Mathematics 1190B Introduction to Sets and Logic
Second Midterm Examination A

November 23, 2012

Instructions: Do all problems. Present your solutions in the accompanying booklet in the order that they appear on this paper. No notes, crib sheets or books are allowed. A calculator is allowed. The test is 50 minutes long.

1. (10 pts) Given a function $f : A \to B$ from a set A to a set B state precisely, using the language of logic, (1) what it means for f to be surjective and (2) what it means for f not to be surjective - but in such a way that the negation symbol immediately modifies only a proposition or a predicate.

   This question was on the first exam. However, recalling the both definitions of injective and surjective :

   $$f \text{ is surjective} \iff \forall y \in B\ \exists x \in A \text{ such that } f(x) = y$$

   $$f \text{ is not surjective} \iff \exists y \in B \text{such that} \forall x \in A,\ f(x) \neq y$$

   $$f \text{ is injective} \iff \forall x_1 \in A \wedge \forall x_2 \in A, (x_1 \neq x_2 \to f(x_1) \neq f(x_2))$$

   $$f \text{ is not injective} \iff \exists x_1 \in A \wedge \exists x_2 \in A : (x_1 \neq x_2) \wedge f(x_1) = f(x_2)$$

2. (10 pts) For the following state in full and complete detail

(a) Bézout's Therorem

(b) The Fundamental Theorem of Arithmetic

3. (12 pts) Perform the following conversions

   (a) decimal 321 to binary

   $321 = 256 + 65 + 1 = 2^8 + 2^6 + 1 = (1\,0100\,0001)_2$

   (b) hexidecimal $D8F_H$ to decimal

   $D8F_H = (13)16^2 + (8)16 + 15 = 3316 + 128 + 15 = 3469$

   (c) binary $(1\,1011)_2$ to decimal

   $1\,1011 = 2^4 + 2^3 + 2 + 1 = 27$

4. (10 pts) Find the least positive integer a such that $a \equiv 42(\text{mod}31)$.

   Note, we must have $a = 42+31k$ for some constant k. With $k = -1$, we have $a = 11$.

5. (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^2$. What is the set $f^{-1}\big([1, 2]\big)$.

   Similar to problem 42(c) Section 2.3, examining the graph of $f(x) = x^2$, it is obvious that $f^{-1}\big([1, 2]\big) = [-\sqrt{2}, -1\,] \bigcup [1, \sqrt{2}\,]$.

6. (8 pts) Using the method of the Chinese Remainder Theorem find the the smallest positive integer that is a simultaneous solution to the set of congruence relations

$$\begin{aligned} x &\equiv 1 \,(\text{mod}\, 3) \\ x &\equiv 4 \,(\text{mod}\, 5) \\ x &\equiv 2 \,(\text{mod}\, 7) \end{aligned}$$

   This is very similar to example 5 page 278 which was treated in detail during class. It is so similar that the values $y_1 = 2, y_2 = 1, y_3 = 1$ are the same. With $M_1 = 35$, $M_2 = 21$, and $M_3 = 15$, and $m = 3 \cdot 5 \cdot 7 = 105$, a solution is then any integer x such that

$$x \equiv 1 \cdot 35 \cdot 2 + 4 \cdot 21 \cdot 1 + 2 \cdot 15 \cdot 1 \,(mod\, 105\,)$$

   or equivalently

$$x \equiv 184 \,(mod\, 105\,)$$

   Any such solution must satisfy $x = 184 + k105$ for some $k \in \mathbb{Z}^+$. With $k = 1$, we see the least such positive solution is $x = 79$.