Mathematics 1190A Discrete Mathematics
Introduction to Sets and Logic

December 10, 2012

Instructions: Do all problems. Present your solutions in the accompanying booklet in the order that they appear on this paper. No notes, crib sheets or books are allowed. Calculators and cellular devices are not allowed. The test is 3 hours long.

1. (12 pts) For the following state in full and complete detail
   (a) The Schröder Bernstein Theorem
   (b) Fermat's Little Theorem
   (c) Bézout's Therorem
   (d) The Chinese Remainder Theorem

2. (12 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be a function.
   (a) Using the syntax of propositional logic, define what it means for f not to be injective.
   (b) The function f is strictly increasing means that for all $x \in \mathbb{R}$ and for all $y \in \mathbb{R}$, if $x < y$, then $f(x) < f(y)$. Using the syntax logic, define what it means for f **not** to be strictly increasing.
   (c) A function $f : \mathbb{R} \to \mathbb{R}$ is continuous at a point $a \in \mathbb{R}$ means:
   $\forall \epsilon > 0, \exists \delta > 0$ such that $|x - a| < \delta \longrightarrow |f(x) - f(a)| < \epsilon$.
   Using the syntax of logic, define what it means for f **not** to be continuous at a point $a \in \mathbb{R}$.

3. (12 pts) Perform the following conversions
   (a) decimal 401 to binary
   (b) hexidecimal $2EF$ to decimal
   (c) binary (111101) to decimal
   (d) decimal 568 to hexadecimal
4. (12 pts) Let $f(x) = \lfloor x^2/3 \rfloor$ and let $g(x) = x^2$
   (a) Find $f(S)$ if $S = \{1, -2, -1, 0, 1, 2, 3\}$.
   (b) Find $f^{-1}([-\frac{1}{2}, 3])$.
   (c) Find $g \circ f(S)$
5. (10 pts) Compute the value of the double sum

$$\sum_{i=1}^{2} \sum_{j=1}^{3} (i + j).$$

6. (10 pts)
   (a) Find the prime factorizations of the numbers 120 and 500.
   (b) Using the prime factorizations above, what is the greatest common divisor of 120 and 500.
7. (12 pts) Let $b \equiv 9 \pmod{13}$ and consider $c \equiv 11b \pmod{13}$
   (a) What are the possible values of b?
   (b) What are the possible values of c?
   (c) What is the least positive value of c?
8. (10 pts) Find the least positive integer x that solves the linear congruence $3x \equiv 4 \pmod{7}$.

9. (a) (5 pts) State the Fundamental Theorem of Arithmetic.

   (b) (10 pts) Using the Fundamental Theorem of Aritmetic, prove that $\sqrt{2}$ is an irrational number.

10. (10 pts) Using Fermat's Little Theorem, find the remainder when dividing 7^{222} by 11. That is, find the least positive number x such that: $7^{222} \equiv x(\operatorname{mod} 11)$.

11. (10 pts) Using mathematical induction, show that 2 divides $n^2 + n$, for every positive integer n.

12. (10 pts)

   (a) Describe the principle of Strong Induction.

   (b) For every integer n let $P(n)$ be the proposition: n is s product of prime numbers. Using Strong Induction, prove $P(n)$ is true for every integer n.