, given the values of the exogenous variables \(B, A_x, A_y^*, \phi_i^*\) and \(\phi_i^*\) and the predetermined values of \(\theta^o, n\) and \(n^*\), the linear approximations of \(ES_x(i^*, P_x)\) and \(ES_{\text{loan}}^*(i^*, P_x)\) about \((\overline{i}, \overline{P})\) are given by

\[ ES_x(i^*, P_x) \approx \lambda_{1P_x}(\overline{i}, \overline{P}) (P_x - \overline{P}) + \lambda_{1i^*}(\overline{i}, \overline{P})(i^* - \overline{i}), \quad \frac{dP_x}{di^*} \bigg|_{ES_x(i^*, P_x) = 0} \approx -\frac{\lambda_{1i^*}}{\lambda_{1P_x}} < 0, \]

\[ ES_{\text{loan}}^*(i^*, P_x) \approx \lambda_{2P_x}(\overline{i}, \overline{P}) (P_x - \overline{P}) + \lambda_{2i^*}(\overline{i}, \overline{P})(i^* - \overline{i}), \quad \frac{dP_x}{di^*} \bigg|_{ES_{\text{loan}}^*(i^*, P_x) = 0} \approx -\frac{\lambda_{2i^*}}{\lambda_{2P_x}} < 0. \]

Figure 3 plots these two linearized relations between \(i^*\) and \(P_x\). Given that \(\lambda_{1P_x} \lambda_{2i^*} - \lambda_{2P_x} \lambda_{1i^*} > 0\) when \(\xi\) is small, and \(\lambda_{1P_x} \lambda_{2i^*} - \lambda_{2P_x} \lambda_{1i^*} < 0\) when \(\xi\) is large, the linearized relation that satisfies \(ES_x = 0\) will be flatter (steeper) than the one satisfying \(ES_{\text{loan}}^* = 0\) if \(\xi\) is small (large).

The equilibrium effects on \(P_x\) and \(i^*\) are given by

\[ \frac{dP_x}{dJ} = -\frac{\lambda_{1J} \lambda_{2i^*} - \lambda_{2J} \lambda_{1i^*}}{\lambda_{1P_x} \lambda_{2i^*} - \lambda_{2P_x} \lambda_{1i^*}}, \quad \text{and} \quad \frac{di^*}{dJ} = -\frac{\lambda_{1P_x} \lambda_{2J} - \lambda_{2P_x} \lambda_{1J}}{\lambda_{1P_x} \lambda_{2i^*} - \lambda_{2P_x} \lambda_{1i^*}}. \]

Together with equations (A7), (A8), and (A10), we can derive the following results and summarize the effects of the temporary shocks in Table 2.
\[
\frac{dX}{1 + i} = \frac{dA_x}{A_x} - \frac{1 - \alpha}{\omega X} \left[ dB + P_x C_x^{*} \frac{d\theta^*}{1 + i^*} \right] + \frac{dP \alpha}{P_x} \Rightarrow \ i(P_x, \ i^*, \ B, \ A_x, \ A_y, \ \phi_i, \ \theta^o, \ n, \ n^*) , \\
+ + - + \ 0 \ 0
\]
\[
\frac{dP_y \alpha}{P_y} = -\frac{dA_y^{*}}{A_y^{*}} + \alpha^{*} \frac{d\theta^*}{1 + i^*} - \left[ (1 - \alpha) P_y C_y^{*} \right] \frac{di}{1 + i} , \Rightarrow \ P_y^{*}(P_x, \ i^*, \ B, \ A_x, \ A_y^{*}, \ \phi_i, \ \theta^o, \ n, \ n^*) , \\
- + + - 0 +
\]
\[
\frac{dT B^*}{T B^*} = \left[ \frac{P_x C_x^{*}}{\tau} - \frac{\alpha q Q_x}{1 - \alpha} \left[ 1 - \xi \left( 1 + \xi^* \right) \right] \frac{di}{1 + i^*} + \frac{d\theta^*}{\phi_i} + \frac{dA_y}{A_x} \right] \frac{\alpha}{1 - \alpha} \left[ \frac{P_y C_y^{*}}{1 - \alpha} \left[ \xi(1 + i) \right] \right] \frac{di}{1 + i} .
\]
\[
\Rightarrow \ TB^*(P_x, \ i^*, \ B, \ A_x, \ A_y^{*}, \ \phi_i, \ \theta^o, \ n, \ n^*) ,
\]
\[
- + + - 0 +
\]

The case with a flexible exchange rate regime

Similar to the derivation in the case with a fixed exchange rate regime, we can examine the \(dES_x\) and \(dES_{loan}\) equations in the case with a flexible exchange rate by using equations (A6') and (A9'), and applying
\[
\frac{de}{\epsilon} = \frac{di}{1 + i} - \frac{n^*(1 + i^*)}{\alpha P_x C_y^{*} \frac{di}{1 + i^*}} \quad \text{and} \quad \frac{di}{1 + i} = \frac{-dB - P_x C_x^{*} \left[ 1 + \frac{n^*(1 + i^*)}{\alpha P_x C_y^{*}} \right] \frac{di}{1 + i^*} + \frac{\omega X}{\tau - \alpha} \frac{dP \alpha}{P_x} + \frac{dA_x}{A_x}}{\omega X - \alpha P_x C_y^{*}} .
\]

We obtain
\[
dES_x = \left\{ P_x C_x^{*} \left[ 1 + \xi q Q_x \right] \left[ 1 - \xi \left( 1 + \xi^* \right) \right] + \left[ (1 + i) w l_x - (1 - \xi) q Q_l \right] \frac{di}{1 + i^*} \right\}
\]
\[
+ \left[ \frac{(1 + i) w l_x + (1 + \xi) q Q_l}{\alpha(1 - \alpha)} \right] - \left[ \frac{(1 + i) w l_x - (1 - \xi) q Q_l}{\alpha(1 - \alpha)} \right] \frac{di}{1 + i^*} \right\}
\]
\[
+ \left[ \frac{w l_x}{\tau - \alpha} \frac{dP \alpha}{P_x} \right] \frac{di}{1 + i^*} + \left[ \frac{w l_x}{\tau - \alpha} \frac{dA_x}{A_x} \right] \frac{di}{1 + i^*} .
\]
\[
dES_{loan} = -\left\{ \frac{\alpha^*}{1 + i^*} \left[ P_y C_y^{*} + P_y C_y^{*} \right] + \frac{w^* l^*_y}{1 + i^*} \left[ 1 - \xi \left( 1 + \xi^* \right) \right] \right\}
\]
\[
+ \left[ \frac{w^* l^*_y}{1 - \alpha} \left( 1 + i^* \right) \right] \frac{di}{1 + i^*} \right\}
\]
\[
- \left[ \frac{w^* l^*_y}{1 - \alpha} \left( 1 + \xi \right) \right] \frac{di}{1 + i^*} \right\}
\]

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and get \( ES_x(P_x, i^*, B, A_x, A_y, \phi^*, \theta^o, n, n^*) \) and \( ES^{\text{loan}}_x(P_x, i^*, B, A_x, A_y, \phi^*, \theta^o, n, n^*) \).

The linear approximation of \( ES_x(i^*, P_x) = 0 \) and \( ES^{\text{loan}}_x(i^*, P_x) = 0 \) around the general equilibrium under a flexible exchange rate regime are both downward sloping, and the linearized relation that satisfies \( ES_x = 0 \) will be flatter than the one satisfying \( ES^{\text{loan}}_x = 0 \). The value of \( \xi \) does not affect the derivatives in a way that can change the relative slope of the two linearized relations qualitatively.

**D. Deriving the Results of the Comparative Static Exercises in Section 4.2**

For simplicity, we assume no uncertainty in \( s \) and have \( \beta(1 + i) = \beta^o(1 + i^*) = 1 \).

The case with no outsourcing \((\theta^o = \overline{\theta})\):

The market-clearing conditions for the markets for home loans, foreign loans, and foreign exchange are,

\[
\begin{align*}
    n + B &= \frac{\alpha a(1 - n)}{1 + i} + \left[ 1 + \frac{\alpha}{1 + i} \right] \overline{\theta}(1 - a^*)(1 - n^*), \\
    n^* &= \frac{\alpha^*a^*(1 - n^*)}{1 + i^*} + \left[ 1 + \frac{\alpha^*}{1 + i^*} \right] \frac{(1 - a)(1 - n)}{\overline{\tau}(1 + i)}, \\
    \frac{(1 - a)(1 - n)}{1 + i} &= \frac{\overline{\tau}(1 - a^*)(1 - n^*)}{1 + i^*} + \tau Z^*.
\end{align*}
\]

It is noted that \( \tilde{Z}^* = 0 \). Assuming that the values of \( a, a^*, \alpha \) and \( \alpha^* \) are sufficiently high. We can show that

\[
\frac{dn}{d\overline{\tau}} > 0, \quad \frac{dn^*}{d\overline{\tau}} < 0 \quad \text{and} \quad \frac{dZ^*}{d\overline{\tau}} = \frac{dTB^*}{d\overline{\tau}} < 0. \quad \text{Hence, we get} \quad \frac{dP^\text{C}_{\text{LR}}}{d\overline{\tau}} < 0, \quad \text{and} \quad \frac{d\left( P^\text{LR}_{\text{LR}} \right)}{d\overline{\tau}} > 0.
\]

The case with international outsourcing \((\theta^o < \overline{\theta})\):

We simplify the analysis further by assuming \( \beta = \beta^o \), which implies \( i = i^* \). Let \( P^\text{C}_{\text{LR}} \equiv P_x/\overline{\tau}, \overline{N} \equiv (1 - n)/\overline{\tau}, \)

\( \overline{N}^* \equiv (1 - n^*), \) and \( \overline{B} \equiv B/\overline{\tau} \). Combining the market clearing conditions for good \( y \), foreign loan, and foreign exchange, denoted respectively by \( (A7), (A9), \) and \( (A10) \), yields

\[
1 - \left[ 1 + \frac{\alpha a^*}{1 + i^*} + \frac{1 - a^*}{1 + i^*} \right] \overline{N}^* - \left( \frac{\alpha}{1 + i^*} \right) \frac{(1 - a)\overline{N}}{1 + i} - z^* = 0. \quad (A11)
\]

We then use \( (A10), (A11) \), and the home firm’s optimal outsourcing condition, \( (22) \), to rewrite the market clearing condition for good \( x \), \( (A6) \), as follows,

\[
\left[ z^* + \frac{(1 - a^*)\overline{N}^*}{1 + i^*} - \frac{(1 - a)\overline{N}}{1 + i} \right] \frac{1 + i^*}{1 + \xi i^*} \left[ \frac{\overline{Z}^*}{\phi^o \theta^o \overline{\tau}} - \theta^o \frac{\overline{Z}^*}{\phi^o \theta^o \overline{\tau}} + (2 - \alpha) \left( \overline{B} - \theta^o \right) \theta^o \frac{1}{\overline{\tau}} \right] \left[ 1 - \frac{(1 + i^*)^n^* \overline{N}^*}{\phi^o \theta^o \overline{\tau} \overline{\tau}} \right] \frac{(1 + \xi i^* \overline{\tau})}{\phi^o \theta^o \overline{\tau} \overline{\tau}} \frac{1}{\overline{\tau}} = \frac{\alpha}{1 + \xi i^*} \left[ a\overline{N} + \frac{(1 - a^*)\overline{N}^*}{1 + i^*} \right] \left[ \frac{\overline{Z}^*}{\phi^o \theta^o \overline{\tau}} - \theta^o \overline{Z}^* \right]. \quad (A12)
\]
Using an expression of \( \frac{\alpha_p^* A_x^*}{\xi_1} \) from equation (A10) and applying equation (A11), we rewrite the home firm’s optimal outsourcing condition, (22), and the market clearing condition for home loans, (A8), respectively as follows.

\[
\frac{\kappa}{A_x} \left[ \frac{1 + \xi i^*}{1 + \nu^*} \right] \left[ (1 + \delta^*) \gamma^* \xi^* \right] \left[ \frac{\theta^*}{\theta^* - \theta^* \gamma^*} \right] = (2 - \alpha) \left[ z^*\left(1 - \frac{(1 - a^*) \alpha}{1 + i^*} \right) \right] \left[ \frac{1 - \frac{(1 - a)^*}{1 + i} N^*}{1 - \alpha} \right]^{1 - \alpha} \left[ 1 - \left( \frac{(1 + i^*) \alpha^* N^*}{\theta^* (1 + \xi^*) \beta^*} \right) \left[ \frac{1 + \xi_1^*}{1 + \nu^*} \right] \right]^{\frac{1}{1 - \alpha}}, \quad (A13)
\]

\[
\left[ \frac{1}{1 - N^* + b} \left( \frac{(1 - a^*) \alpha}{1 + i^*} \right) \right] \left[ (1 + \xi^*) \gamma^* \xi^* \right] \left[ \frac{\theta^*}{\theta^* - \theta^* \gamma^*} \right] = z^* \left(1 - \frac{(1 - a)^*}{1 + i} N^* \right), \quad (A14)
\]

Define \( \delta_1 \equiv \left( \frac{\alpha^*}{1 + i^*} \right) \frac{1 - a}{1 + i} \), \( \delta_2 \equiv 1 + \frac{\alpha^* a^*}{1 + i^*} + \frac{1 - a^*}{1 + i^*} \), \( V \equiv \left[ \frac{(1 + \xi^*) \gamma^* \xi^*}{\theta^* (1 + \xi^*) \beta^*} \right] \left[ \frac{1 + \xi_1^*}{1 + \nu^*} \right]^{\frac{1}{1 - \alpha}} \),

\[
\Theta \equiv \frac{\theta^* \gamma^*}{\theta^* - \theta^* \gamma^*}, \quad \text{and} \quad \Psi \equiv \left[ \left( \frac{1}{1 - \alpha} \right) \left( \frac{(1 - a)^*}{1 + i} N^* \right) \right]^{\frac{1}{1 - \alpha}} \left[ \frac{\theta^*}{\theta^* - \theta^* \gamma^*} \right] \left[ \frac{1 + \xi_1^*}{1 + \nu^*} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{1}{1 - \alpha} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{1 - \alpha}{1 + i} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{1 - \alpha}{1 + i} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{1 - \alpha}{1 + i} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{1 - \alpha}{1 + i} \right]^{\frac{1}{1 - \alpha}} \]
\[ A_{2\theta^o} = \left\{ \left( \frac{2-\alpha}{1-\alpha} \right) \theta^o \left( \frac{2-\alpha}{1-\alpha} \right) + \frac{1}{\alpha(1-\alpha)} \right\} \]

\[ A_{2z^*} = \left\{ \frac{z^* + (1-\alpha) (1-\delta_1 N - \delta_2 N^*)}{\delta_1} - \left( \frac{1-\alpha}{1+\alpha} \right) \left( \frac{N^*}{\alpha(1+\alpha)} \right) \right\} \]

\[ A_{2N} = \frac{1}{\alpha} \]

\[ A_{3N^*} = \left\{ \frac{\alpha}{1-\alpha} \left[ \frac{\delta_1 N + \delta_2 N^*}{\delta_1 N} \right] - \left[ \frac{\delta_1 1}{\xi_1} - \frac{\delta_1 1}{\xi_1} \right] N^* \right\} \]

\[ A_{3\theta^o} = \left[ \frac{\alpha}{1-\alpha} \theta^o \left( \frac{2-\alpha}{1-\alpha} \right) + \frac{1}{\alpha(1-\alpha)} \right] \]

\[ A_{3z^*} = \left\{ \frac{\alpha}{1-\alpha} \left[ \frac{1-\delta_1 N - \delta_2 N^*}{\delta_1 N} \right] - \left[ \frac{1-\delta_1 1}{\xi_1} - \frac{1-\delta_1 1}{\xi_1} \right] \right\} \]

\[ A_{3z^*} = \frac{1}{\alpha} \]

The Effects of Changes in \( \bar{\tau} \)

Holding \( \kappa \) constant, the derivation of the effects of a change in \( \bar{\tau} \) as follows. Using equation (A3), we have

\[ A_{3e} + \left[ A_{3z^*} + A_{3N^*} \frac{dN^*}{d\tau^*} + A_{3\theta^o} \frac{d\theta^o}{d\tau^*} \right] \frac{\bar{\tau} \, dz^*}{\bar{\tau}^2} = 0. \]

Using equations (A1) and (A2), we can show that

\[ \text{sign} [A_{1N^*} - A_{2N^*} - A_{1\theta^o}] = \text{sign} [A_{1z^*} - A_{2z^*} - A_{1\theta^o}] = \text{sign} [A_{1N^*} - A_{2z^*} - A_{1\theta^o}] = - \text{sign} \delta_1. \]

\[ \delta_1 > 0 \Rightarrow \frac{z^* \, dN^*}{N^* \, dz^*} = - \frac{A_{1z^*} - A_{2z^*} - A_{1\theta^o}}{A_{1N^*} - A_{2N^*} \, A_{1\theta^o}} < 0, \quad \text{and} \quad \frac{z^* \, d\theta^o}{\theta^o \, dz^*} = - \frac{A_{1N^*} - A_{2z^*} - A_{1\theta^o}}{A_{1N^*} - A_{2N^*} \, A_{1\theta^o}} < 0. \]

In addition, \( A_{3e} > 0 \) and

\[ \left[ A_{3z^*} + A_{3N^*} \frac{z^* \, dN^*}{N^* \, dz^*} + A_{3\theta^o} \frac{z^* \, d\theta^o}{\theta^o \, dz^*} \right] = \frac{A_{3z^*} [A_{1N^*} - A_{2N^*} - A_{1\theta^o}] - A_{3N^*} [A_{1z^*} - A_{2z^*} - A_{1\theta^o}] - A_{3\theta^o} [A_{1N^*} - A_{2z^*} - A_{1\theta^o}] < 0. \]

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Hence, we have \( \frac{dz^*}{dx} > 0, \quad \frac{d\theta^o}{dx} < 0, \quad \frac{dN^*}{dx} < 0, \quad \frac{dn^*}{dx} > 0, \) and \( \frac{d\left( \frac{P_y C}{\bar{e}} \right)}{dx} < 0 \)

\[
\left[ 1 - z^* \frac{z^*}{N^*} \frac{dN^*}{dz^*} + 1 \right] < 0 \Rightarrow \frac{z^*}{N^*} \frac{dN^*}{dz^*} = \frac{z^*}{N^*} \frac{1 - z^*}{z^*} \frac{dN^*}{dz^*} + 1 < 0.
\]

Using \( P_y C_y = \frac{(1 - a)N}{1 + i}, \quad q^* Q_1 \left[ 1 + \xi \right] = 1 - \left[ 1 + \alpha^* \alpha^* \right] \frac{N^* - 1 + \alpha^*}{1 + i^*} (1 - a)N, \) and

\[
\frac{z^*}{N^*} \frac{dN^*}{dz^*} - \frac{dN^*}{d\bar{e}} < 0, \quad \text{we have} \quad \frac{d q^* Q_1}{d\bar{e}} > 0, \quad \text{and} \quad \frac{d(q^* Q_1 + P_y C_y)}{d\bar{e}} > 0.
\]

The Effects of Changes in \( \kappa \):

Using equation (A3), we get

\[
A_{3e} \frac{de}{\bar{e}} + \left[ A_{3z^*} + A_{3N^*} \frac{z^*}{N^*} \frac{\partial N^*}{\partial z^*} + A_{3\theta^o} \frac{z^*}{\theta^o} \frac{\partial \theta^o}{\partial z^*} \right] \frac{dz^*}{\bar{e}} = 0.
\]

In the case with a flexible exchange rate regime, \( dz^* = 0, \)

\[
\frac{\kappa}{N^*} \frac{dN^*}{d\kappa} \bigg|_{dz^* = 0} = \frac{\kappa}{N^*} \frac{\partial N^*}{\partial \kappa} = \frac{A_{2e}}{A_{1N^*} \cdot A_{2\theta^o} - A_{2N^*} \cdot A_{1\theta^o}}, \quad \frac{dN^*}{d\bar{e}} \bigg|_{dz^* = 0} = 0 < 0.
\]

\[
\frac{\kappa}{\theta^o} \frac{d\theta^o}{d\kappa} \bigg|_{dz^* = 0} = \frac{\kappa}{\theta^o} \frac{\partial \theta^o}{\partial \kappa} = - \frac{A_{2e}}{A_{1N^*} \cdot A_{2\theta^o} - A_{2N^*} \cdot A_{1\theta^o}}, \quad \frac{d\theta^o}{d\bar{e}} \bigg|_{dz^* = 0} = 0 > 0.
\]

\[
A_{2e} > 0, \quad A_{3e} > 0, \quad \text{sign} \left[ A_{3N^*} \cdot A_{1\theta^o} - A_{3\theta^o} \cdot A_{1N^*} \right] = - \text{sign} \left[ A_{1N^*} \cdot A_{2\theta^o} - A_{2N^*} \cdot A_{1\theta^o} \right] = - \text{sign} \delta_i \quad \Rightarrow \quad \left[ A_{3N^*} \frac{dN^*}{N^*} \frac{\kappa}{d\kappa} \bigg|_{dz^* = 0} + A_{3\theta^o} \frac{d\theta^o}{\theta^o} \frac{\kappa}{d\kappa} \bigg|_{dz^* = 0} = \frac{A_{2e} \cdot A_{3N^*} \cdot A_{1\theta^o} - A_{3\theta^o} \cdot A_{1N^*}}{A_{1N^*} \cdot A_{2\theta^o} - A_{2N^*} \cdot A_{1\theta^o}} > 0 \right.
\]

\[
\left. \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{and} \quad \frac{\kappa}{\theta^o} \frac{d\theta^o}{d\kappa} \bigg|_{dz^* = 0} = - \frac{1}{A_{3e}} \left( A_{3N^*} \frac{dN^*}{N^*} \frac{\kappa}{d\kappa} \bigg|_{dz^* = 0} + A_{3\theta^o} \frac{d\theta^o}{\theta^o} \frac{\kappa}{d\kappa} \bigg|_{dz^* = 0} \right) \right) = \frac{A_{2e} \cdot A_{3N^*} \cdot A_{1\theta^o} - A_{3\theta^o} \cdot A_{1N^*}}{A_{1N^*} \cdot A_{2\theta^o} - A_{2N^*} \cdot A_{1\theta^o}} < 0.
\]

When \( e \) is fixed at \( \bar{e} \) by allowing \( z^* \) to adjust, using

\[
\left[ A_{3N^*} \frac{\kappa}{N^*} \frac{\partial N^*}{\partial \kappa} + A_{3\theta^o} \frac{\kappa}{\theta^o} \frac{\partial \theta^o}{\partial \kappa} \right] > 0 \quad \text{and} \quad \left[ A_{3z^*} + A_{3N^*} \frac{z^*}{N^*} \frac{\partial N^*}{\partial z^*} + A_{3\theta^o} \frac{z^*}{\theta^o} \frac{\partial \theta^o}{\partial z^*} \right] < 0, \quad \text{we have}
\]

\[
\frac{\kappa}{z^*} \frac{dz^*}{d\kappa} \bigg|_{de = 0} = \frac{A_{3N^*} \frac{\kappa}{N^*} \frac{\partial N^*}{\partial \kappa} + A_{3\theta^o} \frac{\kappa}{\theta^o} \frac{\partial \theta^o}{\partial \kappa}}{A_{3z^*} + A_{3N^*} \frac{z^*}{N^*} \frac{\partial N^*}{\partial z^*} + A_{3\theta^o} \frac{z^*}{\theta^o} \frac{\partial \theta^o}{\partial z^*}} > 0.
\]
References


The World Bank, the World Integrated Trade Solution and the World Development Indicators Databases, the World Bank.