Mathematics 2320 Discrete Mathematics
MidtermExamination II

November 18,2013

Instructions: Do all problems. Present your solutions in the accompanying booklet in the order that they appear on this paper. No notes, crib sheets or books are allowed. A calculator is allowed. The test is 2 hours long.

1. (12 pts) Let R be a relation on a set S. Define what it means for R to be
   (a) reflexive
   (b) symmetric
   (c) anti-symmetric
   (d) transitive
2. (10 pts) Describe the Tower of Hanoi puzzle and give a recurrence relation that determines the number of necessary moves when there are n disks in terms of the number of moves with $n-1$ disks.
3. (10 pts) What is the solution of the recurrence relation
   $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$.
4. (12 pts)
   (a) Let R be a relation on a set S, define what it means for R to be an equivalence relation.
   (b) For $\preceq$ a relation on a set S, define what it means for $\preceq$ to be a partial order.
   (c) For $\preceq$ a partial order on a set S, define what it means for an element $a \in S$ to be a greatest element of S.

(d) For $\preceq$ a partial order on a set S, define what it means for an element $a \in S$ to be a minimal element of S.

5. (12 pts)

   (a) Define the equivalence relation that determines the equivalence classes of the integers $mod\, 6$, that is $\mathbb{Z}_6$.

   (b) What are the elements of $\mathbb{Z}_5$?

   (c) What are the elements of the equivalence class $[2] \in \mathbb{Z}_6$?

6. (10 pts) Given the set $S = \{3, 5, 9, 12, 15, 60\}$ and the partial order $\mid$ where $a \mid b$ if and only if a divides b

   (a) Draw the Hasse diagram that represents the relation.

   (b) Find the maximal elements.

   (c) Is there a greatest element?

   (d) Find all upper bounds of $\{3, 5\}$.

   (e) Find the least upper bound of $\{3, 5\}$.

7. (10 (pts)

   (a) Given an undirected graph and a vertex v. What is the definition of the degree of v?

   (b) What are the degrees of the vertices in the following diagram

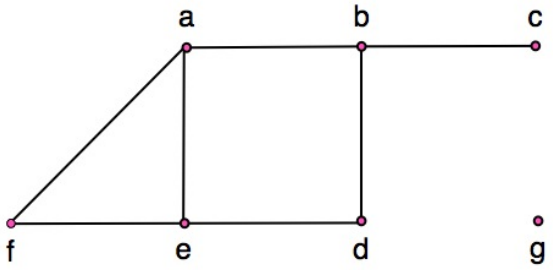


Figure 1: Find degree of each vertex