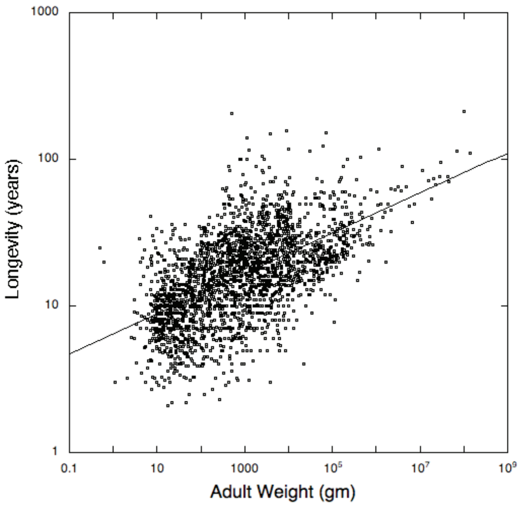
SC/BIOL 2090.02— Current Topics in Biophysics — **TERM TEST ONE**

There are three questions. You must complete all of them. Ensure that you show your work (that is, equations and calculations).

QUESTION ONE

Many organismal functions (for example, metabolic rates and rates of biomass accumulation) scale as the power 0.75 (rate ~ $\text{mass}^{0.75}$), and the ratios of surface area to volume scale as the power 0.67 (A/V ~ $\text{mass}^{0.67}$) (for mammals). But other functions scale differently. For example, longevity versus organismal mass (left) scales as the power 0.137 (longevity ~ $\text{mass}^{0.137}$) (for vertebrates). In fact, within the mammalian clade, all species die after approximately $3.3 \bullet 10^8$ breath cycles and $1.5 \bullet 10^9$ heart beats, regardless of size (that is, small animals with brief life spans have a faster heart rate). Propose a reason why all mammals, regardless of size, have the same total breath cycles and heart beats, bearing in mind that both functions are directly linked to repetitive cycles of muscle contraction.



QUESTION TWO

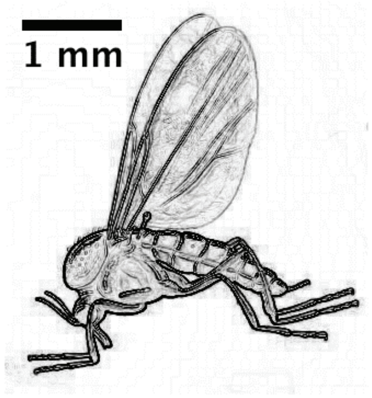
The integral forms for the first and second moments of either mass or area —

$$\int r\,dm \quad \int r^2\,dm \quad \int y\,dA \quad \int y^2\,dA$$

— play surprisingly important and fundamental roles in diverse biological processes. Select one and only one of the four integral equations and explain its biophysical application(s). Excessive length is not encouraged, but clarity is.

QUESTION THREE

The 'universe' of low Reynolds number involves not only bacteria but other organisms as well. Compare the Reynolds number for a gnat (right) flying at about 30 mm sec^{-1} in air (*J Exp Biol 208:2963, 2005*) and a human free-falling in the same medium (say, 60 m sec^{-1}). Values for air viscosity are given in the equations/constants appendix. Please ensure that your estimates of other parameters are realistic.



KEY: Question One.

To a first approximation, the work of a heart beat or a breath cycle is independent of mass. By analogy to the similar jump heights of a flea and a human:

$$\text{work } (w) \propto m \cdot h \quad \& \quad \text{work } (w) \propto V_{muscle}. \quad 25\%$$

$$(\text{or } \propto F \cdot d)$$ equating w, mass cancels out

the work of a heart beat should be independent of the mass of the heart (or diaphragm) & therefore independent of the organismal (mammal) mass.

So, if the total number of heart beats (or breath cycles) are the same in any mammalian species, 25%
it follows that the total work output in a lifespan is also independent of mammalian mass.

Since a mouse with a short lifespan has a much higher heart beat rate than a large elephant, the total work output must be completed sooner. 25%

That is, "work" $\left[\left(\underset{\text{(watts)}}{\text{joules sec}^{-1}}\right) \cdot \text{lifespan}\right]$ is constant.

To say that a mouse lives "faster" than an elephant is reasonable.

The underlying causes are likely related to the homeothermic nature of mammalians, where the faster metabolic rate of a mouse, coupled with its ~~[illegible]~~ higher $\frac{\text{surface area}}{\text{vol}}$. 25%
result in the completion of its total work output sooner than for an elephant with its slower metabolic rate and lower $\frac{\text{surface area}}{\text{volume}}$.

KEY: Question Two

$\int r\,dm$ used to evaluate an organism's center of mass $\frac{\int r\,dm}{\int dm}$

$\int r^2\,dm$ used to evaluate an organism's inertial moment.

$\int y\,dA$ used to evaluate an organisms area. or "center of area" (centroid) $\frac{\int y\,dA}{\int dA}$

$\int y^2\,dA$ used to evaluate the squared distribution of area. Best known for its utility in evaluating the strength of biomaterials (such as bones & trees)

in a tree, the outer perimeter contributes more to the strength.

KEY: Question Three $[\text{gm cm}^{-1}\text{sec}^{-1}]$

$$Re = \frac{v \cdot L \cdot \rho}{\eta} \qquad \frac{\frac{3}{6000}\,(\text{cm sec}^{-1}) \cdot \frac{0.02}{200}\,(\text{cm}) \cdot 1\ \text{gm cm}^{-3}}{\eta}$$

from the tables.

$$\eta_{air} = 1.813 \times 10^{-5}\ \text{Pa} \cdot \text{sec}\ (20°\text{C})$$

conversions

$\text{Pa} = \text{N m}^{-2}$

$\text{N} = \text{kg} \cdot \text{m} \cdot \text{sec}^{-2}$

$$\eta_{air} = 1.813 \times 10^{-5}\ \text{kg} \cdot \text{m}^{-1}\text{sec}^{-1}\ \frac{10^3\ \text{gm}}{1\ \text{kg}} \cdot \frac{\text{m}}{10^2\ \text{cm}}$$

$$= 1.813 \times 10^{-4}\ \text{gm cm}^{-1}\text{sec}^{-1}$$

= gnat 333 $(\sim 10^2)$

human $6.67 \cdot 10^9$ $(\sim 10^{10})$

6 orders of magnitude difference.

$$N_T = N_0 \bullet 2^{(T/g)}$$

as time increases, $t/g = 1, 2, 3 \ldots$, thus 2^1, 2^2, 2^3, etc.

g is the generation time

N_0 is the number of cells at time $T = 0$

N_T is the number of cells at time T

$$\frac{dM}{dt} = \mu M \quad \mu \text{ is the specific growth rate and M the mass}$$

$$\frac{dM}{dt} = \mu M - \mu \frac{M^2}{G} \quad \text{G is the maximal attainable population}$$

A cube has a surface area of $6 \bullet x^2$. Its volume is x^3.
For a sphere, the surface area is $4 \bullet \pi \bullet r^2$, and the volume is $\pi \bullet r^3$.
For a cylinder, the surface area is $2 \bullet \pi \bullet r \bullet h$ (plus $2\pi r^2$ for the top and bottom of the cylinder), the volume is $\pi \bullet r^2 \bullet h$.
For a rectangle, the surface area is $2(d \bullet w) + 2(l \bullet d) + 2(l \bullet w)$, the volume is $d \bullet w \bullet l$.

Heat conduction rates are defined by the relation:

$$P_{cond} = Q / t = k \bullet A \bullet [(T_a - T_b) / L]$$

where P_{cond} is the rate of conduction (transferred heat, Q, divided by time, t); k is the thermal conductivity; T_a and T_b are the temperatures of the two heat reservoirs a and b; A is the area; and L is the distance. Thermal conductivities of water and air are about 0.6 and 0024 W m^{-1} K^{-1}, respectively.

Thermal radiation is defined by the relation:

$$P_{rad} = \sigma \bullet \varepsilon \bullet A \bullet T^4$$

where P_{rad} is the rate of radiation; σ is the Stefan-Boltzmann constant ($5.6703 \bullet 10^{-8}$ W m^{-2} K^{-4}; ε is the emissivity (varies from 0 to 1 for a blackbody radiator); A is the area; and T is the temperature (in Kelvins). The *net* radiative emission or absorption will depend upon the difference in temperature:

$$P_{net} = \sigma \bullet \varepsilon \bullet A \bullet (T^4_{body} - T^4_{ambient})$$

$w \propto m \bullet h$ w is the work, m is the mass, and h is the height

$w \propto V_{muscle}$ V_{muscle} is the muscle volume

$$F = m \bullet a,\ F = m\frac{\Delta v}{t},\ F \bullet t\ =\ m \bullet \Delta v,\ w\ =\ F \bullet d$$

$\int r \bullet dm$ and $\int r^2 \bullet dm$ first and second moments of mass

$\int y \bullet dA$ and $\int y^2 \bullet dA$ first and second moments of area

$$\text{compression} = \rho \bullet h$$

$$h_{critical} = \frac{\text{compression}}{\rho}$$

$$I_x = \frac{\pi \bullet r^4}{4}$$

$$F_{cr} = \frac{E \bullet I \bullet \pi^2}{L_{eff}^2}$$

$$h = 0.851 \bullet \left[\frac{E}{\rho}\right]^{1/3} \bullet r^{2/3}$$

common name	species	diameter meters	height meters	Modulus of Rupture $GN \cdot m^{-2}$	Modulus of Elasticity $GN \cdot M^{-2}$	density kg m^{-3}	compression parallel to grain $MN \cdot m^{-2}$
Redwood	Sequoia sempervirens	7.6808	97.8408	0.074	9.40	436	42.4
Eastern Hemlock	Tsuga canadensis	1.6332	50.2920	0.059	8.30	431	21.2
Trembling Aspen	Populus tremuloides	0.9702	41.4528	0.059	8.22	401	14.8
White Pine	Pinus strobus	2.4174	40.2336	0.061	8.81	373	16.8
Sugar Maple	Acer saccharum	1.8030	35.0520	0.108	12.65	676	27.7
Yellow Poplar	Liriodendron tulipfera	3.0238	33.8328	0.064	10.38	427	18.3
Yellow Birch	Betula lutea	1.5038	31.6992	0.117	14.53	668	23.3
Black Locust	Robina pseudoacacia	2.5225	28.6512	0.134	14.20	798	70.2
Eastern Cottonwood	Populus deltoides	3.5898	28.3464	0.060	9.53	433	15.7
Hornbeam	Ostrya virginiana	0.9298	21.3360	0.100	11.76	762	n/a
Common Apple	Malus sylvestris	1.1400	21.3360	0.088	8.77	745	n/a
Dogwood	Cornus florida	0.8894	10.0584	0.105	10.64	796	n/a
			means	0.086	10.599	578.8	27.822

$$\Psi_{wv} = \frac{RT}{\overline{V}_w}\ln\left(\frac{\%\text{ relative humidity}}{100}\right) + \rho_w g h$$

where R is the gas constant (8.314 m^3 Pa mol^{-1} $^{o}K^{-1}$), T is the temperature (^{o}K), V_w is the partial molal volume of water ($1.805 \bullet 10^{-5}$ m^3 mol^{-1} at 20ºC [293ºK]). At 20ºC, the term RT/V_w is 135 MPa. The second term is the gravitational potential: ρ_w is the density of water (998.2 kg m^{-3} at 20ºC), g is the gravitational constant (9.807 m sec^{-2}) and h is the height.

The basic equation describing the flow velocity of a liquid through a tube is the Poiseuille equation[1]:

velocity (meters sec^{-1})

pressure difference
(Pascal = 1 kg m^{-1} sec^{-1})

Tube radius

$$v = \left(\frac{\Delta p}{l}\right)\left(\frac{1}{4 \bullet \eta}\right)(R^2 - r^2)$$

Distance from
center of tube

distance (meters)

viscosity (water = 0.01 gm cm^{-1} sec^{-1}, or Pa sec)

The fastest velocity is at the center of the tube (r = 0):

$$v = \left(\frac{\Delta p}{l}\right)\left(\frac{1}{4 \bullet \eta}\right) R^2$$

$$J_v = \left(\frac{\Delta p}{1}\right)\left(\frac{\pi}{8 \bullet \eta}\right) \bullet R^4$$

density (water = 1 gm cm^{-3})

velocity (cm sec^{-1})

tube diameter (cm)

$$R_e = \frac{\rho \bullet v \bullet l}{\eta}$$

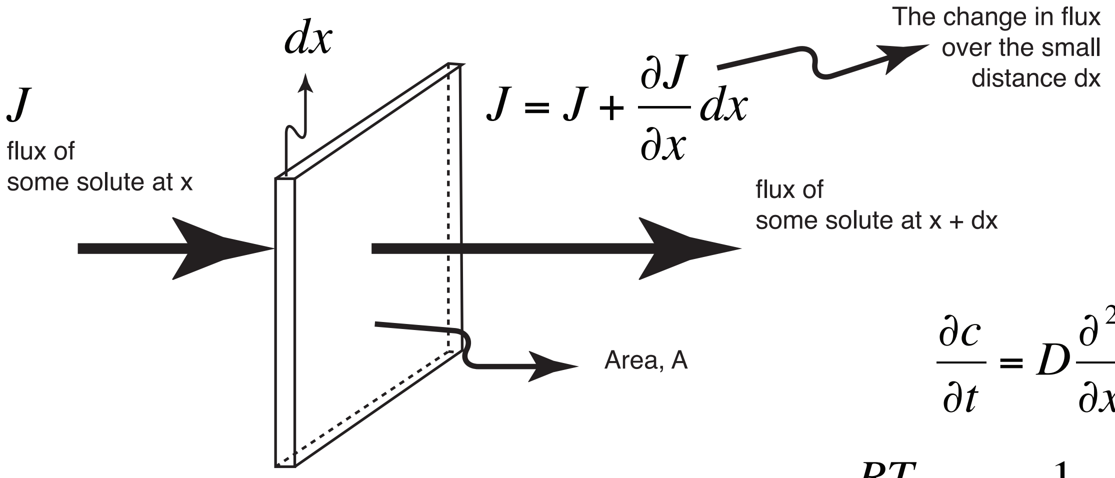
viscosity (water = 0.01 gm cm^{-1} sec^{-1})

flux, J (mol cm^{-2} sec^{-1})

concentration gradient, dc/dx with units of (mol cm^{-3})/(cm), or mol cm^{-4}.

$$J = -D \bullet \frac{dc}{dx} \Rightarrow \left(\frac{cm^2}{sec}\right)\left(\frac{mol}{cm^4}\right) \Rightarrow \left(\frac{mol}{sec \bullet cm^2}\right)$$

Diffusion coefficient with units of cm^2 sec^{-1}



$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

$$D = \frac{RT}{N} \bullet \frac{1}{6 \bullet \pi \bullet \eta \bullet r}$$

$$< x^2(t) >= 2Dt$$

$$\mathrm{r} = \sqrt{6 \bullet \mathrm{D} \bullet \mathrm{t}}$$

$$\mathrm{J} = \mathrm{D}\frac{\mathrm{K_p}}{\mathrm{d}}[c_{outside} - c_{inside}]$$

where $D\frac{K_p}{d} = P$, the permeability coefficient with

units of $\frac{\frac{cm^2}{sec}}{cm}$, or cm•sec^{-1}

$$\frac{F_{inertial}}{F_{frictional}} = \frac{l^3 \bullet \rho \bullet \nu^2 / r}{\eta \bullet l^3 \bullet \left(\nu / r^2\right)} = \frac{\rho \bullet \nu \bullet r}{\eta} = R_e$$

The drag (D) on an object is defined by:

fluid density

velocity

$$D = \frac{1}{2} \bullet \rho \bullet V^2 \bullet A \bullet C_D$$

frontal area

drag coefficient
(shape-dependent)

$$m\left(-\frac{d\upsilon}{dt}\right) = 6 \bullet \pi \bullet \eta \bullet r \bullet \upsilon$$

$$\upsilon(t) = \upsilon_0 e^{\left(-\frac{6 \bullet \pi \bullet \eta \bullet r}{m} \bullet t\right)}$$

$$p_0(t) = e^{(-\lambda t)}$$

$$p(k,\mu) = \frac{\mu^k}{k!} e^{-\mu}$$

$$\text{Rotor}_{(n)} + mH^+_{\text{outside}} \longleftrightarrow \text{Rotor}_{(n+1)} + mH^+_{\text{inside}}$$

$$\text{ADP} + \text{P}_\text{i} + \text{mH}^+_{\text{outside}} \longleftrightarrow \text{ATP} + \text{mH}^+_{\text{inside}}$$

gas constant

Faraday constant

Voltage

$$\mu = \mu^{\circ} + \text{RT}\ln(a_{H^+}) + zF\Psi$$

temperature

activity of protons

$$\Delta G_{ATP} = \Delta G^{\circ}_{ATP} + \text{RT}\ln\left(\frac{[ATP]}{[ADP][P_i]}\right)$$

$$\Delta \mathrm{G}_{\text{total}} = n \bullet \Delta\mu_{H^+} + \Delta G_{ATP} = 0$$

$$n \bullet \left(RT\ln\left(\frac{a_{H^+}^{inside}}{a_{H^+}^{outside}}\right) + F\Delta\Psi\right) + \Delta G^{o}_{ATP} + RT\ln\left(\frac{[ATP]}{[ADP][P_i]}\right) = 0$$

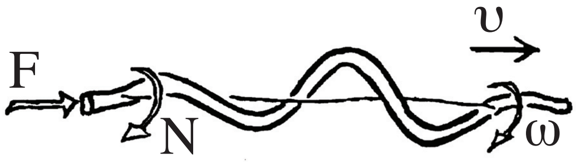
$$\Delta\mu_{H^+} = \frac{RT}{F}\ln\left(\frac{a_{H^+}^{inside}}{a_{H^+}^{outside}}\right) + \Delta\Psi \quad \text{(units: mV)}$$

RT/F is about 25 mV at 20ºC.

Both velocity and rotation contribute to both the force and torque.

$$F = A\upsilon + B\omega$$

$$N = C\upsilon + D\omega$$



$$\mathbf{P} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

The constants A, B, C and D (proportional to the viscosity and the size and shape of the ‘propellor’) comprise the propulsion matrix **P**.

APPENDIX I

Numerical Values of Constants and Coefficients

Symbol	Description	Magnitude
c	speed of light in vacuum	2.998×10^8 m s^{-1}
c^*_{wv}	saturation concentration of water vapor (i.e., at 100% relative humidity)	See pp. 548–550 for values from −30°C to 60°C.
C_P^{water}	volumetric heat capacity of water at constant pressure (1 atmosphere, 0.1013 MPa)	4.217 MJ m^{-3} °C^{-1} at 0°C 4.175 MJ m^{-3} °C^{-1} at 20°C 4.146 MJ m^{-3} °C^{-1} at 40°C
C_P^{air}	volumetric heat capacity of dry air at constant pressure (1 atmosphere)	1.300 kJ m^{-3} °C^{-1} at 0°C 1.212 kJ m^{-3} °C^{-1} at 20°C 1.136 kJ m^{-3} °C^{-1} at 40°C
D_{CO_2}	diffusion coefficient of CO_2 in air (1 atmosphere, 0.1013 MPa)	1.33×10^{-5} m^2 s^{-1} at 0°C 1.42×10^{-5} m^2 s^{-1} at 10°C 1.51×10^{-5} m^2 s^{-1} at 20°C 1.60×10^{-5} m^2 s^{-1} at 30°C 1.70×10^{-5} m^2 s^{-1} at 40°C
D_{O_2}	diffusion coefficient of O_2 in air (1 atmosphere, 0.1013 MPa)	1.95×10^{-5} m^2 s^{-1} at 20°C
D_{wv}	diffusion coefficient of water vapor in air (1 atmosphere, 0.1013 MPa)	2.13×10^{-5} m^2 s^{-1} at 0°C 2.27×10^{-5} m^2 s^{-1} at 10°C 2.42×10^{-5} m^2 s^{-1} at 20°C 2.57×10^{-5} m^2 s^{-1} at 30°C 2.72×10^{-5} m^2 s^{-1} at 40°C
e	base for natural logarithm	2.71828 ($1/e = 0.368$)
	electronic charge	1.602×10^{-19} C
F	Faraday's constant	9.649×10^4 coulomb mol^{-1} 9.649×10^4 J mol^{-1} V^{-1} 2.306×10^4 cal mol^{-1} V^{-1} 23.06 kcal mol^{-1} V^{-1}

Symbol	Description	Magnitude
g	gravitational acceleration	9.780 m s^{-2} (sea level[1], 0° latitude)
		9.807 m s^{-2} (sea level[1], 45° latitude)
		9.832 m s^{-2} (sea level[1], 90° latitude)
		978.0 cm s^{-2} (sea level[1], 0° latitude)
		980.7 cm s^{-2} (sea level[1], 45° latitude)
		983.2 cm s^{-2} (sea level[1], 90° latitude)
h	Planck's constant	6.626×10^{-34} J s
		6.626×10^{-27} erg s
		0.4136×10^{-14} eV s
		1.584×10^{-37} kcal s
hc		1.986×10^{-25} J m
		1 240 eV nm
H_{sub}	heat of sublimation of water	51.37 kJ mol^{-1} (2.847 MJ kg^{-1}) at -10°C
		51.17 kJ mol^{-1} (2.835 MJ kg^{-1}) at -5°C
		51.00 kJ mol^{-1} (2.826 MJ kg^{-1}) at 0°C
		12.27 kcal mol^{-1} (680 cal g^{-1}) at -10°C
		12.22 kcal mol^{-1} (677 cal g^{-1}) at -5°C
		12.18 kcal mol^{-1} (675 cal g^{-1}) at 0°C
H_{vap}	heat of vaporization of water	45.06 kJ mol^{-1} (2.501 MJ kg^{-1}) at 0°C
		44.63 kJ mol^{-1} (2.477 MJ kg^{-1}) at 10°C
		44.21 kJ mol^{-1} (2.454 MJ kg^{-1}) at 20°C
		44.00 kJ mol^{-1} (2.442 MJ kg^{-1}) at 25°C
		43.78 kJ mol^{-1} (2.430 MJ kg^{-1}) at 30°C
		43.35 kJ mol^{-1} (2.406 MJ kg^{-1}) at 40°C
		42.91 kJ mol^{-1} (2.382 MJ kg^{-1}) at 50°C
		40.68 kJ mol^{-1} (2.258 MJ kg^{-1}) at 100°C
k	Boltzmann's constant	1.381×10^{-23} J molecule^{-1} K^{-1}
		1.381×10^{-16} erg molecule^{-1} K^{-1}
		8.617×10^{-5} eV molecule^{-1} K^{-1}
kT		0.02354 eV molecule^{-1} at 0°C
		0.02526 eV molecule^{-1} at 20°C
		0.02569 eV molecule^{-1} at 25°C
		0.02699 eV molecule^{-1} at 40°C
K^{air}	thermal conductivity coefficient of dry air (1 atmosphere)[2]	0.0237 W m^{-1} °C^{-1} at -10°C
		0.0243 W m^{-1} °C^{-1} at 0°C
		0.0250 W m^{-1} °C^{-1} at 10°C
		0.0257 W m^{-1} °C^{-1} at 20°C
		0.0264 W m^{-1} °C^{-1} at 30°C
		0.0270 W m^{-1} °C^{-1} at 40°C
		0.0277 W m^{-1} °C^{-1} at 50°C

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1. The correction for height above sea level is -3.09×10^{-6} m s^{-2} per m of altitude.

2. The pressure sensitivity is very slight, K^{air} increasing only about 0.0001 W m^{-1} °C^{-1} per atmosphere (0.1013 MPa) increase in pressure.

Symbol	Description	Magnitude
	thermal conductivity coefficient of moist air (100% relative humidity, 1 atmosphere)	0.0242 W m^{-1} °C^{-1} at 0°C 0.0255 W m^{-1} °C^{-1} at 20°C 0.0264 W m^{-1} °C^{-1} at 40°C
K^{water}	thermal conductivity coefficient of water	0.565 W m^{-1} °C^{-1} at 0°C 0.599 W m^{-1} °C^{-1} at 20°C 0.627 W m^{-1} °C^{-1} at 40°C
ln 2		0.6931
N	Avogadro's number	6.0220×10^{23} entities mol^{-1}
Nhc		0.1196 J mol^{-1} m 119 600 kJ mol^{-1} nm 28.60 kcal mol^{-1} μm 28 600 kcal mol^{-1} nm
N^*_{wv}	saturation mole fraction of water vapor (i.e., at 100% relative humidity) at 1 atmosphere (0.1013 MPa)	See pp. 548–550 for values from −30°C to 60°C.
P^*_{wv}	saturation vapor pressure of water	See pp. 548–500 for values from −30°C to 60°C.
	protonic charge	1.602×10^{-19} C
R	gas constant	8.314 J mol^{-1} K^{-1} 1.987 cal mol^{-1} K^{-1} 8.314 m^3 Pa mol^{-1} K^{-1} 8.314×10^{-6} m^3 MPa mol^{-1} K^{-1} 0.08205 litre atmosphere mol^{-1} K^{-1} 0.08314 litre bar mol^{-1} K^{-1} 83.14 cm^3 bar mol^{-1} K^{-1}
RT		2.271×10^3 J mol^{-1} (m^3 Pa mol^{-1}) at 0°C 2.437×10^3 J mol^{-1} (m^3 Pa mol^{-1}) at 20°C 2.479×10^3 J mol^{-1} (m^3 Pa mol^{-1}) at 25°C 2.271×10^{-3} m^3 MPa mol^{-1} at 0°C 2.437×10^{-3} m^3 MPa mol^{-1} at 20°C 2.479×10^{-3} m^3 MPa mol^{-1} at 25°C 542.4 cal mol^{-1} at 0°C 582.2 cal mol^{-1} at 20°C 2.271 litre MPa mol^{-1} at 0°C 2.437 litre MPa mol^{-1} at 20°C 22.71 litre bar mol^{-1} at 0°C 24.37 litre bar mol^{-1} at 20°C 22 710 cm^3 bar mol^{-1} at 0°C 24 370 cm^3 bar mol^{-1} at 20°C 22.41 litre atmosphere mol^{-1} at 0°C 24.05 litre atmosphere mol^{-1} at 20°C
2.303 RT		5.612 kJ mol^{-1} at 20°C 5.708 kJ mol^{-1} at 25°C 1.342 kcal mol^{-1} at 20°C 1.364 kcal mol^{-1} at 25°C 56 120 cm^3 bar mol^{-1} at 20°C

Symbol	Description	Magnitude
RT/F		25.3 mV at 20°C 25.7 mV at 25 °C
2.303 RT/F		58.2 mV at 20°C 59.2 mV at 25°C 60.2 mV at 30°C
$RT/\overline{V}_w$		135.0 MPa at 20°C 137.3 MPa at 25°C 32.31 cal cm^{-3} at 20°C 135.0 J cm^{-3} at 20°C 1 350 bars at 20°C 1 330 atmospheres at 20°C
2.303 $RT/\overline{V}_w$		310.9 MPa at 20°C 316.2 MPa at 25°C 3 063 atmospheres at 20°C 3 109 bars at 20 °C
	solar constant	1 368 W m^{-2} 1.960 cal cm^{-2} min^{-1} 1.368×10^5 erg cm^{-2} s^{-1} 0.1368 W cm^{-2}
	thermal capacity of water (mass basis)	4 218 J kg^{-1} $°\text{C}^{-1}$ at 0°C 4 182 J kg^{-1} $°\text{C}^{-1}$ at 20°C 4 179 J kg^{-1} $°\text{C}^{-1}$ at 40°C 1.0074 cal g^{-1} $°\text{C}^{-1}$ at 0°C 0.9988 cal g^{-1} $°\text{C}^{-1}$ at 20°C 0.9980 cal g^{-1} $°\text{C}^{-1}$ at 40°C
	thermal capacity of water (mole basis)	75.99 J mol^{-1} $°\text{C}^{-1}$ at 0°C 75.34 J mol^{-1} $°\text{C}^{-1}$ at 20°C 75.28 J mol^{-1} $°\text{C}^{-1}$ at 40°C 18.14 cal mol^{-1} $°\text{C}^{-1}$ at 0°C 17.99 cal mol^{-1} $°\text{C}^{-1}$ at 20°C 17.98 cal mol^{-1} $°\text{C}^{-1}$ at 40°C
$\overline{V}_w$	partial molal volume of water	1.805×10^{-5} m^3 mol^{-1} at 20°C 18.05 cm^3 mol^{-1} at 20°C
ε_0	permittivity of a vacuum	8.854×10^{-12} coulomb^2 m^{-2} N^{-1} 8.854×10^{-12} coulomb m^{-1} V^{-1}
η_{air}	viscosity of air	1.716×10^{-5} Pa s at 0°C 1.813×10^{-5} Pa s at 20°C 1.907×10^{-5} Pa s at 40°C
η_w	viscosity of water	1.787×10^{-3} Pa s at 0°C 1.307×10^{-3} Pa s at 10°C 1.002×10^{-3} Pa s at 20°C 0.798×10^{-3} Pa s at 30°C 0.653×10^{-3} Pa s at 40°C 0.547×10^{-3} Pa s at 50°C 0.01002 dyn s cm^{-2} at 20°C 0.01002 poise at 20°C

Symbol	Description	Magnitude
ν_{air}	kinematic viscosity of air (dry, 1 atmosphere)	1.327×10^{-5} m^2 s^{-1} at 0°C
		1.505×10^{-5} m^2 s^{-1} at 20°C
		1.691×10^{-5} m^2 s^{-1} at 40°C
ν_w	kinematic viscosity of water	1.787×10^{-6} m^2 s^{-1} at 0°C
		1.004×10^{-6} m^2 s^{-1} at 20°C
		0.658×10^{-6} m^2 s^{-1} at 40°C
π	circumference/diameter of circle	3.14159
ρ_{air}	density of dry air (1 atmosphere, 0.1013 MPa)	1.293 kg m^{-3} at 0°C
		1.205 kg m^{-3} at 20°C
		1.128 kg m^{-3} at 40°C
	density of saturated air (1 atmosphere)[3]	1.290 kg m^{-3} at 0°C
		1.194 kg m^{-3} at 20°C
		1.097 kg m^{-3} at 40°C
ρ_w	density of water	999.8 kg m^{-3} (0.9998 g cm^{-3}) at 0°C
		1 000.0 kg m^{-3} (1.0000 g cm^{-3}) at 4°C
		999.7 kg m^{-3} (0.9997 g cm^{-3}) at 10°C
		998.2 kg m^{-3} (0.9982 g cm^{-3}) at 20°C
		995.6 kg m^{-3} (0.9956 g cm^{-3}) at 30°C
		992.2 kg m^{-3} (0.9922 g cm^{-3}) at 40°C
$\rho_w g$		0.00979 MPa m^{-1} (20°C, sea level, 45° latitude)
		0.0979 bar m^{-1} (20°C, sea level, 45° latitude)
		979 dyn cm^{-3} (20°C, sea level, 45° latitude)
		0.0966 atmosphere m^{-1} (20°C, sea level, 45° latitude)
σ	Stefan–Boltzmann constant	5.670×10^{-8} W m^{-2} K^{-4}
		5.670×10^{-12} W cm^{-2} K^{-4}
		8.130×10^{-11} cal cm^{-2} min^{-1} K^{-4}
		5.670×10^{-5} erg cm^{-2} s^{-1} K^{-4}
σ_w	surface tension of water	0.0756 N m^{-1} (Pa m) at 0°C
		0.0742 N m^{-1} (Pa m) at 10°C
		0.0728 N m^{-1} (Pa m) at 20°C
		0.0712 N m^{-1} (Pa m) at 30°C
		0.0696 N m^{-1} (Pa m) at 40°C
		7.28×10^{-8} MPa m at 20°C
		72.8 dyn cm^{-1} at 20°C
		7.18×10^{-5} atmosphere cm at 20°C
		7.28×10^{-5} bar cm at 20°C

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[3] Moist air is less dense than dry air at the same temperature and pressure, because the molecular weight of water (18.0) is less than the average for air (29.0).