Equations relevant to Membrane Transport: Geometry, Diffusion and Flux

Sphere Area: $4 \cdot \pi \cdot r^2$ Sphere Volume: $\frac{4}{3} \cdot \pi \cdot r^3$

Cylinder Area: $4 \cdot \pi \cdot r \cdot h$ Cylinder Volume: $\pi \cdot r^2 \cdot h$

Cube Area: $6 \cdot \mathrm{h}^2$ Cube Volume: h^3

Fick – 1[st] Diffusion Law: $J = D \cdot \frac{dc}{dx}$ Fick – 2[d] Diffusion Law: $\frac{dc}{dt} = D \cdot \frac{d^2c}{dx^2}$

Einstein – Random Walks: $D = \frac{1}{2} \cdot \frac{\Delta^2}{\tau}$, $\langle x^2 \rangle = 2 \cdot D \cdot t$, and $\langle r^2 \rangle = 6 \cdot D \cdot t$

Membrane Diffusion: $J = P \cdot \left(c_{outside} - c_{inside}\right)$, $\mathrm{P} = \mathrm{D}\frac{\mathrm{k_p}}{\mathrm{d}}$

Membrane Diffusion: $J = -(uRT) \cdot \frac{dc}{dx} - (zFuc) \cdot \frac{d\Psi}{dx}$

Membrane Diffusion: $J = -P \cdot \left(\frac{zF\Psi}{RT}\right) \cdot \left(\frac{c_o - c_i \cdot e^{zF\Psi/RT}}{1 - e^{zF\Psi/RT}}\right)$

Nernst Equation: $\Psi = \left(\frac{RT}{zF}\right) \cdot \ln\left(\frac{c_o}{c_i}\right)$

Ohms Law: $V = I \cdot R$, $I = g \cdot V$, $R = \rho \cdot \left(l/A\right)$, and $J = I/(zF)$

Radial Diffusion: $C(r) = C_\infty \cdot \left(1 - \frac{a}{r}\right)$, and $J(r) = -D \cdot C_\infty \cdot \left(\frac{a}{r^2}\right)$

Radial Currents: $I_m = 4 \cdot \pi \cdot a^2 \cdot \beta$, and $I_d = 4 \cdot \pi \cdot a \cdot D \cdot C_\infty$

Dimensionless relations $P_e = \frac{2 \bullet a \bullet \nu}{D}$ and $R_e = \frac{\rho \bullet \nu \bullet l}{\eta}$

Goldman - Hodgkin - Katz (GHK) equation

$$\Psi = \frac{RT}{F} \ln\left(\frac{P_H c_H^o + P_{Na} c_{Na}^o + P_K c_K^o + P_{Cl} c_{Cl}^i}{P_H c_H^i + P_{Na} c_{Na}^i + P_K c_K^i + P_{Cl} c_{Cl}^o}\right)$$

Equations relevant to Bioenergetics

For the vectorial chemical reaction:

$$\text{ATP} + \text{n} \bullet \text{ion}^{\text{i}} \leftrightarrow ADP + P_i + \text{n} \bullet \text{ion}^{\text{o}}$$

(n is the stoichiometry)

$$\text{At equilibrium: } \Delta \text{G}_{\text{total}} = n \sum \Delta \mu_{ion} + \Delta G_{ATP}$$

$$\Delta G_{ATP} = \Delta G^o_{ATP} + RT \ln \frac{[ADP][P_i]}{[ATP]}$$

$$\Delta \mu_{ion} = RT \ln \frac{c^o_{ion}}{c^i_{ion}} + zF\Delta\Psi$$

Note that ΔG^o_{ATP} varies with pH and $[Mg^{2+}]$. For our purposes, specifying –10 kcal mole^{-1} is a reasonable estimate.

Equations relevant to membrane capacitance

$$Q = C \cdot \Delta E \ \text{(coulombs)} = \text{(coulombs/volt) (volt)}$$

Charge (Q) for a spherical cell of radius r:

$$Q = \tfrac{4}{3} \cdot \pi \cdot r^3 \cdot c \cdot F$$

c is the concentration of net charge.

Capacitance of a spherical cell of radius r:

$$C = 4 \cdot \pi \cdot r^2 \cdot \text{C}'$$ C' is the capacitance per unit area

(about 1 microFarad per square centimeter for cells).

Symbol	Value	Units	Comments
GAS CONSTANT			
R	8.314	J mol^{-1} K^{-1}	R is the Boltzmann constant times Avogadro's Number ($6.023 \cdot 10^{23}$)
	1.987	cal mol^{-1} K^{-1}	
	8.314	m^{-3} Pa mol^{-1} K^{-1}	
RT	$2.437 \bullet 10^{3}$	J mol^{-1}	At 20 ºC (293 ºK)
	$5.833 \bullet 10^{2}$	cal mol^{-1}	At 20 ºC (293 ºK)
	$2.437 \bullet 10^{-3}$	liter MPa mol^{-1}	At 20 ºC (293 ºK)
RT/F	25.3	mV	At 20 ºC (293 ºK)
2.303 • RT	5.612	kJ mol^{-1}	At 20 ºC (293 ºK)
	1.342	kcal mol^{-1}	At 20 ºC (293 ºK)
FARADAY CONSTANT			
F	$9.649 \bullet 10^{4}$	coulombs mol^{-1}	F is the electric charge times Avogadro's Number
	$9.649 \bullet 10^{4}$	J mol^{-1} V^{-1}	
	23.06	kcal mol^{-1} V^{-1}	
CONVERSIONS			
kcal	4.187	kJ (kiloJoules)	Joules is an energy unit (equal to 1 Newton•meter)
Watt	1	J sec^{-1}	
Volt	1	J $coulomb^{-1}$	
Amperes	1	coulomb sec^{-1}	
Pascal (Pa)	1	Newton $meter^{-2}$	Pascal is a pressure unit (equal to 10^{-5} bars)
Siemens	1	Ohm^{-1}	Siemens (S) is conductance, the inverse of resistance (Ohm)
PHYSICAL PROPERTIES			
η_w	$1.004 \bullet 10^{-3}$	Pa sec	viscosity of water at 20 ºC
ν_w	$1.004 \bullet 10^{-6}$	m^2 sec^{-1}	kinematic viscosity of water at 20 ºC (viscosity/density)

Source: Nobel, Park S (1991) Physicochemical and Environmental Physiology

Table 9. System of the van der Waals radii (Å) of elements

Li 2.2 2.63	Be 1.9 2.23											B 1.8 2.05	C 1.7 1.96	N 1.6 1.79	O 1.55 1.71	F 1.5 1.65
Na 2.4 2.77	Mg 2.2 2.42											Al 2.1 2.40	Si 2.1 2.26	P 1.95 2.14	S 1.8 2.06	Cl 1.8 2.05
K 2.8 3.02	Ca 2.4 2.78	Sc 2.3 2.62	Ti 2.15 2.44	V 2.05 2.27	Cr 2.05 2.23	Mn 2.05 2.25	Fe 2.05 2.27	Co 2.0 2.25	Ni 2.0 2.23	Cu 2.0 2.27	Zn 2.1 2.24	Ga 2.1 2.41	Ge 2.1 2.32	As 2.05 2.25	Se 1.9 2.18	Br 1.9 2.10
Rb 2.9 3.15	Sr 2.55 2.94	Y 2.4 2.71	Zr 2.3 2.57	Nb 2.15 2.46	Mo 2.1 2.39	Tc 2.05 2.37	Ru 2.05 2.37	Rh 2.0 2.32	Pd 2.05 2.35	Ag 2.1 2.37	Cd 2.2 2.37	In 2.2 2.53	Sn 2.25 2.46	Sb 2.2 2.41	Te 2.1 2.36	I 2.1 2.22
Cs 3.0 3.30	Ba 2.7 3.05	La 2.5 2.81	Hf 2.25 2.52	Ta 2.2 2.42	W 2.1 2.36	Re 2.05 2.35	Os 2.0 2.33	Ir 2.0 2.34	Pt 2.05 2.37	Au 2.1 2.41	Hg 2.05 2.25	Tl 2.2 2.53	Pb 2.3 2.53	Bi 2.3 3.52	Po	At
		Th 2.4 2.75	U 2.3 2.65													

Table 9 presents the set of the recommended crystallographic (upper numbers) and equilibrium (lower numbers) van der Waals radii. The equilibrium radii are where there is a minimum in the potential of van der Waals interaction between two isolated atoms.
Source: Batsanov SS (2001) Van der Waals radii of elements. *Inorganic Materials*. 37(9):871–885.

Tabulated data for atomic radii, enthalpies of hydration and mobilities:

ATOMIC RADII, HYDRATION ENTHALPIES AND MOBILITIES

Ion	Atomic Radii (Å)	Enthalpies of Hydration (kcal/mole)	Mobility (10^{-4}) (cm/sec)/(V/cm)
Tl^+	1.44	71	7.74
H^+	.	-269	36.3
NH_4^+	1.48	.	7.52
Cs^+	1.69	-72	8.01
Rb^+	1.48	-79.2	8.06
K^+	1.33	-85.8	7.62
Na^+	0.95	-104.6	5.19
Li^+	0.60	-131.2	4.01
Cl^-	1.81	-82	7.92
F^-	1.36	-114	5.74
Br^-	1.95	-79	8.09
I^-	2.16	-65	7.96
NO_3^-	2.90	.	7.41
Mg^{2+}	0.65	-476	2.75
Ca^{2+}	0.99	-397	3.08
Sr^{2+}	1.13	-362	3.08
Mn^{2+}	0.80	-458	.
Ba^{2+}	1.35	-328	3.30
Co^{2+}	0.74	-502	.
Ni^{2+}	0.72	-517	.
Zn^{2+}	0.74	-505	.

Source: Hille, B (1991) Ionic Channels of Excitable Membranes. Sinauer Associates. pp. 157 & 166.