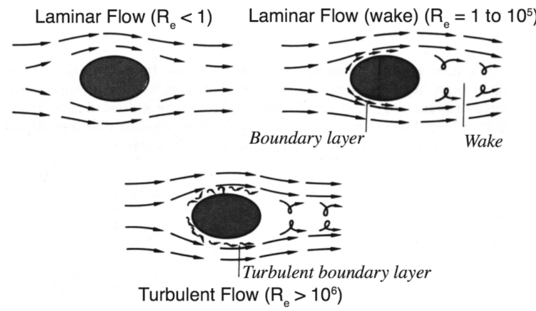
Name:___________KEY____________ SC/BPHS2090 First Term Test (Lew) 9 October 2014
Student ID:________________________

Be sure to write your name and student ID above. Read the three questions carefully, think, then write your answers in the lined space (front and back of this page). When finished, please hand your answer in.

QUESTION ONE:

The Wikipedia article on Reynolds number (R_e) mentions that the transition from laminar flow to a turbulent wake occurs at a R_e of 10^3 for hydraulic flow through a cylindrical pipe (such as xylem vessels or arteries/veins) but that the laminar to turbulent wake transition for a cell swimming in aqueous solution occurs at a much lower R_e of about 10^{-1}. Under circumstances where the velocity, density, diameter and viscosity are *all the same*, why would the laminar to turbulent wake transition R_e be so different (10^4-fold)?



Two factors may be at play. The first is due to a fundamental difference in movement of a sphere (a swimming bacteria) through an 'open solution' and the movement of a fluid through a tube. In the first case, the fluid must be displaced as the sphere moves. The volume element displacement occurs in three directions:

$$\text{the volume element } l^3 \frac{dv}{dt} \text{ must move around the sphere: } l^3 \frac{d}{dt}\frac{dx}{dt}, \; l^3 \frac{d}{dt}\frac{dy}{dt}, \text{ and } l^3 \frac{d}{dt}\frac{dz}{dt}$$

For the tube, the volume displacement occurs solely in the x-direction (explicitly noted in the course notes, which assume the dP/dy and dP/dz are zero):

$$\text{the volume element } l^3 \frac{dv}{dt} \text{ only moves in the x-direction: } l^3 \frac{d}{dt}\frac{dx}{dt}$$

So, there are two additional accelerative forces on the particle that could contribute to the appearance of turbulent wake at lower Re for the sphere in aqueous solution.

Another possible explanation has to do with the boundary conditions: Hydraulic flow through the tube is bounded to zero velocity at the walls of the cylinder. No such constraint exists for a sphere in aqueous solution. Thus volume displacements extend a far greater distance away from the sphere compared to the cylindrical pipe. This action at a distance may encourage the appearance of a turbulent wake.

scoring:

base: 25 (lower if minimal effort)
3-d acceleration: 12.5
constrained (v=0) boundary: 12.5

(/50 pt)

(Continued on Back of Page)

QUESTION TWO:

Bacteria are often of greatest interest (from a clinical viewpoint) when they adhere to a surface. From a physical perspective (that is, unrelated to any non-random movement of the bacteria), would you expect the motion of bacteria near a surface to differ from its motion in an open aqueous space? Explain using a random walk model.

In a random walk, there is an equal probability that the particle will walk 'left' or 'right'. If there is a surface on the left side, any time the bacterium walks 'left', it will return to its starting point. Thus, a bacterium will spend a relatively long period of time at the surface, solely on the basis of a random walk. Furthermore, if the bacterium is far away from the surface, but randomly walks to the surface, once there, it will tend to spend time there, as the 'left' walk causes it to stay at the same place. This purely physical phenomenon works in favor of the bacterium, because the extended dwell time at the surface increases the probability of an adhesion event and eventual bio-film formation.

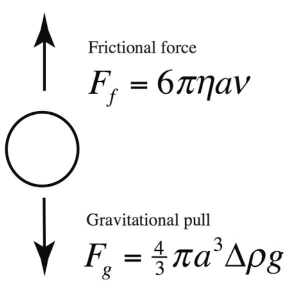
scoring:

base: 12.5
dwell at surface: 6.25
return to surface: 6.25

(/25 pt)

QUESTION THREE:

Stokes relation defines the steady state velocity of a particle when frictional forces are equal to gravitational forces. At low Reynolds number, would the velocity be affected if the particle fell inside a narrow column? If so, how far away would the column walls have to be to allow unimpeded fall of a particle of radius a? Show your work.



There are a number of ways to approach this problem. The easiest may be to consider volume displacement as the particle falls inside the narrow column. Simplifying to consider the relative areas: The particle area would be πa^2, the area of the cylinder πk^2, the displacement area with increasing cylinder radius would be $\pi k^2 - \pi a^2$. Setting a = 1, even at k=2, the displacement area would be 47 compared to 3.14 for the particle. So, if the cylinder has a radius greater than 2–fold the radius of the particle, the area available for volume displacement is voluminous.

An alternative way to approach the problem is to consider laminar flow lines as they are affected by the frictional gradient $dF_{frictional}/dx$. The gradient would scale with the inverse of distance. So a distance 2a would decrease frictional drag due to the stationary cylinder wall by 0.5, 4a by 0.25, 8a by 0.125. With this approach, a cylinder radius 8–fold the radius of the particle would minimize frictional impedance very effectively.

scoring:

base: 12.5
volume displacement: 7.5
frictional gradient: 7.5

(__ /25 pts)

(End)

Symbol	Value	Units	Comments
GAS CONSTANT			
R	8.314	J mol^{-1} K^{-1}	R is the Boltzmann constant times Avogadro's Number (6.023•10^{23})
	1.987	cal mol^{-1} K^{-1}	
	8.314	m^3 Pa mol^{-1} K^{-1}	
RT	2.437 • 10^3	J mol^{-1}	At 20 ºC (293 ºK)
	5.822 • 10^2	cal mol^{-1}	At 20 ºC (293 ºK)
	2.437	liter MPa mol^{-1}	At 20 ºC (293 ºK)
RT/F	25.3	mV	At 20 ºC (293 ºK)
2.303 • RT	5.612	kJ mol^{-1}	At 20 ºC (293 ºK)
	1.342	kcal mol^{-1}	At 20 ºC (293 ºK)
k_B	1.381 • 10^{-23}	J molecule^{-1} K^{-1}	Boltzmann's constant
FARADAY CONSTANT			
F	9.649 • 10^4	coulombs mol^{-1}	F is the electric charge times Avogadro's Number
	9.649 • 10^4	J mol^{-1} V^{-1}	
	23.06	kcal mol^{-1} V^{-1}	
CONVERSIONS			
kcal	4.187	kJ (kiloJoules)	Joules is an energy unit (equal to 1 Newton•meter)
Watt	1	J sec^{-1}	
Volt	1	J coulomb^{-1}	
Amperes	1	coulomb sec^{-1}	
Pascal (Pa)	1	Newton meter^{-2}	Pascal is a pressure unit (equal to 10^{-5} bars)
Radians	Radians•(180º/π)	degrees	Conversion of radians to degrees
PHYSICAL PROPERTIES			
η_w	1.002 • 10^{-3}	Pa sec	viscosity of water at 20 ºC
ν_w	1.002 • 10^{-6}	m^2 sec^{-1}	kinematic viscosity of water at 20 ºC (viscosity/density)
V_w	1.805 • 10^{-5}	m^3 mol^{-1}	partial molal volume of water at 20 ºC

Source: Nobel, Park S (1991) Physicochemical and Environmental Physiology

$N_T = N_0 \bullet 2^{(T/g)}$

as time increases, t/g = 1, 2, 3 ...,
thus 2^1, 2^2, 2^3, etc.

g is the generation time

N_0 is the number of cells at time T = 0

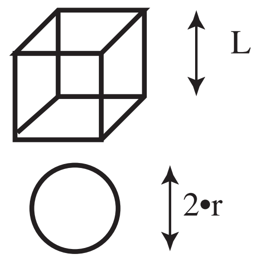
N_T is the number of cells at time T

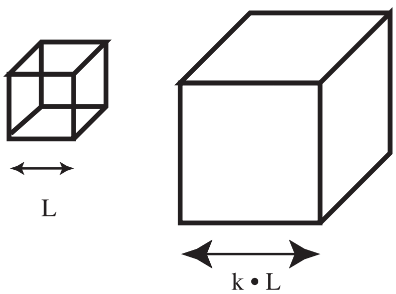
Logistic growth curve:

$$N_T = \frac{K \bullet N_0 \bullet e^{T/g}}{K + N_0(e^{T/g} - 1)}$$

K is the carrying capacity

A cube has a surface area of $6 \bullet L^2$. Its volume is L^3. As long as the shape is constant, the ratio of suraface area to volume will always be $(6 \bullet L^2) / L^3$, or 6/L.
For a sphere, the surface area is $4 \bullet \pi \bullet r^2$, and the volume is $\pi \bullet r^3$; the corresponding ratio of surface area to volume is 4/r.





(area) $A_1 = 6 \bullet L^2$ $A_k = 6 \bullet (k \bullet L)^2$ $A_k = 6 \bullet k^2 \bullet L^2$ $(= k^2 \bullet A_1)$
(volume) $V_1 = L^3$ $V_k = (k \bullet L)^3$ $V_k = k^3 \bullet L^3$ $(= k^3 \bullet V_1)$
The scaling coefficient is different for area (k^2) and for volume (k^3).

Heat conduction rates are defined by the relation: $P_{cond} = Q / t = k \bullet A \bullet [(T_a - T_b) / L]$ where P_{cond} is the rate of conduction (transferred heat, Q, divided by time, t); k is the thermal conductivity; T_a and T_b are the temperatures of the two heat reservoirs a and b; A is the area; and L is the distance. Thermal conductivities of water and air are about 0.6 and 0024 W m^{-1} K^{-1}, respectively.
Thermal radiation is defined by the relation: $P_{rad} = \sigma \bullet \varepsilon \bullet A \bullet T^4$
where P_{rad} is the rate of radiation; σ is the Stefan-Boltzmann constant ($5.6703 \bullet 10^{-8}$ W m^{-2} K^{-4}; ε is the emissivity (varies from 0 to 1, where 1 is for a blackbody radiator); A is the area; and T is the temperature (in Kelvins). The *net* radiative emission or absorption will depend upon the difference in temperature: $P_{net} = \sigma \bullet \varepsilon \bullet A \bullet (T^4_{body} - T^4_{ambient})$

$$\text{compression} = \rho \bullet h \quad F_{cr} = \frac{E \bullet I \bullet \pi^2}{L_{eff}^2} \quad \Psi_{wv} = \frac{RT}{\overline{V}_w} \ln\left(\frac{\%\text{ relative humidity}}{100}\right) + \rho_w g h$$

$$F_{cr} = \frac{E \bullet \dfrac{\pi \bullet r}{4} \bullet \pi^2}{(2 \bullet h)^2}, \text{ and } F_{cr} = \rho \bullet \pi \bullet r^2 \bullet h$$

velocity (meters sec^{-1})
pressure difference (Pascal = 1 kg m^{-1} sec^{-1})
tube radius

$$v = \left(\frac{\Delta p}{l}\right)\left(\frac{1}{4 \bullet \eta}\right)(R^2 - r^2)$$

distance (meters)
viscosity (water = 0.01 gm cm^{-1} sec^{-1}, or Pa sec)
distance from center of tube

density (water = 1 gm cm^{-3})
velocity (cm sec^{-1})
tube diameter (cm)

$$R_e = \frac{\rho \bullet \nu \bullet l}{\eta}$$

viscosity (water = 0.01 gm cm^{-1} sec^{-1})

$$v = \left(\frac{\Delta p}{l}\right)\left(\frac{1}{4 \bullet \eta}\right) R^2$$

$$J_v = \left(\frac{\Delta p}{1}\right)\left(\frac{\pi}{8 \bullet \eta}\right) \bullet R^4$$

$$J_V = -\frac{r^2}{8 \bullet \eta} \bullet \frac{\partial P}{\partial x}$$

$$J = -D\frac{\partial c}{\partial x}$$

Fick's First Law of Diffusion: The flux is proportional to the concentration gradient

$$\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x}$$

Fick's Second Law of Diffusion: Changes in concentration over time depend upon the flux gradient

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

$$J = -\frac{1}{2} \bullet \frac{\Delta^2}{\tau} \bullet \frac{dC}{dx}$$

$$\nabla \nu = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

velocity vector —the notation grad ν is sometimes used

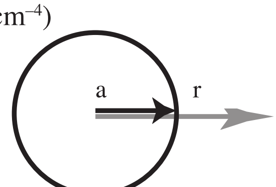
with velocity components, u, v, and w, in the three dimensions, x, y, and z.

$$D = \frac{1}{2} \bullet \frac{\Delta^2}{\tau}$$

units: moles cm^{-2} sec^{-1}

$$J_x = -D\frac{\partial c}{\partial x} + v_x \bullet c$$

(cm^2 sec^{-1})(moles cm^{-4})
(cm sec^{-1})(moles cm^{-3})



Fick's First law : $J_r = -D\frac{\partial C}{\partial r}$

Fick's Second Law : $\frac{\partial C}{\partial t} = D\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C}{\partial r}\right) = 0$

$$C(r) = C_0\left(1 - \frac{a}{r}\right)$$

(steady state)

$$J_r(a) = -D \bullet C_0 \bullet 4 \bullet \pi \bullet a = I_D \text{ (diffusive current)}$$

(mole cm^{-2} sec^{-1})

(units of mole sec^{-1})

$$P_e = \frac{2 \bullet a \bullet u}{D}$$

$$I_m = 4 \bullet \pi \bullet a^2 \bullet \beta \text{ (metabolic current)}$$

(cm^2) (units of mole sec^{-1})

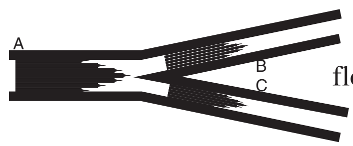
$$\frac{\partial C}{\partial t} = u \bullet \frac{\partial C}{\partial r} \bullet C + D\frac{\partial^2 C}{\partial r^2}$$

flow velocity

concentration gradient

concentration

$$Q = \frac{\Delta p}{l}\frac{\pi a^4}{8\eta}$$



$$\mu_j^{liquid} = \mu_j^* + RT\ln a_j + \overline{V}_j P + z_j FE + m_j gh$$

fluid density

velocity

$$D = \frac{1}{2} \bullet \rho \bullet V^2 \bullet A \bullet C_D$$

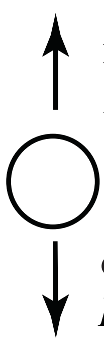
frontal area

drag coefficient
(shape-dependent)

$$m\left(-\frac{d\upsilon}{dt}\right) = 6 \bullet \pi \bullet \eta \bullet r \bullet \upsilon$$

$$\upsilon(t) = \upsilon_0 e^{\left(-\frac{6 \bullet \pi \bullet \eta \bullet r}{m} \bullet t\right)}$$

$$V_{\text{terminal}} = \sqrt{\frac{2mg}{\rho A C_D}}$$



Frictional force

$$F_f = 6\pi\eta a v$$

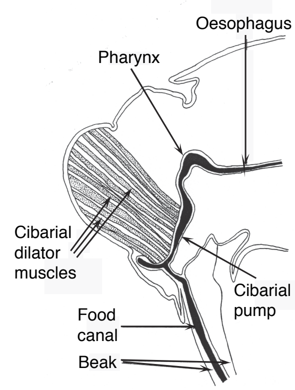
Where the frictional and gravitational forces are balenced, the velocity reaches a steady state.

Gravitational pull

$$F_g = \tfrac{4}{3}\pi a^3 \Delta\rho g$$

The energetic details of the pumping mechanism are shown below for *Rhodnius* (a blood sucking insect) and spittlebugs (*Philaenus*)[1].

	Rhodnius	*Philaenus*
Muscle tension (maximum)	600 kPa	600 kPa
Pump stroke frequency	3 Hz	1.7 Hz
Muscle contraction rate (muscle lengths per second)	1 s^{-1}	0.5 s^{-1}
Ratio of muscle/piston	2.5	10.0
Maximum muscle tension	–300 kPa	–2400 kPa



$$\mathrm{Rotor}_{(n)} + mH^{+}_{\mathrm{outside}} \longleftrightarrow \mathrm{Rotor}_{(n+1)} + mH^{+}_{\mathrm{inside}}$$

$$\mathrm{ADP} + \mathrm{P_i} + \mathrm{mH}^{+}_{\mathrm{outside}} \longleftrightarrow \mathrm{ATP} + \mathrm{mH}^{+}_{\mathrm{inside}}$$

gas constant

Faraday constant

Voltage

$$\mu = \mu^{\circ} + \mathrm{RT}\ln(a_{H^+}) + zF\Psi$$

temperature

activity of protons

$$\Delta \mathrm{G}_{\mathrm{ATP}} = \Delta \mathrm{G}^{\circ}_{\mathrm{ATP}} + \mathrm{RT}\ln\left(\frac{[ATP]}{[ADP][P_i]}\right)$$

$$\Delta \mathrm{G}_{\mathrm{total}} = n \bullet \Delta\mu_{H^+} + \Delta G_{ATP} = 0$$

$$n \bullet \left(RT\ln\left(\frac{a^{inside}_{H^+}}{a^{outside}_{H^+}}\right) + F\Delta\Psi\right) + \Delta G^{o}_{ATP} + RT\ln\left(\frac{[ATP]}{[ADP][P_i]}\right) = 0$$

$$\Delta\mu_{H^+} = \frac{RT}{F}\ln\left(\frac{a^{inside}_{H^+}}{a^{outside}_{H^+}}\right) + \Delta\Psi$$

(units: mV)

RT/F is about 25 mV at 20°C.

The work exerted will depend upon the speed of the contraction, and the cross-sectional area of the muscle times its length. Muscle contraction speeds are normally in the range of 3 milliseconds. The initial velocity will equal the impulse force divided by the mass ($\nu = F_{\mathrm{impulse}}/\mathrm{mass}$).

The work done in the leap is proportional to mass and the height of the leap ($W \propto mH$), while the work of the muscles is proportional to the mass of the muscle (or the whole organism) ($W \propto m$). It follows then, that the total work is related solely to the height, since the organism's mass cancels out. Thus, the height of the leap is not proportional to the organisms's size, but rather is similar for any organism. D'Arcy Thompson describes this as an example of the Principle of Biological Similitude.

$$\mu_j^{liquid} = \mu_j^{*} + RT\ln a_j + \overline{V}_j P + z_j FE + m_j gh$$

$$RT\ln a_j + \overline{V}_j P + m_j gh$$

gravitational potential

$$a_j = \gamma_j c_j$$

The activity of water (a_j) is the product of the activity coefficient and the concentration of water

$$\overline{V}_j = \left(\frac{\partial V}{\partial n_j}\right)_{n_i, T, P, E, h}$$

The partial molal volume of species j is the incremental increase in volume with the addition of species j. For water, it is 18.0 X 10^{-6} $\mathrm{m^3\ mol^{-1}}$.

$$RT\ln a_j = \overline{V}_j \Pi$$

osmotic pressure

$$\Pi_s = RT\sum_j c_j$$

Van't Hoff relation

The terms inter-relate various properties of water: changes in its activity with the addition of solutes, and the relation to pressure.

$$F = A\upsilon + B\omega$$

$$N = C\upsilon + D\omega$$

That is, both velocity and rotation contribute to both the force and torque.