

Mathematics 1190A Introduction to Sets and Logic

Midterm Examination A

October 19, 2012

Instructions: Do all problems. Present your solutions in the accompanying booklet in the order that they appear on this paper. No notes, crib sheets or books are allowed. A calculator is allowed. Questions are of equal weight. The test is 50 minutes long.

1. Given a function $f : A \rightarrow B$ from a set A to a set B state precisely, using the language of logic, (1) what it means for f to be surjective and (2) what it means for f not to be surjective - but in such a way that the negation symbol immediately modifies only a proposition or a predicate.
2. (a) Given two sets A and B and if $|A|$ and $|B|$ denote the cardinality of A and B respectively, what it meant by $|A| < |B|$.
(b) Give a proof showing that the odd numbers have cardinality $\aleph_0 = |\mathbb{Z}^+|$.
3. (a) What is the negation of the proposition $(p \vee \neg q) \rightarrow q$. Form your answer in such a way that the negation symbol immediately modifies only a proposition.
(b) Construct the truth table for the compound proposition above.
4. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$, if $A_i = (0, \infty) = \{x \in \mathbb{R} : 0 < x < 1\}$
5. Prove that if x an irrational number and y is rational number, then $x + y$ is an irrational number.
6. Let a, r be numbers with $r \neq 0$ and $r \neq 1$. For a positive integer n what is the sum of the series $\sum_{j=0}^n ar^j$. Using this result above find the sum $\sum_{j=1}^8 2^j$.