

**HOW TO MEASURE MILITARY WORTH  
(AT LEAST IN THEORY)**

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## ABSTRACT

Three axioms lead to a unique way to measure the importance of a weapon within the context of the other armaments its owner possesses. The method also can measure the importance of different features of a given weapon. It implies, for example, that in a well-defined way, the command and control centre of a strategic triad is twelve times as important as a single leg, and that missile accuracy is three times as important as the yield of the warhead. The method is based on the Shapley value of an n-person coalitional game. It is compared to other ways of defining importance in military operations research. The present approach is not practical in the sense of yielding numbers to guide specific decisions, but it helps us understand the limits of the usual modes of thinking about these issues and corrects some conceptual fallacies.

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## 1. Introduction

The notion of measuring a weapon's importance arises in several contexts. To determine whether an arms control agreement is fair, one could estimate the value of the weapons that each side is dismantling, and compare the results. This is usually done as an intuitive guess, although Kent (1986) and Drell (1982) have proposed exact numerical schemes that would govern an arms build-down agreement. In 1976 discussions on strategic arms limits, Andrei Gromyko proposed that each B-1 bomber count three units against a ceiling of 1320 units where a B-52 or a ballistic missile launcher which would count only one (Talbot, 1979). This approach may finally be implemented in a START (Strategic Arms Reduction Talks) Treaty. In the May 1990 joint draft text, each U.S. bomber equipped with cruise missiles counts for ten missile warheads, and each Soviet bomber counts for eight (Hardenbergh, 1990).

A measure of importance might be especially valuable in the context of conventional arms control. The discussions on Conventional Armed Forces in Europe (CFE) deal with weapons of many types. Whereas START negotiators handle ten types in the category of ICBMs, for example, tanks and armoured combat vehicles alone come in over 90 types (Leavitt, 1990), plus there are other classes of weapons, each subdivided into several zones depending on their proximity to Central Europe. Although no formal methods are used now, a measure of value for groups might make negotiating simpler.

The notion of importance arises also in military planning. An early application of military worths was investigated at the RAND Corporation in the late 1940's, when Ed Paxson, the founder of the systems analysis division, used a formal measure to guide the design of strategic offense and defense (Digby, 1990). Another use of measures is to judge the "military balance," where analysts assess the value of individual items and add up the holdings in each side's arsenal. A large literature has developed indices comparing NATO and the Warsaw Pact, for example. Many automated war games use numerical rules to decide how to allocate forces, how much one side should sacrifice to eliminate some adversary units, or how to tabulate scores at the end of the session.

Another example of the use of measures in military analysis involves the tenet of a defender's innate advantage, that the attacker typically needs three times the defender's strength to win. Does the historical record support this? The claim presumes some way to calculate the force ratio. If the adversaries had only one type of weapon in two different amounts, calculating the ratio would be easy, but if they possess various sorts in different proportions an analyst must score individual items and add.

This paper presents a theoretical way to calculate the "importances" of weapons, and illustrates its behaviour through examples. The measure takes as given the benefits of having a set of weapons, and tells how to allocate the credit among them. It is very general, applicable to any problem where the values for the outcomes of the conflict can be specified. If one accepts the three postulates as compelling, it is the only way to measure importance.

In some applications it can also measure the importance of weapon qualities, such as firepower versus vulnerability.

It takes as data the benefits of possessing all combinations, and this requirement prevents its use for immediate practical decisions. The purpose of the paper is *not* to suggest a measure for use in some of the arms control and warplanning applications mentioned above. The contribution it hopes to make is to clarify intuitive patterns of thought. The definition should show what a logically correct concept of importance would be like, bringing out any inconsistencies in current measures. A further aim is to understand the images people use in their thinking, which are probably more influential in determining actions than any analysis. We hear about who is "ahead," whether the other is "catching up," whether there is a "military balance" or a "gap," what is each side's "strength" compared to the other's, but these metaphors are fundamentally misleading, in my view. They are especially inappropriate in the context of nuclear weapons, which in no sense balance off against each other. The paper will try to clarify the notion of importance of weapons in way that can be compared with people's natural thought processes.

Section 2 develops the definition of importance, Section 3 illustrates it with examples and extends it to continuous features of weapons, and Section 4 compares it with some categories of operations research techniques for assessing weapons values. The problems with present formal methods and informal metaphors turn out to be the same: they ignore synergy and redundancy among weapons, and they assign values to weapons based on their individual characteristics without considering the benefits these weapons confer on the possessor.

## 2. The Method

The method is based on Shapley's value of coalitional games (1953), supplemented by Young's recent discoveries about the Shapley value (1985). There is no game theory in this paper however, since there is no strategizing or choosing moves. Shapley's mathematical structure is borrowed without the content.

Suppose my weapons are a finite set  $W$ , and suppose that I can measure my benefit from having these weapons and also measure my benefit from having any subset of  $W$ . This latter is a hard requirement in practice but the idea is simple: How good or bad would it be to have  $1/2$  as many ships and  $2/3$  as many tanks, and so on, for all fractions? If  $S$  is some subset of the total arsenal  $W$ , the number  $b(S)$  will designate my benefit if only weapons  $S$  exist, other things equal. The function  $b$  will be called the *benefit function*.

The importance of the weapon  $x$ , given that the benefit function is  $b$ , will be written  $I_b(x)$ , and the "right" or "appropriate" function  $I_b$  is what we are trying to find. Each of the following three principles restrict  $I_b$  and, taken together, they determine it uniquely.

This principle says that the sum of the importances of one's weapons equals the benefit derived from having them. It seems most natural to talk of importances as additive, and it is hard to think of a simple alternative. Principle 1 seems intuitively compelling.

The next principle involves the concept of equivalent weapons: two weapons  $x$  and  $y$  are *equivalent* if replacing one by the other in any set  $S$  leaves  $b(S)$  unchanged. That is, no matter what others are extant, replacing one by the other does not change the benefit you receive. Weapons are equivalent if they are physically identical, for example, and in general two different weapons are equivalent if the benefit function  $b$  treats them the same in all situations.

*Principle 2:* If  $x$  and  $y$  are equivalent for  $b$ , then  $I_b(x) = I_b(y)$ .

The final principle is the most subtle. It stems from the vague notion that the importance  $I_b(x)$  of a weapon  $x$  ought to reflect the *added good* you gained from acquiring  $x$ . At first thought this would suggest defining importance simply as the marginal contribution  $b(W) - b(W - \{x\})$ . However this would clash with either Principle 1 or 2. Suppose, for example, you possess two weapons  $x$  and  $y$ , either one of which would achieve your ends, so that one is redundant. (We will arbitrarily take the values of achieving and not achieving your ends to be 1 and 0.) The marginal benefit method would assign each an importance 0, since acquiring it when you have the other does you no further good. But this conclusion is unacceptable: they cannot both be worthless because their importances have to sum to 1.

We might try to fix this problem by giving one weapon a value 1 and the other (the "redundant one") a value 0, but this would violate Principle 2. The two weapons are the same but one gets all the credit -- this tank is valuable but this one is worthless even though they are identical.

As often happens, an intuition which seems right is indeed basically correct, but has to be formalized in just the proper way. Identifying importance with marginal contribution led to a problem because the added value of each was 0 *given* the other weapon was in the arsenal. But each weapon is in fact worth something because it would make an important marginal contribution *if* the other were not there. This suggests the rule: Do not look at the marginal contribution only to the complete arsenal, consider what the weapon adds to all subsets. Accordingly for each  $x$  we will define a function, the marginal contribution, on the subsets of  $W$  (where the notion of subset includes  $W$  itself),

$$\begin{aligned} b_x'(S) &= b(S) - b(S - \{x\}) && \text{if } x \text{ is in } S, \\ &= b(S \cup \{x\}) - b(S) && \text{if } x \text{ is not in } S. \end{aligned}$$

Thus  $b_x'(S)$  is a series of numbers that shows what benefit  $x$  is adding to or could add to  $S$ . As the notation suggests, it is like a derivative of  $S$  with respect to  $x$ . For the two-weapon situation above, for example,  $b_x'(\emptyset) = b_x'(x) = 1$ , where  $\emptyset$  is the empty set containing no weapons, and  $b_x'(y) = b_x'(xy) = 0$ . Principle 3 states that  $b'$  is sufficient to determine the importances. Even comparing two different benefit functions, identical marginal benefits mean identical importances.

*Principle 3:* For a given weapon  $x$ ,  $I_b(x)$  is determined by the marginal contributions  $b_x'$ .

Young's surprising result is:

*Theorem.* Only the following function satisfies the three principles:

Here  $w$  is defined as the number of weapons in  $W$  and  $s$  as the number of weapons in  $S$ . So the importance of  $x$  is a weighted average of the marginal contributions of the weapon  $x$ , the weights depending on the number of weapons in the subset.

### 3. Examples and Extensions

*Example 1.* A command centre and two units.

Suppose that we have a command/supply centre  $C$  and two units  $J$  and  $K$  that attack the enemy directly.  $C$  is the "tail" and  $J$  and  $K$  are the "tooth." Assume that a command centre plus either unit will succeed, but with no engagement units you will lose and with no command centre you will lose. With values 0 and 1 for losing and winning the benefit function for this game is:  $b(CJ) = b(CK) = b(CJK) = 1$ ,  $b(S) = 0$  for other  $S$ .

The formula of the theorem gives  $I_b(C) = 2/3$  for the command centre and  $I_b(J) = I_b(K) = 1/6$  for each engagement unit. (For  $n$  units the command centre has value  $n/(n+1)$ .)

Why not  $1/2$ ,  $1/4$ ,  $1/4$  or something else? Applying the logic of Young's proof answers this question for this particular example. Call the original benefit function  $b_1$  and define a second situation:  $b_2(CJ) = 1$ ,  $b_2(S) = 0$  for other  $S$ . Now only  $C$  and  $J$  succeed, and adding  $K$  in fact nullifies them, since  $b_2(CJK) = 0$ . Unit  $J$ 's contributions in  $b_1$  and  $b_2$  are the same since in both it contributes 1 to the set  $C$ , 1 to  $CJ$ , and 0 to others. Principle 3 then requires that  $J$ 's worth must be the same in the two. Define a third situation:  $b_3(CJK) = -1$ ,  $b_3(S) = 0$  for other  $S$ . In  $b_2$  and  $b_3$   $K$ 's contributions are the same: -1 to  $CJ$  and  $CJK$  and 0 to other subgroups, and thus  $K$ 's worth is the same. But worths in  $b_3$  can be assigned by Principles 1 and 2 alone since the  $C$ ,  $J$  and  $K$  are equivalent weapons in  $b_3$ , and the values must sum to -1. Therefore  $K$  gets  $-1/3$  in  $b_3$  and thus also in  $b_2$ . Since  $C$  and  $J$  are equivalent in  $b_2$  and the sum of the three worths must be  $b_2(CJK) - b_2(\phi) = 0$  by Principle 1, then  $J$  must be assigned  $1/6$  in  $b_2$  and hence also in  $b_1$ . The values of the other two follow since  $J$  is equivalent to  $K$  and the three sum to 1. Young showed that this type of chain sequence of  $b$ -functions can always be constructed, leading from the original to a situation solvable by its symmetry alone.

*Example 2. Deterrence with triads.*

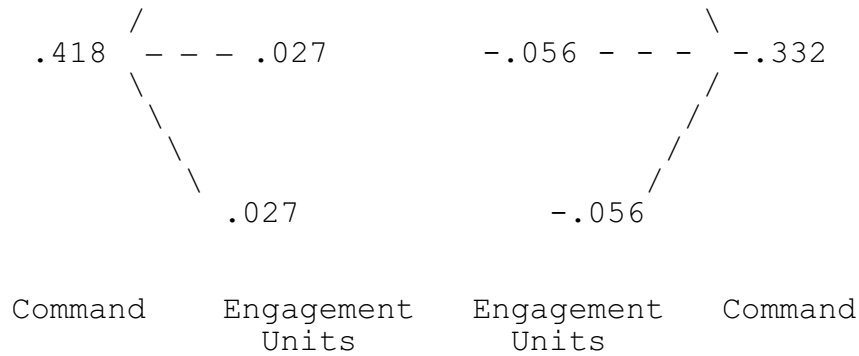
If Example 1 is modified to include three units instead of two, it resembles the triad of strategic nuclear forces, the units being the so-called "three legs of the triad," intercontinental missiles, submarine-launched missiles and bombers. The metaphor's suggestion is that with only two legs the system would topple, but this is misleading, of course, since the explicit goal of having a triad is that each unit is sufficient by itself. The next example involves triads on both sides. Suppose there are two sides, each with a command centre and three engagement forces. A side is defined to have the *ability to attack* if it has the command centre plus one or more of the three forces. A side loses if it cannot maintain deterrence, i.e., if it does not have the ability to attack and the other does. Otherwise it wins.

One notable difference from the last example is that here importances are affixed to your own *and* the adversary's weapons. The latter are measured from your viewpoint and therefore will be negative. Another new aspect of this example is the suggestion of common interests. You win if both have the ability to attack, or if neither does, implying that if we were to define payoffs for the other, both could win at once. It is a model of importances of weapons for deterrence and assumes that their only value comes from use in war.

A simple computer program could be used to calculate the formula above. It would involve determining the outcome of all  $2^8$  subgroups, weighting each and adding. In fact a faster method devised by Hart and Mas-Collel (1988), was programmed, with the result as shown below.

Notice that your weapons' scores total  $1/2$  and the other's  $-1/2$ . By Principle 1 the sum must be zero since within the model a world with all the weapons is as beneficial to you as one with none. Your command centre turns out to be more valuable to you than the other's is threatening to you (.418 versus .332). Whether there is an intuition behind this I do not know, but there is no inherent reason they should be equal.





### *Evaluating Continuous Features of Weapons*

Examples 3, 5 and 6, to follow, involve the Lanchester theory of combat. Lanchester models typically use differential equations and so assume a continuously variable quantity of forces on both sides. The Shapley value generalized to continuous sets is called the *Aumann-Shapley value*. (Owen (1983) gives an introduction and some simple calculations, and Tauman (1988) has written a recent survey.) The importances of individual weapons will generally be zero but we can ask for the importance per a certain measured quantity of the weapons, or the importance for the whole of one subgroup of similar weapons.

The definition of the Aumann-Shapley value in this context is as follows. Suppose I possess  $n$  continuous items in quantities  $a_1, a_2, \dots, a_n$  where my benefit from possessing each to degree  $t_1 a_1, t_2 a_2, \dots, t_n a_n$  is  $f(t_1, t_2, \dots, t_n)$ . The Aumann-Shapley value of the  $i$ 'th item is defined to be

where  $f_i$  is the derivative of  $f$  with respect to its  $i$ 'th variable. We are looking at the rate of marginal contribution of the  $i$ 'th item only for subsets that contain the weapons in the same proportions as the whole. Just when this value exists is an intricate subject and here we will omit precise statements of the conditions, say simply that they are satisfied by the examples. Young (1988) shows that it follows from continuous versions of Principles 1 to 3 plus a further weak axiom.

### *Example 3. Numbers versus quality in aimed fire*

Suppose two sides X and Y have homogenous forces  $x(t)$  and  $y(t)$  that destroy each other according to the equations:

$$dx(t)/dt = -ay(t), \quad dy(t)/dt = -bx(t).$$

These would be appropriate for aimed fire, that is, a context where the two sides can take target specific remaining forces on the other side rather than barrage the opponent. The constants  $a$  and  $b$  are Y and X's fire effectiveness per weapon respectively, involving the rate of fire and proportion of shots that destroy the target. Lanchester's original 1914 paper offered a proof that the quantity  $bx^2(t) - ay^2(t)$  is invariant as combat progresses, a proposition known as "Lanchester's Square Law."

The Square Law has been cited in the non-mathematical literature on procurement as support for buying quantity instead of high technology (Canby, 1984; Perry, 1984). It suggests that numbers are more important than quality since it would take a fourfold increase in quality to match a simple doubling of the numbers. What happens when we compare quality and numbers using the Aumann-Shapley value? Note that we are asking for the importance of weapons attributes rather of entire weapons, but the axioms for the definition of importance are also sensible for this purpose.

One issue must be decided before invoking the measure: How should we gauge the benefit of having certain forces left after the battle? Should we take simply the size of the surviving force, or should we include the quality of the forces? If we assume that the forces have no value in themselves, are inanimate objects, we might use some increasing function of  $ax^2$ , since if remaining weapons are employed elsewhere as the only weapons in a further battle, this function is the one that determines X's success. Somewhat arbitrarily, we will then take  $ax^2$ , so the benefit to X when the two sides possess  $x(0)$  and  $y(0)$  is  $bx^2(0) - ay^2(0)$ , but the result below depends very much on this choice.

Applying the formula for the Aumann-Shapley value, we calculate that the total importance to X of X's firepower effectiveness is  $ax^2(0)/3$ , and of X's numbers is  $2ax^2(0)/3$ . The importance (to X) of Y's firepower effectiveness is  $-by^2(0)/3$ , and of Y's numbers  $-2by^2(0)$ . Thus, taken in total, numbers are twice as important as quality both for one's own and for the adversary's weapons.

#### *Example 4. Warhead yield and accuracy*

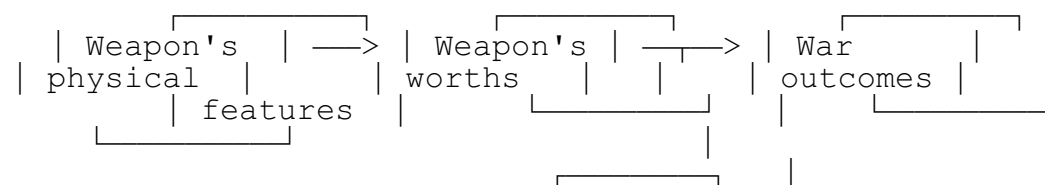
The importances derived in Example 3 are instances of a general proposition, easy to derive from the Aumann-Shapley formula, that if the benefit function has the form of a product of powers  $x_1^{r_1} x_2^{r_2} \dots x_n^{r_n}$ , then factor  $x_i$ 's total importance is proportional to  $r_i$ . Another application of this rule is to the countersilo effectiveness of a ballistic missile warhead. Considering the two factors, weapon yield  $y$  and accuracy  $a$  (defined as the inverse of the circular error probable), within the standard model the likelihood of destruction of a targeted silo is proportional to  $y^{2/3}a^2$  (Bennett, 1981). The rule then implies that accuracy is three times as important as yield.

During debates on military buildups, one often hears that the "destructive power" of strategic weapons has been decreasing in recent years, and the claim is sometimes accompanied by a graph showing the total megatons falling with time (e.g., United States Department of Defense, 1988, p.110; President's Commission on Integrated Long-term Strategy, 1988, p.39). This is misleading in that it ignores the development of missile accuracy, which has been rising rapidly (MacKenzie, 1990), and is important since many weapons are aimed at hardened military installations. We can now say with mathematical precision just how misleading the claim is: the factor that it neglects is three times as important as the one it includes.

#### **4. Approaches to Measuring Importance of Weapons**

We have chosen some existing models that define a force measure, and will compare our measure with the one defined in the literature.

##### *1) The Additive Features Approach*



The additive features approach rates each weapon based on its physical attributes, then adds the numbers to get an overall score for a side. The two sides' scores are typically compared overall to see if there is a "balance," or for a specific type of weapon to decide whether there is a "gap."

A sophisticated example of the additive features approach is the Armored Division Equivalents (ADE) method of the U.S. Army Concepts Analysis Agency, 1974, 1979; Mako, 1983), also called the WEI/WUV ("wee-wuv") scoring system. The basic method is used by U.S. government agencies to report the military state of affairs (e.g., Department of Defense, 1988, p.31). The original system, WEI/WUV I, divides conventional weapons into nine categories: tanks, armed helicopters, mortars, etc. Each category score depends on several "performance factors" that such a weapon should have, such as firepower, mobility, or survivability (Figure 1). Each of these is analyzed into objective physical features, or "performance characteristics," which are measured, normalized for a standard weapon of that type, weighted for their importance as a contributor to that performance factor, then added to get the factor score. For undesirable features, like presented-target area, the inverse is used. The scores on the factors are then weighted for importance and added to derive the Weapons Effectiveness Index (WEI) for the particular weapon. The various weights are estimated by a panel of experts, and differ depending whether the weapon is used offensively or defensively. The WEI values are multiplied by further weights portraying the importance of the category (of tanks, helicopters, etc.) and totalled for all forces to get a Weighted Unit Value (WUV). The WUV is normalized by dividing by 48,743 (in the defensive case), the nominal total for an American armoured division. This gives the Armored Division Equivalent (ADE) score, a measure of overall strength.

This approach has the advantage that the scores are based partly on objective technical data, but it can be criticized for the subjectivity involved in judging the weights and in selecting the features to include. There is also a deeper structural problem, however, involving whether one can combine weapons' features or different weapons by addition. Adding assumes independence. It assumes that an additional weapon makes the same contribution whatever was there before. However weapons are generally not independent, nor are features. Some weapons are synergistic, such as the command centre and one leg of the triad of Example 2, and others are largely redundant, such as one leg in relation to another. (These criticisms apply equally to certain systems for measuring the "strategic balance" of nuclear weapons such as Equivalent Weapons discussed by Bennett, 1980.)

The WEI/WUV I system can give odd results when the weapons or features possess synergy or redundancy. If a weapon's features are combined by addition, a high enough value on one feature can appear to compensate for poor scores on the others, in contrast to reality where both features are absolutely necessary. The author estimated the WEI score of his all-terrain bicycle regarded as an Armoured Personnel Carrier (APC). (Although the normalizing values are kept secret, some can be estimated from known data about the standard weapon, the M113A1, and most others are irrelevant since the bicycle usually scored near zero.) On two it excelled: presented-target area and gross weight. These contributed so much (since when normalized by the measure of the standard APC the bike's gross weight factor is about 60 instead of the typical 1) that they compensated for low troop capacity, no armour or firepower at all, etc., and the bicycle's Weapons Effectiveness Index was double that of the Swiss MOWAG 3M1, the best APC in the report.

A defender of the ADE method might counter that a bicycle is not an armoured personnel carrier and should not be scored as one. Every approach has its range of valid application, and this method is intended as an aid to common sense, not a substitute for it. I think this response is largely valid, but the above counterexample should be a warning. Additive

rules make are more sensible for some pairs of features or weapons than for others, and one should consider this when employing such measures.

A later version, WEI/WUV III, tries to prevent the domination of the WEI score by a few characteristics. It limits the maximum contribution any characteristic can make. This mitigates bizarre counterexamples but does not eliminate them: the bicycle scores 56% to 74% of current APCs indicating that it would take a few more bicycles to do their job. More significantly it does not correct the source of the problem and weapons may well be ranked incorrectly even though they are too similar to make the error blatant.

As well as producing counterexamples based on features of a single weapon, we could have mixed an odd concoction of weapons to make up a unit just to get a high score. A recent report on the "conventional balance" in Europe by the Congressional Budget Office (1988) tries to answer this criticism, that the method may implicitly call for a very unbalanced mix of weapons,

Finally the WEI/WUV method assumes that the added benefit of additional weapons is linear -- that is more weapons of any kind continue to provide the same additional capability as the first such weapon. This assumption is called 'constant marginal utility' in economic jargon and ignores the fact that, beyond a certain point, additional weapons of one kind might be redundant and therefore of no added utility. For this reason WEI/WUV scores should not be used by themselves to determine the optimal mix of weapons in a division. . . . Rather, the scores should be used to suggest how one mix of weapons deemed plausible by military experts might perform against another plausible mix.

The authors do not confront the inconsistency: military experts have chosen to mix weapons in certain proportions, whereas WEI/WUV scores suggest that other mixes would do better. The ideal mix according to WEI/WUV may be implausible, but reasonable appearing mixes shifted in that direction would still be misrated as better. Why then should we believe the WEI/WUV score?

An example of one approach to deal with synergy is that of the IDAGAM and COMBAT computer models of battle, devised at the Institute for Defense Analyses (Anderson and Miercort, 1989). The programs group weapons into classes, according to whether weapons of one class require weapons of another class for their protection. In case a side's forces are unbalanced, in the sense that the protecting weapons are lacking, the program COMBAT regards some of the weapons as ineligible to participate in the battle, and so reduces its estimate of the side's effectiveness. Although this approach appears somewhat too dichotomous in the way it portrays protection, it seems an interesting and promising practical solution.

Military balance analyses neglect the context and dynamics in which the war might be fought. There may be a first-strike advantage as with strategic missiles, or the opposite, a relative advantage to being the second mover, if each side has adopted defensive positions and is hoping the other will leave its trenches. The WEI-WUV methodology compensates for this partially by calculating both an offensive and a defensive value. Usually however only one is reported. Another difficulty is that during the course of a war one side may switch from defense to counterattack. (This question is also addressed by Anderson and Miercort.) The metaphor of a balance is itself misleading. Nuclear systems for example do not balance off against each other: matching the other side's first-strike capable weapons by adding similar weapons of one's own can make the situation more unstable, not more balanced. In our method these contextual factors affect the importance through the benefit function  $b$ . A mutual first-strike advantage, for example, would induce a higher probability of war and be included in the model through lower values of the benefit function.

Although the WEI-WUV system is flawed, so are all practicable ones, and I believe the scores are a good guide to force strength under some circumstances, in particular, if the features are positively associated with one another as would happen if the various features reflected more or less money spent, or a lower versus higher level of technology. Also a

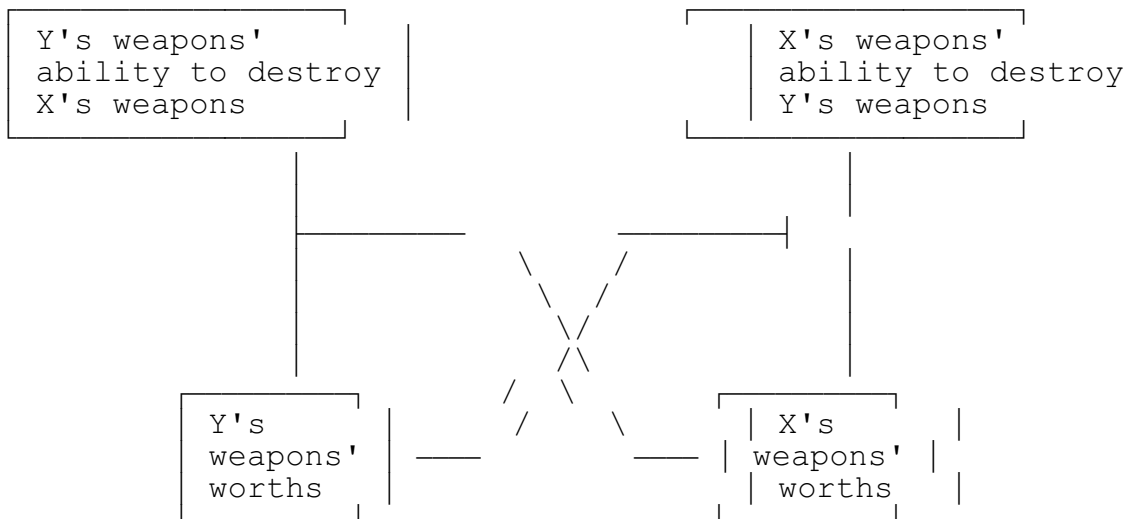
system using estimates of the relative values of the various categories is an improvement if two armies possess categories in roughly the same proportions.

*Example 5. Balancing weights versus importance measures.*

An attraction of the additive features approach is the simple rule used to determine whether there is a "military balance." It avoids specifying the dynamics of possible wars. Of course, this simplicity is a virtue only if the additive assumption is accurate. This example asks: If the notion of a balance *were* true in a hypothetical situation, how would the numbers that determine the balance relate to the importances? Would they necessarily be identical? The answer is no.

Suppose side X has  $m$  weapons with associated weights  $x_1, \dots, x_m$  and Y has  $n$  weapons with weights  $y_1, \dots, y_n$ . The meaning of the weights will be that they determine whether the system is balanced. Looking at the situation from X's viewpoint we will assign  $b(S) = 1$  if the sum of the weights of X's weapons in  $S$  is at least as great as Y's. Then X's weapons "balance" Y's. Otherwise  $b(S) = 0$ . An example shows that balancing weights and importance values are different. Suppose X has three weapons with weights 2, 1, and 1, and Y has an identical set of three. The importances to X are calculated to be 29/60, 18/60, 18/60, -29/60, -18/60 and -18/60. Thus  $x_1$ 's importance (29/60) does not cancel  $y_2$  and  $y_3$ 's combined importances (36/60), even though  $x_1$  is enough to balance those two weapons.

2) *The Antipotential-potential Approach:*



Several writers (see Spudich, n.d.; Howes and Thrall, 1973; Dare and James, 1973; Holter, 1971; Anderson, 1979, and the excellent summary by Taylor, 1983) have defined weapons' lethality recursively by relating X's values to the rate at which its weapons destroy Y's valued systems, the latter being defined in turn by the rate at which they destroy X's value. Anderson aptly named the idea the *antipotential potential* (APP). It leads to calculating eigenvalues of matrices involving the attrition rates of each weapon versus each of the opponent's.

In a clever and sensible way this approach makes values dependent on other existing weapons, both one's own and the adversary's, as the diagram shows. However, we see two problems, the first being that it can be used in only in contexts when attrition is linear and when the allocations of weapons to types of targets are fixed throughout the combat. The second difficulty is that APP values of the weapons are defined without reference to the value of the benefits to the

possessors. In essence the approach sidesteps Principles 1 and 3 by not defining benefits. One consequence is that the APP values are not completely determined. The values can be found up to comparison of ratios within one side, but the ratio of weapons between sides is arbitrary. That is, if  $v_1, v_2$ , are one side's values and  $v_3, v_4$  the other's, then for any positive  $k$ ,  $v_1, v_2, kv_3, kv_4$  will satisfy the conditions as well. The constant  $k$  then has to be chosen using some argument different than the notion that value is the ability to destroy value. The choice of  $k$  is critical for computer simulations such as IDAGAM and TACWAR (Hoerber, 1981) that define a side's total force as numbers times corresponding APP scores, and use this for force ratio comparisons. Different authors have suggested various ways of determining the parameter  $k$ , but no convincing argument has distinguished itself, and some have encountered paradoxes. Farrell (1975) notes that increasing the firepower of a Y-side weapon sometimes causes the force ratio to move in X's favour, and Anderson (1975) has shown that dividing a weapon into two subtypes that are effectively identical, can greatly increase one's rated force ratio within the method of Howes and Thrall. Clearly neither of these changes reflects the benefits for a side in a conflict, but APP scores are not defined by benefits, and so this questionable behaviour can arise. For the importance measure presented here, Principle 3 guarantees that a desirable improvement makes a weapon more important.

### 3) The Optimal Allocation Rule Approach



Here one sets up a model of combat and finds the rule for optimally allocating weapons to the opponent's forces. Sometimes it will involve an index for the opponent's weapons that states which should be given priority, and this constitutes a measure of their importance. Concepts like this often arise in models of strategic exchanges (Congressional Research Service, 1985) or missile defense (Karr, 1981), but the following example is based in Lanchester theory.

#### Example 6. The Isbell-Marlow fire allocation problem.

Side X has two systems  $X_1$  and  $X_2$  in quantities  $x_1(t)$  and  $x_2(t)$ , and Y has one system of amount  $y(t)$ . X's systems attack Y's and Y's system can divide its fire between  $X_1$  and  $X_2$  in proportions  $c$  and  $1-c$ . Unlike Example 4 the allocation between the two targets is under the Y's control and can be changed over time. The equations of attrition are:

$$dx_1/dt = -ca_1y, \quad dx_2/dt = -(1-c)a_2y, \quad dy/dt = -b_1x_1 - b_2x_2.$$

The combat ends when one side has no forces left. Which side will win depends on the initial conditions and the parameters. Here we will look only at the case in which Y can win, partly because the other is more complicated and also because we avoid having to make assumptions about Y's preference for different mixes of  $X_1$  and  $X_2$  remaining after Y loses. (The condition for Y to win is  $a_1b_1y(0)^2 > (b_1x_1(0)+b_2x_2(0))^2 + (a_1b_1/a_2b_2 - 1) (b_2x_2(0))^2$ . Since the Aumann-Shapley value looks at only subsets with the same proportions as the entirety, the condition holds for all the subsets that arise.) Taylor (1973) showed that to maximize Y's remaining weapons, Y should fire at whichever of X's weapons has a higher value of  $a_i b_i$  until that one is gone, then switch to the other. Accordingly he suggested measuring the value of  $X_i$  to Y by  $a_i b_i$ , a combination of the threat from  $X_i$  and Y's ability to counter it. The rule then becomes simply: fire at the more valuable weapon first. To calculate Y's payoff we use Taylor's generalized form of Lanchester's square law stating that if  $c$  is constant from time  $t_1$  to  $t_2$ , then

$$u^2(t_1) - u^2(t_2) = [ca_1b_1 + (1-c)a_2b_2] [y^2(t_1)-y^2(t_2)] \text{ where } u(t) = b_1x_1(t) + b_2x_2(t).$$

For simplicity and consistency with Example 3 we will assign the benefit  $b(S)$  of an initial set  $S$  of weapons to be the square of the number of the weapons  $Y$  has left. Effectiveness is not an issue here since we are not evaluating its importance and in any case it is different for the two targets. Assuming from now on that  $a_1b_1 > a_2b_2$ , meaning that by definition  $X_1$  are the weapons  $Y$  shoots at first, we get benefit

$$b[y(0), x_1(0), x_2(0)] = y(0)^2 - x_1(0)^2 b_1/a_1 - 2x_1(0)x_2(0)b_2/a_1 - x_2(0)^2 b_2/a_2.$$

The resulting Aumann-Shapley importances for the totals of each type of weapon are

$$\begin{aligned} Y: & y(0)^2 \\ X_1: & -x_1(0)^2 b_1/a_1 - x_1(0)x_2(0)b_2/a_1 \\ X_2: & -x_2(0)^2 b_2/a_2 - x_1(0)x_2(0)b_2/a_1 \end{aligned}$$

or, for importance measured per weapon,

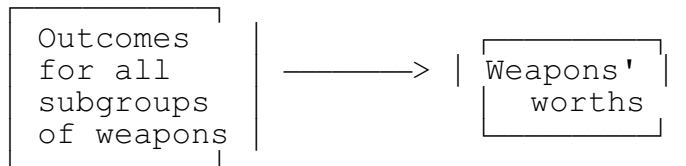
$$\begin{aligned} Y: & y(0) \\ X_1: & -x_1(0) b_1/a_1 - x_2(0)b_2/a_1 \\ X_2: & -x_2(0) b_2/a_2 - x_1(0)b_2/a_1. \end{aligned}$$

The formulae above are too complicated to comprehend entirely but we can extract some of their meaning by noting which factors they include. Weapon  $X_1$ , for example, has a (negative) importance to  $Y$  due to  $x_1$ ,  $a_1$ ,  $b_1$ ,  $x_2$ , and  $b_2$ , but  $a_2$  or  $y$  have no effect on it. One can see why each of the factors included should have an influence. Importance of an individual  $X_1$  weapon is aggravated by the number  $x_1$  of such weapons in all, since under the aimed fire hypothesis the other  $X_1$  weapons divert  $Y$ 's fire from that particular item. Naturally, an  $X_1$  weapon is worse for  $Y$  for its own firepower  $b_1$  and its invulnerability  $1/a_1$ . The number  $x_2$  and effectiveness  $b_2$  of  $X_2$  weapons matter because they are firing on  $Y$  while  $Y$  is attacking  $X_1$  and are lowering  $y$ , thus promoting the survival of an  $X_1$  item and consequently harming  $Y$  through  $X_1$  weapons. However  $X_2$ 's invulnerability  $1/a_2$  should not matter because  $Y$  is not firing on  $X_2$  while under attack by  $X_1$ , so this invulnerability play a role only when  $Y$  turns to attack  $X_2$ . Weapons in  $X_2$  are in a different position, however, and indeed according to the formulae, the invulnerability of  $X_1$  weapons affects the importance to  $Y$  of  $X_2$ .

Taylor's measures,  $a_1b_1$  and  $a_2b_2$ , are different from ours as they should be, since he is talking about values to  $Y$  for *attacking* the weapons, not the latters' values per se. His concept should involve the threat they pose but not how much  $Y$  is able to counter it.

The strong point of the allocation rule approach is that it is tied in to a well-defined decision. A shortcoming is the narrow context in which it applies. In Example 5,  $Y$  had only one weapon, but if two weapons systems faced two others, and one were especially good against one and the other especially good against the other, no single set of numbers assigned to the adversary's weapons could represent their value.

4) *The Credit Apportioning Approach*



Here one must know the outcomes and the approach tells how to allocate the credit among weapons. The theory in this paper belongs here. It would be more satisfying if we could state in a phrase what these importance numbers mean, beyond that they are the ones that satisfy the axioms. Certain interpretations of the numbers that were available in the original application of the axioms are implausible in the weapons context. Shapley's theory assigned values to people, players of a game, and the values showed players' a priori expectation from participating in the game. When we turn from evaluating people to non-goal-seeking objects, the axioms of the Shapley value still seem to apply to the undefined idea of "importance," but the interpretation of the numbers give us nonsense: which type of weapon an individual weapon should prefer to be. Fortunately, Roth and Verrecchia (1979) have suggested an alternative interpretation of Shapley values when they are applied to cost allocation problems, that the value gives the expected benefits to an item's program manager who is about to enter bargaining over a budget. This is plausible for weapons all owned by one side, but it is difficult to accept when the adversary's weapons are involved as well as in Example 2. In the end the importance of the theory is the structure it sets up, its potential for clarifying concepts. The numbers are interesting but not the goal. Most other approaches to weapons importances come up with numbers that guide a specific decision or expectation, what to procure, how to allocate, who will win. However the present theory has suggested reasons to be sceptical about their internal validity. Its aim is to straighten out our thinking, to avoid logical fallacies, and clarify the meaning of other theories.

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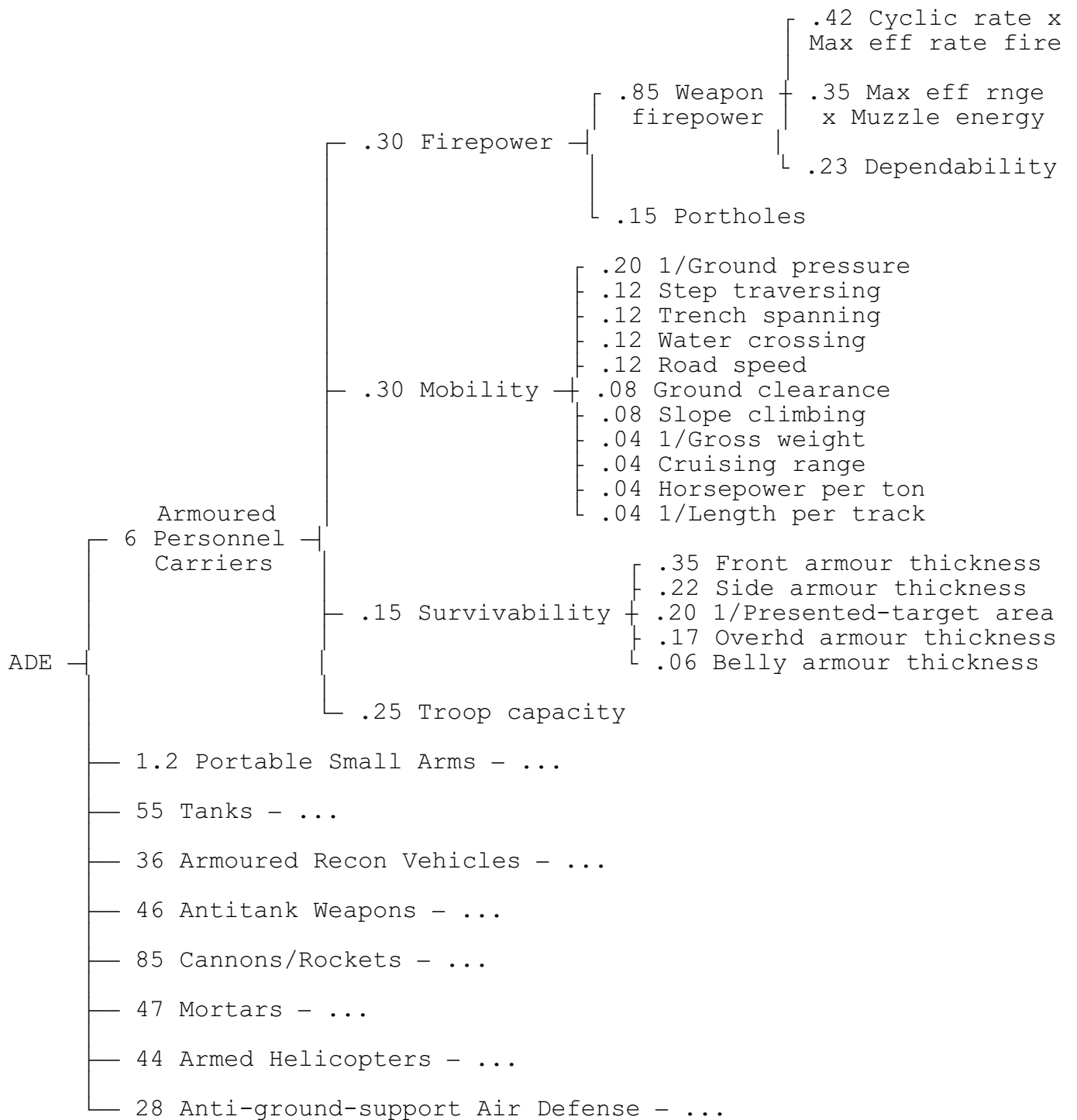


Figure 1. The calculation of Armored Division Equivalents according to WEI/WUV I (defensive). Detail is given only for APC's. Scores at branching points are weighted linear combinations of the branches. Features are normalized so that a typical weapon has value 1, then added according to the weights and numbers in the force. The final result would be the Weighted Unit Value.