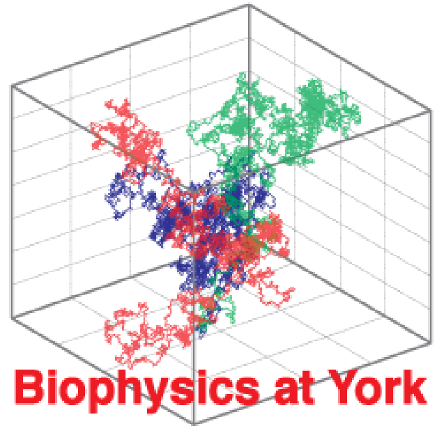
Name:

Student ID:

SC/BPHS2090 Final Exam (Lew)

Be sure to write your name and student ID above. Read the questions carefully, think, then write your answers in the lined space. When finished, please hand your answer in, separate from your exam booklet.

QUESTION ONE: Purcell[1] asserted that bacterial swimming "accomplishes nothing" when it comes to obtaining food from the surrounding medium. Explain why Purcell is right, so that even a non-physicist (like Dr. Lew) can understand. How large would a bacterium have to be for Purcell to be wrong? Show your work.

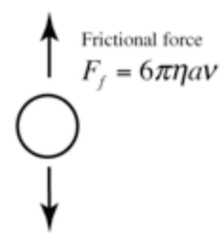


The most straightforward way to approach the question is to compare the mixing caused by advection *versus* mixing caused by diffusion. Purcell provides the starting point for the comparison: the dimensionless Sherwood number: (velocity • length) / Diffusion coefficient. If the Sherwood number is greater than one, than advection is better than diffusion. For the parameters appropriate for a bacterium, the Sherwood number is less than one, so Purcell is right, swimming "accomplishes nothing".

[1] Purcell, E.M. (1977) Life at low Reynolds number. American Journal of Physics 45:3–11.

QUESTION TWO: Purcell seems unperturbed by the low efficiency of bacterial motility. Explain why so that even a non-physicist (like Dr. Lew) will understand. What percentage of the basal metabolic rate of a human would be required for propulsion if we had flagella and existed at a Reynolds number similar to that of a bacterium? Show your work.

The force that has to be overcome to maintain a constant velocity is a function of the viscosity of the solution and size of the object, Stokes Relation:



This drag is relatively small (compared to the higher drag that occurs at Reynolds numbers greater than 1, where turbulent drag starts to be a factor). And, since diffusion supplies more than sufficient energy to the bacterium, motility is not so metabolically expensive that the bacterium will starve.

To compare to humans, scale the energy requirement of 0.5 W per kilogram: about 35 W for a 70 kilogram human. A Watt is equal to a joule per second. There are (60•60•24) [86400 sec/day] • 35 joules per second equals 3024000 joules per day. A kcal is equal to 4187 joules, so, about 725 kcal per day, compared to a basal metabolism of about 2000 kcal per day. It is significant for a human.

QUESTION THREE: At low Reynolds number, a jet propulsion mechanism won't work. Why? Propose an alternative mechanism —excluding flagella-based or pili twitching.

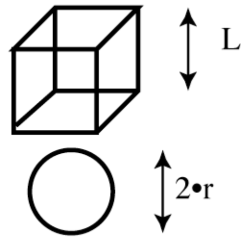
Because you have to 'pull' solution into the jet engine to eject it, it's reversible movement at low Reynolds number. The closest analogy in Purcell's paper is the scallop. Alternatives? This is a direct copy and paste from Purcell, who explores a number of mechanisms.

(End)

Symbol	Value	Units	Comments
GAS CONSTANT			
R	8.314	J mol^{-1} K^{-1}	R is the Boltzmann constant times Avogadro's Number ($6.023 \cdot 10^{23}$)
	1.987	cal mol^{-1} K^{-1}	
	8.314	m^{-3} Pa mol^{-1} K^{-1}	
RT	$2.437 \bullet 10^3$	J mol^{-1}	At 20 ºC (293 ºK)
	$5.833 \bullet 10^2$	cal mol^{-1}	At 20 ºC (293 ºK)
	2.437	liter MPa mol^{-1}	At 20 ºC (293 ºK)
RT/F	25.3	mV	At 20 ºC (293 ºK)
2.303 • RT	5.612	kJ mol^{-1}	At 20 ºC (293 ºK)
	1.342	kcal mol^{-1}	At 20 ºC (293 ºK)
k_B	$1.381 \bullet 10^{-23}$	J K^{-1}	Boltzmann constant
FARADAY CONSTANT			
F	$9.649 \bullet 10^4$	coulombs mol^{-1}	F is the electric charge times Avogadro's Number
	$9.649 \bullet 10^4$	J mol^{-1} V^{-1}	
	23.06	kcal mol^{-1} V^{-1}	
CONVERSIONS			
kcal	4.187	kJ (kiloJoules)	Joules is an energy unit (equal to 1 Newton•meter)
erg ($g \cdot cm^2/s^2$)	$1 \bullet 10^{-7}$	J (Joules)	(see above)
Watt	1	J sec^{-1}	
Volt	1	J $coulomb^{-1}$	
Amperes	1	coulomb sec^{-1}	
Pascal (Pa)	1	Newton $meter^{-2}$	Pascal is a pressure unit (equal to 10^{-5} bars)
Radians	Radians•(180º/π)	degrees	Conversion of radians to degrees
PHYSICAL PROPERTIES			
η_w	$1.004 \bullet 10^{-3}$	Pa sec	viscosity of water at 20 ºC
ν_w	$1.004 \bullet 10^{-6}$	m^2 sec^{-1}	kinematic viscosity of water at 20 ºC (viscosity/density)
V_w	$1.805 \bullet 10^{-5}$	m^3 mol^{-1}	partial molal volume of water at 20 ºC

Source: mostly Nobel, Park S (1991) Physicochemical and Environmental Physiology

A cube has a surface area of $6 \bullet L^2$. Its volume is L^3. As long as the shape is constant, the ratio of suraface area to volume will always be $(6 \bullet L^2) / L^3$, or 6/L.
For a sphere, the surface area is $4 \bullet \pi \bullet r^2$, and the volume is $\pi \bullet r^3$; the corresponding ratio of surface area to volume is 4/r.



density (water = 1 gm cm^{-3}) velocity (cm sec^{-1})

Reynolds number

$$R_e = \frac{\rho \bullet \nu \bullet l}{\eta}$$

length (cm)

viscosity (water = 0.01 gm cm^{-1} sec^{-1})

$$m\left(-\frac{d\upsilon}{dt}\right) = 6 \bullet \pi \bullet \eta \bullet r \bullet \upsilon$$

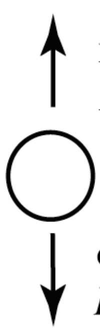
$$\upsilon(t) = \upsilon_0 e^{\left(-\frac{6 \bullet \pi \bullet \eta \bullet r}{m} \bullet t\right)}$$

$$F = A\upsilon + B\omega$$
$$N = C\upsilon + D\omega$$

That is, both velocity and rotation contribute to both the force and torque.

$$J_x = -D\frac{\partial c}{\partial x} + v_x \bullet c$$

units: moles cm^{-2} sec^{-1} (cm^2 sec^{-1})(moles cm^{-4}) (cm sec^{-1})(moles cm^{-3})



Frictional force

$$F_f = 6\pi\eta a v$$

Where the frictional and gravitational forces are balenced, the velocity reaches a steady state.

Gravitational pull

$$F_g = \tfrac{4}{3}\pi a^3 \Delta\rho g$$

Sherwood number

velocity (cm sec^{-1})

$$S_h = \frac{\nu \bullet l}{D}$$

length (cm)

Diffusion (cm^2 sec^{-1})