

# TERM TEST ONE: KEY

## QUESTION ONE

### Proportion and Scaling of a 10-fold taller human

If a human being were 10 times taller than normal, how thick would the legs have to be to provide the same strength as a human of normal height?



The problem has been explored previously in course notes, lecture and assignments. Essentially, compressive strength scales per area, weight scales with volume ( $V^{2/3}$  is proportional to  $A$ ). If the human is 10 times taller, mass will be 10 times more, area must be increased  $10^{3/2} = 31$  to maintain the same compressive strength. Other alternatives (compressive  $h_{\text{critical}}$  and Euler's column) are awkward at best.

[ $A^{3/2}$  (8/8); partial credit for effort or creativity]

## QUESTION TWO

### Physical Meanings

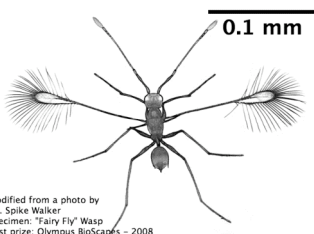
What is a Pascal? Remember that Dr. Lew is not a physicist, and he believes that units are important, in addition to your explanation.

Apparently, pressure ( $P$ ) is Force (Newtons) per area ( $m^2$ ) [ $P = N \cdot m^{-2}$ ]. Newtons (N) are  $kg \cdot m \cdot s^{-2}$ . So, in fundamental units,  $P = kg \cdot m^{-1} \cdot s^{-2}$ .

[ $P = N \cdot m^{-2}$  (7/8);  $kg \cdot m^{-1} \cdot s^{-2}$  (1/8); partial credit for effort]

## QUESTION THREE

### Reynolds number



Modified from a photo by Mr. Spike Walker  
Specimen: "Fairy Fly" Wasp  
First prize: Olympus BioScapes - 2008

The fairy fly is reputedly the smallest flying insect. In fact, the feathery appendages can barely be considered wings. How is it that a fairy fly can fly?

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[formula setup (2/8); units (4/8); rational answer (2/8)]

Hints:

A comparative approach may be most useful, considering the physical environment of a fairy fly compared to that of a normal size bird. You can approximate their relative flight velocities as 2000 body lengths per minute. Terminal velocity may also be helpful, and is defined by:

$$V_t = \sqrt{\frac{2mg}{\rho A C_d}}$$

where  $V_t$  is the terminal velocity,  $m$  is the mass,  $g$  the acceleration due to gravity ( $9.81 \text{ m/s}^2$ ),  $\rho$  the density of air,  $A$  the effective area and  $C_d$  the drag coefficient (assume it is  $24/R_e$ ).

Reynolds Number  $Re = \frac{\rho \cdot v \cdot r}{\eta}$

Fairy Fly: 
$$\frac{(10^3 \text{ kg m}^{-3})(0.3 \times 10^{-2} \text{ m s}^{-1})(0.01 \times 10^{-2} \text{ m})}{1.813 \times 10^{-5} \text{ Pa s}}$$
 Annotations:   
 -  $0.01 \text{ cm} \times 2000 / 60 \text{ sec} \times 10^{-2} \text{ for m s}^{-1}$  (pointing to velocity)   
 -  $0.01 \text{ cm} \times 10^{-2} \text{ for m}$  (pointing to radius)   
 -  $\left(\frac{\text{kg} \cdot \text{m}}{\text{m s}}\right)$  (pointing to the numerator)

$Re = 16$   
 $C_D = \frac{24}{16} = 1.5$

=  
 bird 
$$\frac{(10^3 \text{ kg m}^{-3})(3.3 \text{ m s}^{-1})(0.1 \text{ m})}{1.813 \times 10^{-5} \text{ Pa s}}$$
 Annotations:   
 -  $10 \text{ cm} \times 2000 / 60 \text{ sec} \times 10^{-2} \text{ for m s}^{-1}$  (pointing to velocity)   
 -  $10 \text{ cm}$  (pointing to radius)

$Re = 1.8 \times 10^5$   
 $C_D = \frac{24}{1.8 \times 10^5} = 1.3 \times 10^{-4}$

=  
 Terminal Velocity  $\sqrt{\frac{2mg}{\rho A C_D}}$  (Annotation:  $(0.01 \times 10^{-2} \text{ m})^3 \cdot (10^3 \text{ kg m}^{-3})$ )

Fairy Fly 
$$\sqrt{\frac{(2)(1 \times 10^{-9} \text{ kg})(9.81 \text{ m s}^{-2})}{(10^3 \text{ kg m}^{-3})(1 \times 10^{-8} \text{ m}^2)(1.5)}} = V_t = 0.04 \text{ m s}^{-1}$$
 Annotations:   
 -  $(0.01 \times 10^{-2} \text{ m})^2$  (pointing to the area term in the denominator)

bird 
$$\sqrt{\frac{(2)(0.001 \text{ kg})(9.81 \text{ m s}^{-2})}{(10^3 \text{ kg m}^{-3})(1 \times 10^{-4} \text{ m}^2)(1.3 \times 10^{-4})}} = V_t = 39 \text{ m s}^{-1}$$

Basically, the Fairy Fly barely falls compared to a normal bird.