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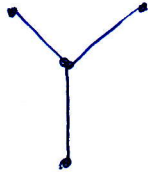
Definition: A graph consists of 2 sets
a set V of vertices (nodes) and a set E
of "edges". An edge has associated with it either
(i) 1 vertex - in which case the edge is a loop
(ii) 2 vertices - called endpoints.
if $u \in V$ and $v \in V$ are associated with an edge e
then u & v are said to be adjacent or
neighbors.

Definition A directed graph is a graph (V, E)
in which each edge is assigned is assigned
an "initial vertex" and a "terminal vertex".
If $e \in E$ is an edge, and v_0 and v_1 are the
associated vertices, and v_0 is the initial
vertex and v_1 the terminal vertex - it may
happen that $v_0 = v_1$ - and the edge is then a
"loop".

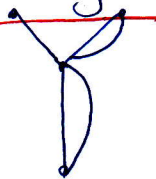
[A directed graph (V, E) may be considered as
a relation R on V , where $\forall v \in V \forall u \in V$,
 $u R v \Leftrightarrow$ there is an edge $e \in E$ such that
 u is the initial vertex and v is the terminal vertex

Various graphs

Simple - no "multiple edges" - undirected - no loops



Simple multigraph - multiple edges allowed - no loops
undirected



Pseudo graph - undirected, loops & multiple edges allowed



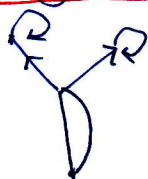
Simple directed - no loops - no multiple edges



Directed multigraph - loops & multiple edges allowed



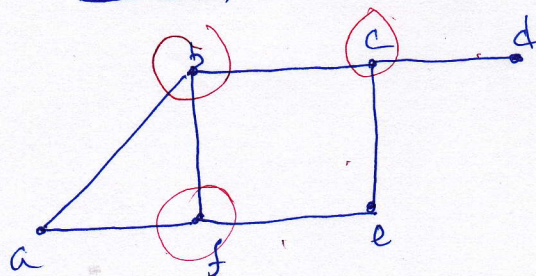
Mixed graph - some edges are directed - loops & multiple edges allowed



Given graph (V, E) undirected

- 1) two vertices $u, v \in V$ are adjacent or neighbors if $\exists$ an edge connecting, then
i.e. $u, v \in V$ are the endpoints of the edge -
such an edge is incident with u, v .
- 2) The degree of a vertex v , $\deg(v) =$
the number of edges incident with v . A loop counts for 2 edges.
- 3) Let $v \in V$. The set of all vertices adjacent to v is the neighborhood of v - written $N(v)$.
If $A \subset V$, $N(A) = \bigcup_{v \in A} N(v)$.
- 4) if $\deg(v) = 0$, then v is an isolated vertex.
if $\deg(v) = 1$, then v is pendant or hanging.

Example



$$V = \{a, b, c, d, e, f\}$$

$$A = \{b, c, f\}$$

$$\deg(a) = 2$$

$$\deg(b) = 3$$

$$\deg(c) = 3$$

$$\deg(d) = 1$$

$$\deg(e) = 2$$

$$+ \deg(f) = 3$$

$$\hline 14$$

$$N(A) =$$

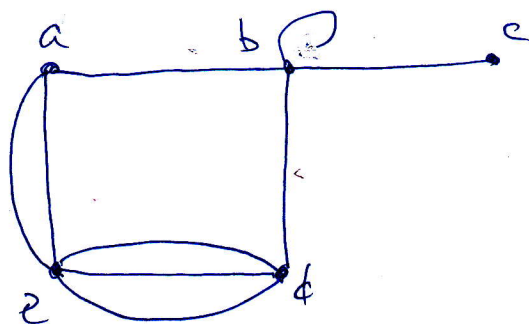
$$\{a, b, c, f, e\}$$

note: there are 7 edges and

$$2 \cdot 7 = 14 = \sum_{v \in V} \deg(v)$$

(5)

$$V = \{a, b, c, d\}$$



$$\begin{aligned} \deg(a) &= 3 \\ \deg(b) &= 5 \\ \deg(c) &= 1 \\ \deg(d) &= 4 \\ + \deg(e) &= 5 \\ \hline &18 \end{aligned}$$

There are 9 edges

$$2 \cdot 9 = 18 = \sum_{v \in V} \deg(v)$$

In general: for undirected graph (V, E)

each edge has 2 associated vertices

If for each edge we add the degrees of associated vertices, we count the degree of each vertex twice.

Thus, if m = number of edges,

$$m = \sum_{v \in V} \deg(v)$$

$$\text{ie: } 2m = \sum_{v \in V} \deg(v)$$

As a result, there must be an even number of vertices of odd degree

The directed graph version.

Let (V, E) be a directed graph

Let $\deg^+(v)$ = number of edges with v as initial vertex
 $\deg^-(v)$ = _____ terminal vertex

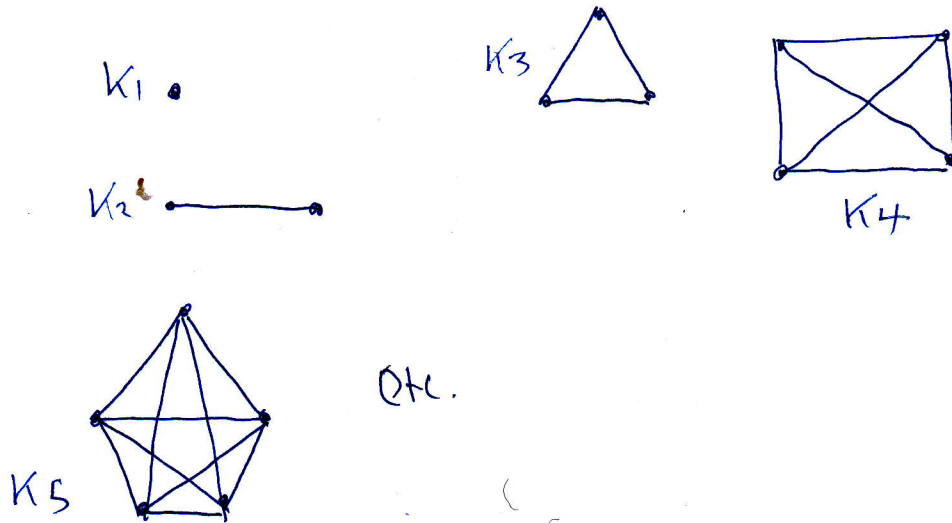
Then since each edge has an initial and a terminal vertex,

$$\sum_{v \in V} \deg^-(v) + \sum_{v \in V} \deg^+(v) = m$$

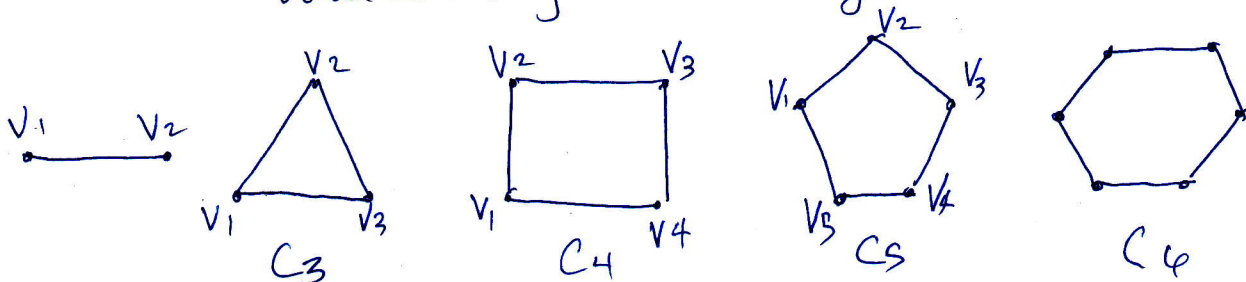
where m = number of edges.

Special Simple Graphs

Complete graphs on n vertices - called K_n
has exactly 1 edge between any 2 vertices

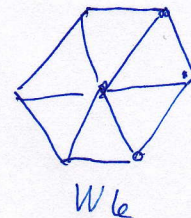
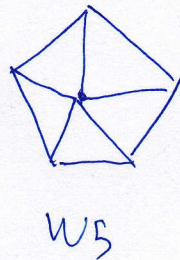
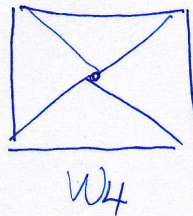
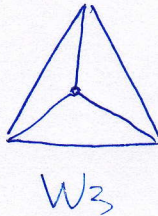


A cycle graph C_n ^{$n \geq 3$} consists of an linear ordering of the vertices: $v_1, v_2, \dots, v_n$ and an edge connecting v_i with v_j $i=2, \dots, n$.

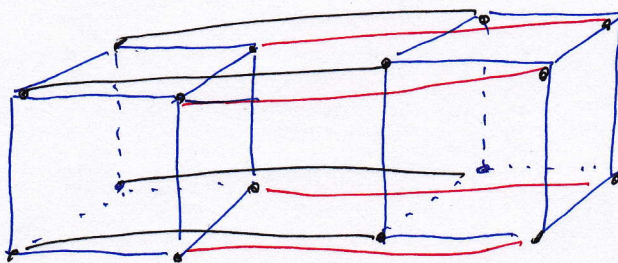
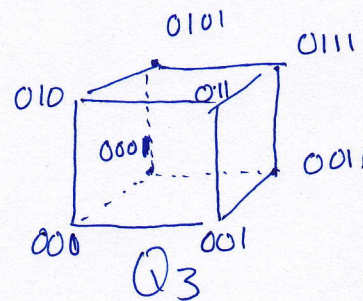
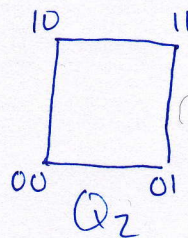
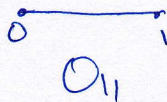


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A wheel graph W_n is constructed from a cycle graph C_n by adding an additional vertex



An n -cube graph ^{Q_n} can be constructed inductively



OR

