Mathematics 1190A Introduction to Sets and Logic Midterm Examination B

October 18, 2012

Instructions: Do all problems. Present your solutions in the accompanying booklet in the order that they appear on this paper. No notes, crib sheets or books are allowed. A calculator is allowed. Questions are of equal weight. The test is 50 minutes long.

1. (a) For $1 \leq i \leq \infty$, let A_i be a set. Describe using set notation what is meant by $\bigcap_{i=1}^{\infty} A_i$. see page 134. Or from your class notes you will see that $\bigcap_{i=1}^{\infty} A_i = \{x : \forall i \in \mathbb{Z}^+, x \in A_i\}$

   (b) Let $f : A \to B$ be a function from a set A to a set B. Let S be a subset of B. Define the set $f^{-1}(S)$
   problem 43 section 2.3 was assigned. To have done this problem it would have been necessary to read the text between problems 41 and 42 - where you can find the answer. Or from your class notes you can see $f^{-1}(S) = \{x \in A : f(x) \in S\}$.

   (c) Define what it means for two sets to have the same cardinality.
   see definition 1 p. 170

2. Determine the truth value of each of the following statements in which the domain of each variable is the real numbers. If false, provide a counter-example.see definition 1 p. 170

   (a) $\forall x \exists y\ (x^2 = y)$ true

   (b) $\exists x \forall y\ x^2 = y)$ false - if $x \neq 0$ the statement says every y is a square. for the statement to be true If $x = 0$ every y would have to be 0 for the statement to be true.

3. (a) What is the negation of the proposition $(p \wedge q) \rightarrow (q \rightarrow p)$. Form your answer in such a way that the negation symbol only the propositions p or q.

   $(p \wedge q) \wedge (q \wedge \neg p) \equiv \mathbf{F}$

   (b) Construct the truth table for the compound proposition above.

p	q	$p \wedge q$	$q \rightarrow p$	$(p \wedge q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T

4. For $i \in \mathbb{Z}^+ A$, for $A_i = [i, \infty)$, namely the set $\{x \in \mathbb{R} : i \leq x\}$. Describe the sets $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

   $\bigcup_{i=1}^{\infty} A_i = [1, \infty)$

   $\bigcap_{i=1}^{\infty} A_i = \emptyset$

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Find (see comment for problem 1)

   (a) $f^{-1}(\{1\})$

   $f^{-1}(\{1\}) = \{-1, 1\}$

   (b) $f^{-1}(\{x : 4 < x\})$

   $f^{-1}(\{x : 4 < x\}) = (-\infty, 2) \bigcup (2, \infty)$

6. Prove that if $5n + 2$is odd, then so is n.

   Proof: Strategy - prove the contrapositive : Thus assume n is eve, say $n = 2k$. Thus $5n + 2 = 10k + 2 = 2(5k + 1)$. Thus $5n + 2$ is even. Done!