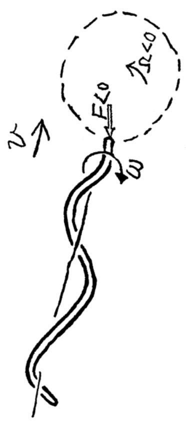
SC/BIOL 2090.02— Current Topics in Biophysics — **TERM TEST ONE**

There are three questions. You must complete all of them. Ensure that you show your work (that is, equations, calculations and units). Excessive length is not encouraged.

QUESTION ONE

The Reynolds number is a crucial dimensionless number that defines physical constraints on organismal motility as well as a myriad of other biological functions.



Compare the Reynolds number of a human and a bacterium (use *reasonable* estimates of length, mass and velocity: bacteria are about 1 μm in effective length and swim at about 20 μm sec^{-1}, humans walk at about 1.8 m sec^{-1}).

For humans to experience the same Reynolds number as a bacteria, what would the viscosity of air have to be?

QUESTION TWO

What is a Newton? Remember that Dr. Lew is not a physicist, and he believes that units are important.

QUESTION THREE

The Wikipedia article on Reynolds number mentions that the transition from laminar flow to turbulent flow occurs at an R_e of 10^3 for hydraulic flow through a cylindrical pipe (such as a xylem vessel or artery/vein) but that the laminar to turbulent transition for a cell swimming in aqueous solution occurs at a much lower R_e of about 10^{-1}. Under circumstances where the velocity, density, diameter and viscosity are *all the same*, why would the laminar to turbulent transition R_e be so different (10^4-fold)?

Viscosities (and other data) for various liquids (and air)

Fluid	**Density, ρ ($kg \cdot m^{-3}$)**	**Viscosity, η (Pa • sec) ($kg \cdot m^{-1} \cdot sec^{-1}$)**	**Viscous critical force ($f_{critical}$) (N)**
Air	1	$2 \cdot 10^{-5}$	$4 \cdot 10^{-10}$
Water	1000	$9 \cdot 10^{-4}$	$8 \cdot 10^{-10}$
Olive Oil	900	$8 \cdot 10^{-2}$	$7 \cdot 10^{-6}$
Glycerine	1300	1	$8 \cdot 10^{-4}$
Corn Syrup	1000	5	$3 \cdot 10^{-2}$

Nota bene. The viscous critical force, $f_{critical} = \eta^2/\rho$, is a measure of the force required to shift from laminar flow to turbulent flow. It depends on viscosity and density, but is not a dimension-less number (like the Reynolds Number R_e).

Nota bene. Kinematic viscosity is sometimes used, and is equal to η/ρ (with units of m^2 sec^{-1}).

Nota bene. Two other units are sometimes used to describe viscosity. One is the poise (with cgs units of g cm^{-1} sec^{-1}). The other is the stoke, for kinematic viscosity (with cgs units of cm^2 sec^{-1}).

Source: Philip Nelson. Biological Physics. pp. 165.

KEY

Question One

$$Re\ (\text{bacteria}) = \frac{10^3\ kg/m^3 \cdot 20 \times 10^{-6}\ m/s \cdot 1 \times 10^{-6}\ m}{9 \times 10^{-4}\ kg/m \cdot s} \quad (\eta\ \text{water})$$

(30) $= 2.2 \times 10^{-5}$ (unit-less)

human inertia depends on human density

I think the height, rather than width is a preferred "effective" size

$$Re\ (\text{human}) = \frac{10^3\ kg/m^3 \cdot 1.8\ m/s \cdot 1.5\ m}{2 \times 10^{-5}\ kg/m \cdot s} \quad (\eta\ \text{air})$$

(30) $= 1.4 \times 10^8$ (unit-less)

For a human to experience the bacterial "universe", the viscosity of air would have to be:

(30) $$\eta = \frac{10^3\ kg/m^3 \cdot 1.8\ m/s \cdot 1.5\ m}{2.2 \times 10^{-5}} \quad (Re[\text{bacteria}])$$

$= 1.2 \times 10^8\ kg/m \cdot s$ very very high.

(10) For comparative purposes, molten rock has a viscosity of $\sim 10^{12}\ kg/m \cdot s$. Molten glass ranges from 10^4 to $10^6\ kg/m \cdot s$

KEY

Question Two

From the fundamental relation $F = m \cdot a$, where force (F) is equal to a mass (m) being accelerated by m/s^2 ($a = d/dt \cdot v$, where $v = dx/dt$).

The force required to accelerate a mass is standardized as a Newton (N), equal to 1 $kg \cdot m/s^2$.

Newtons are used in dynamic systems where mass is accelerated (or de-celerated in frictional forces), and is a part of other "forces" such as pressure ($Pa = N/m^2$). Thus, we have seen Newtons in many guises in our Biophysics lectures.

KEY

Question Three

The differences in the laminar to turbulent transitions for flow through a tube ($\sim 10^3$) and particle movement (or the concordant flow around a particle) ($\sim 10^{-1}$) are experimentally verifiable

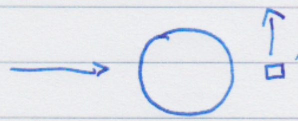
It's hard to envision this as a "mistake" in the calculation of Re. If this were true, then we could normalize Re on the basis of the transition. But, if we did, a tube of 100 um diameter would be equivalent to a particle 10 nm in diameter (10^4-fold smaller). That doesn't seem realistic.

Instead, it may be more useful to explore fundamental differences that are not embedded in the Re equation That is, neither velocity (and therefore not the Poiseulle equation) nor size.

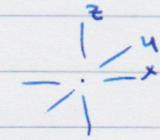
Two fundamental differences may be causal.

one is the vector(s) of accelerative force.

For a sphere, the volume displacement:



volume element, $l^3 \cdot \frac{dv}{dt}$



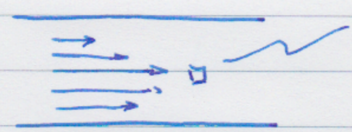
must move around the sphere:

$$l^3 \frac{d}{dt} \cdot \frac{dx}{dt}, \; l^3 \frac{d}{dt} \cdot \frac{dy}{dt} \; \& \; l^3 \frac{d}{dt} \cdot \frac{dz}{dt}.$$

KEY

Question Three (continued)

For a tube, the volume displacement occurs solely in the x-direction



volume element $l^3 \cdot \frac{d}{dt} \cdot \frac{dx}{dt}$

(this is noted explicitly in the course notes. The Poiseuille equation assumes dP/dy and dP/dz are zero.)

So, there are two additional acceleration vectors for a particle. that could contribute to a transition to turbulent flow at a lower Re for the particle.

The second fundamental difference is the boundary conditions. For a tube, the velocity is zero at the walls. For a particle, volume displacements will extend far away from the sphere. Certainly, 2 to 4 times the sphere diameter. This 'action at a distance' may also contribute to the laminar to turbulent transition at lower Re for the sphere.