Mathematics 1190A Introduction to Sets and Logic
Midterm Examination A

October 18, 2012

Instructions: Do all problems. Present your solutions in the accompanying booklet in the order that they appear on this paper. No notes, crib sheets or books are allowed. A calculator is allowed. Questions are of equal weight. The test is 50 minutes long.

1. (a) For $1 \leq i \leq \infty$, let A_i be a set. Describe using set notation what is meant by $\bigcup_{i=1}^{\infty} A_i$. see page 134. Or from your class notes you will see that $\bigcup_{i=1}^{\infty} A_i = \{x : \exists i \in \mathbb{Z}^+, x \in A_i\}$

   (b) Let $f : A \to B$ be a function from a set A to a set B. Let $y \in B$. Define the set $f^{-1}(y)$
   problem 43 section 2.3 was assigned. To have done this problem it would have been necessary to read the text between problems 41 and 42 - where you can find the answer. Or from your class notes you can see $f^{-1}(S) = \{x \in A : f(x) = 1\}$.

   (c) Define what it means for a set to be countable.
   see definition 3 p. 171

2. Determine the truth value of each of the following statements in which the domain of each variable is the real numbers. If false, provide a counter-example. see definition 1 p. 170

   (a) $\forall x \exists y\ (x = y^2)$ F - suppose $x = 2$, there then is no integer y such that $y^2 = 2$.

   (b) $\exists x \forall y\ (x = y^2)$ F - says that no matter what x we pick, it must be a square.

3. (a) What is the negation of the proposition $(p \vee q) \to (q \to p)$. Form your answer

in such a way that the negation symbol modifies only the propositions p or q.

$(p \vee q) \wedge (q \wedge \neg p)$

(b) Construct the truth table for the compound proposition above.

p	q	$p \vee q$	$q \to p$	$(p \vee q) \to (q \to p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	T

4. For $i \in \mathbb{Z}^+ A$, for $A_i = (0, i)$, namely the set $\{x \in \mathbb{R} : 0 < x < i)\}$. Describe the sets $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

$\bigcup_{i=1}^{\infty} A_i = (0, \infty)$

$\bigcap_{i=1}^{\infty} A_i = (0, 1) = A_1$

5. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Find (see comment for problem 1)

   (a) $f^{-1}(\{1\})$ $f^{-1}(\{1\}) = \{-1, 1\}$

   (b) $f^{-1}(\{x : 0 < x < 1\})$

   Note $\{x : 0 < x < 1\}$ is the open interval $(0, 1)$. Then $f^{-1}((0, 1)) = (-1, 0) \bigcup (0, 1)$

6. Prove that if $3n + 2$is odd, then so is n.

Proof: Strategy - prove the contrapositive : Thus assume n is even - say $n = 2k$. Thus $3n + 2 = 6k + 2 = 2(3k + 1)$. Thus $3n + 2$ is even. Done!