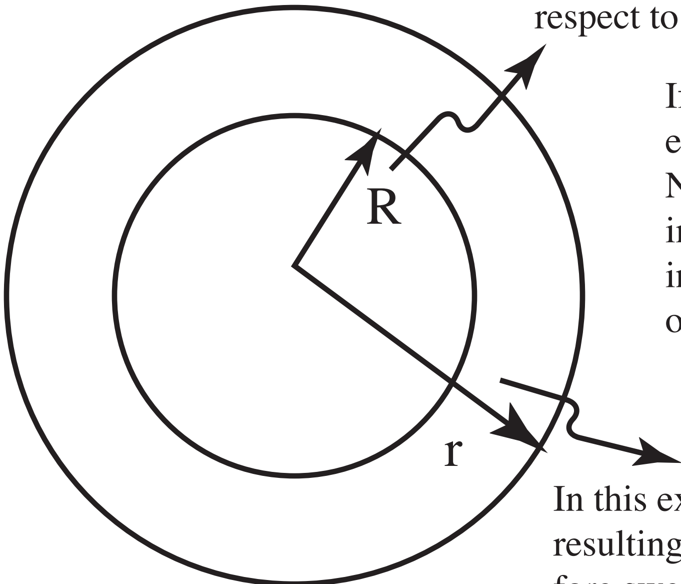
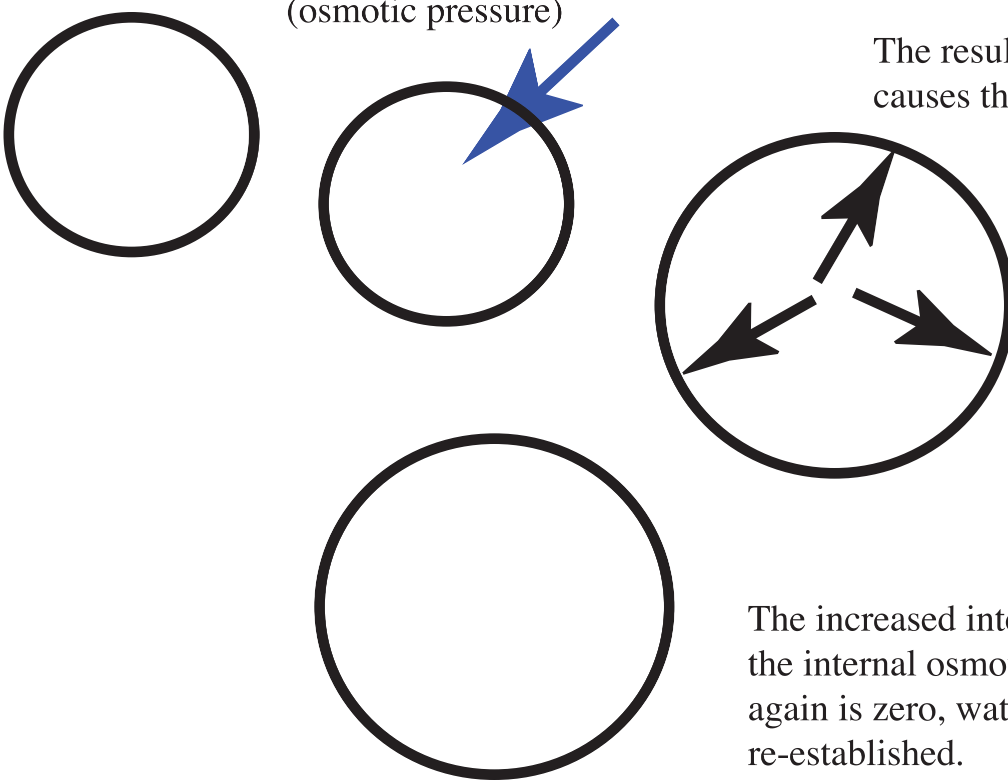
It is easiest to consider an isolated cell to consider how pressure and volume changes affect the well-being of the cell.

In the initial state, the cell is in equilibrium with respect to water flow (Radius is R and $J_V = 0$).



If the external osmolarity is changed (for example, by adding an osmolyte such as NaCl or sucrose), there is an osmotic imbalence which effects water flow, either into or out of the cell, to balence the osmotic difference.

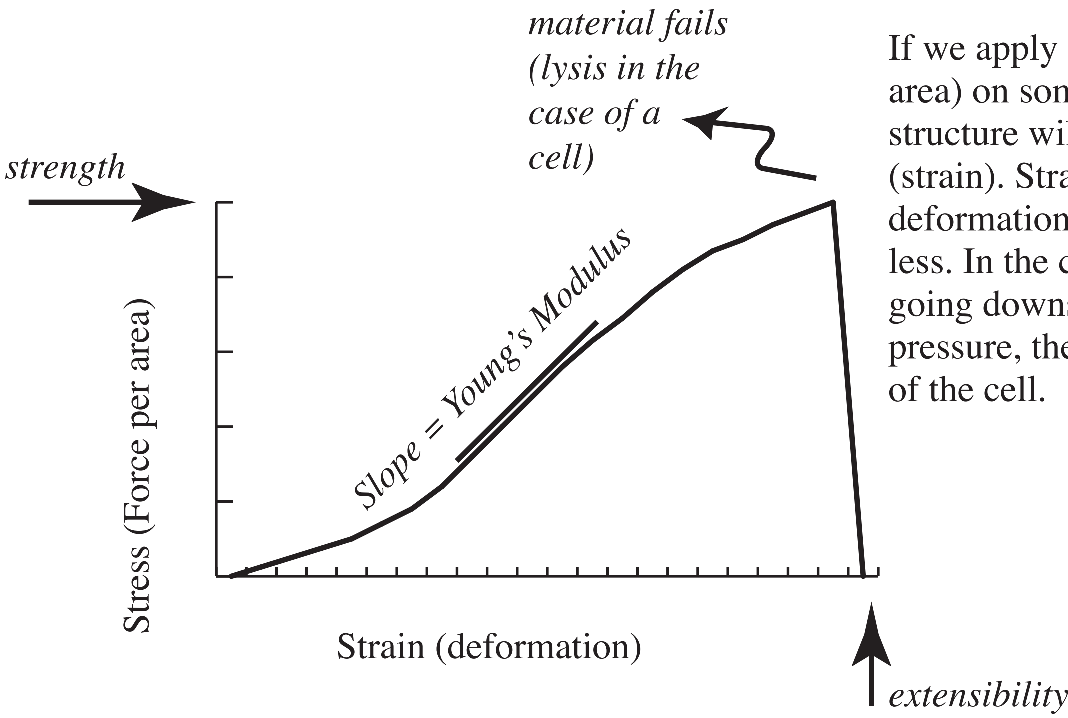
In this example, the external osmoticum is lower, resulting in water flow into the cell, which therefore swells to the radius r.



The resulting internal pressure causes the cell to swell.

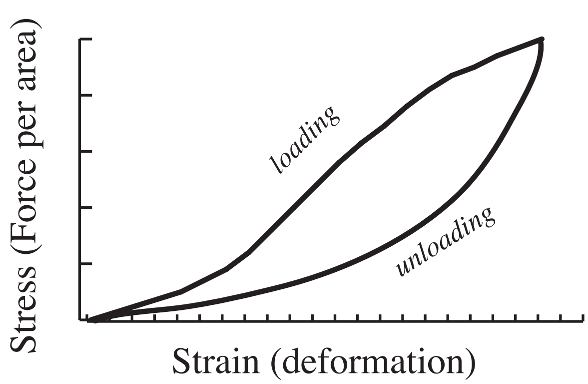
The increased internal volume dilutes the internal osmoticum so that ΔP once again is zero, water flow equilibrium is re-established.

The extent of the cell swelling caused by lower external osmolarity (a *downshock*, in the parlance of volume regulation researchers) depends upon the nature of the outer cell wall of the cell. All cells have a wall, of some sort or another. It may be a glycocalyx, a polysaccharide coat, a chitin fibrillar network, a cellulosic framework or something else. Even silica can be used as an outer wall or coat. No matter the composition, it will resist the swelling caused by the pressure change. This is known as the modulus of elasticity.



If we apply a stress (a force per area) on some material, the structure will be deformed (strain). Strain is a fractional deformation, hence dimensionless. In the case of a cell undergoing downshock, the stress is pressure, the strain is swelling of the cell.

The steeper the slope (the greater the value of Young's modulus, in units of force per area), the stiffer (less deformable) the material.



Experimentally, materials may be 'viscoelastic', leading to a stress-strain relation in which load and unloading of the force follow different paths of deformation.

[1]Source: Vogel, Steven (1988) Life's Devices. The physical world of animals and plants. Princeton University Press. pp. 183–184.

The mathematical envelope underlying the changes in osmolarity, pressure and volume is complex, but exceedingly important for the survival of a cell (or organism for that matter)[1].

Integration is required to determine P(t), V(t) and π(t)

$$J_V = -\frac{1}{A} \bullet \frac{dV}{dt} = L_p \bullet \left[P(t) + \pi^o + \Delta\pi^o - \pi^i(t) \right]$$

- J_V: water flow
- A: area
- $\frac{dV}{dt}$: change in volume over time
- L_p: hydraulic conductivity
- $P(t)$: change in pressure over time
- π^o: the external osmotic pressure
- $\Delta\pi^o$: the change in external osmotic pressure
- $\pi^i(t)$: the change in internal osmotic pressure over time

Note that in the absence of a change in external osmolarity, the equation has the form:

$$J_V = -\frac{1}{A} \bullet \frac{dV}{dt} = L_p \bullet \left[P - RT(c^i - c^o) \right] = L_p \bullet \Delta\Psi$$

where $RT(c^i - c^o) = \pi^i - \pi^o$

When the pressure and difference in osmotic pressure are equal: $P = RT(c^i - c^o)$
Then the water flow is zero.

This is also true if the hydraulic conductivity (L_p) is zero or close enough to allay rapid change in cell volume.

[1]Source: Zimmerman (1991) Pressure Probe Techniques. private publication.

The dependence of pressure on volume, P(V), is described by the modulus of elasticity described earlier[1]:

elastic coefficient of the cell

$$\frac{dP}{dV} = \frac{\varepsilon}{V} \approx \frac{\Delta P}{\Delta V} = \frac{P - P_0}{V - V_0}$$

The dependence of internal osmotic potential ($RT \bullet c^i$) on volume, $\pi(V)$, can be described by:[1]:

osmotic potential and volume at time zero.

$$\frac{d\pi^i}{dV} \approx \frac{\Delta\pi^i}{\Delta V} = \frac{\pi^i - \pi_0^i}{V - V_0}$$

Combining these results and integrating over the boundary conditions of P_0 at time zero and P_e at 'infinite' time:

$$P_0 - P_e = \frac{\varepsilon}{\varepsilon + \pi_0^i} \bullet \Delta\pi^o$$

and P(t):

$$P(t) = (P_{initial} - P_e) \bullet e^{\left(-L_p \bullet A \bullet \frac{\varepsilon + \pi^i}{V} \bullet t\right)}$$

That is, an exponential change in pressure dependent on internal osmolarity, volume, cell area, and hydraulic conductivity.

[1]Source: Zimmerman (1991) Pressure Probe Techniques. private publication.