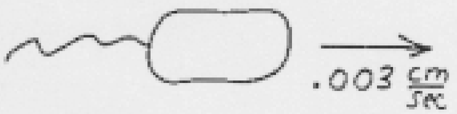
Be sure to write your name above. Read the three sub-questions carefully, think, then write your answers in the lined space (front and back of this page). When finished, please hand your answer in, separate from your exam booklet.

QUESTION: Purcell[1] observed that a bacteria moving at 30 μm s^{-1} would require 2 • 10^{-8} erg s^{-1} if the propulsion efficiency was 1% (for students far younger than their professors, an erg is 10^{-7} joules). Berg[2] provides a lower estimate of 8 • 10^{-11} erg s^{-1} based on Stoke's drag: the power required to maintain a velocity of 20 μm s^{-1} being: $6\pi\eta a v^2$ where η is the viscosity (10^{-2} g/cm s), a is the radius (10^{-4} cm) of the spherical bacterium, and v is the velocity (0.002 cm/sec).



Energy required, if efficiency of propulsion is 1%:

2×10^{-8} erg/sec,

or $\frac{1}{2}$ watt/kilogram

Glucose is a common energy source for a bacteria and the Gibbs free energy change for conversion to H_2O and CO_2 is 2870 kJ/mole (36–38 ATP molecules are formed from one glucose molecule in aerobic respiration).

• How many glucose molecules would be required by a bacteria (per second) to 'fuel' its motility using Purcell's estimate (2 • 10^{-8} erg s^{-1})? How many ATP molecules (per second)?

$$2870 \frac{\text{kJoule}}{\text{mole}} \cdot \frac{1}{6.02 \times 10^{23} \text{ molecule/mole}} = 4.767 \times 10^{-18} \text{ J/molecule}$$

$$2 \times 10^{-8} \frac{\text{erg}}{\text{sec}} \cdot 10^{-7} \frac{\text{joules}}{\text{erg}} = 2 \times 10^{-15} \frac{\text{joules}}{\text{sec}}$$

$$2 \times 10^{-15} \frac{\text{joules}}{\text{sec}} \frac{1}{4.767 \times 10^{-18} \text{ J/molecule}} = 420 \text{ glucose molecules}$$

$$37 \frac{\text{ATP molecules}}{\text{glucose molecule}} \times 420 \text{ glucose molecule} = 15540 \text{ ATP molecules}$$

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[1] Purcell EM (1977) Life at low Reynolds number. American Journal of Physics 45:3–10.
[2] Berg HC (1993) Random Walks in Biology. Princeton University Press. pp. 77

• A bacterial cell is geometrically similar to a rectangle with dimensions of 1 by 1 by 2 μm (10^{-6} m). The normal ATP concentration is about 3 mM (mMoles/liter). What percentage of the normal concentration would be required for motility, per second?

bacteria volume $(1 \times 10^{-6} m)(1 \times 10^{-6} m)(2 \times 10^{-6} m) = 2 \times 10^{-18} m^3$

$10^{-3} m^3 = 1$ liter $\quad 2 \times 10^{-15}$ l

$15540 \text{ ATP molecules} \times \frac{1}{6.02 \times 10^{23} \text{ molecules/mole}} = \frac{2.58 \times 10^{-20} M}{2 \times 10^{-15} l} = 12.9 \mu M$

$100 \cdot \frac{0.0129}{3} = 0.43\%$ (very low)

• Recall that power is equal to force times velocity. Explain to a non-physicist (like Dr. Lew) why power required for bacterial motility is described by $6\pi\eta av^2$?

We need to consider the factors that contribute to drag when Reynolds number is so low that turbulence "need not apply".

The viscosity (η) will have an impact. The higher the viscosity, the greater the drag. The larger the bacteria (a), the slower the bacteria, but what about velocity (v)? The faster, the greater the drag.

(End)