Equations relevant to Membrane Transport: Geometry, Diffusion and Flux

$$\text{Sphere Area: } 4 \bullet \pi \bullet r^2 \qquad \text{Sphere Volume: } \frac{4}{3} \bullet \pi \bullet r^3$$

$$\text{Cylinder Area: } 4 \bullet \pi \bullet r \bullet h \qquad \text{Cylinder Volume: } \pi \bullet r^2 \bullet h$$

$$\text{Cube Area: } 6 \bullet \mathrm{h}^2 \qquad \text{Cube Volume: } h^3$$

$$\text{Fick's Diffusion: } J = D \bullet \frac{dc}{dx} \quad \text{Fick's Diffusion: } \frac{dc}{dt} = D \bullet \frac{d^2c}{dx^2}$$

$$\text{Einstein's Random Walks: } D = \frac{1}{2} \bullet \frac{\Delta^2}{\tau},\ \langle x^2 \rangle = 2 \bullet D \bullet t \text{ , and } \langle r^2 \rangle = 6 \bullet D \bullet t$$

$$\text{Membrane Diffusion: } J = P \bullet \left(c_{outside} - c_{inside}\right)$$

$$\text{Membrane Diffusion: } J = -(uRT) \bullet \frac{dc}{dx} - (zFuc) \bullet \frac{d\Psi}{dx}$$

$$\text{Membrane Diffusion: } J = -P \bullet \left(\frac{zF\Psi}{RT}\right) \bullet \left(\frac{c_o - c_i \bullet e^{zF\Psi/RT}}{1 - e^{zF\Psi/RT}}\right)$$

$$\text{Nernst Equation: } \Psi = \left(\frac{RT}{zF}\right) \bullet \ln\left(\frac{c_o}{c_i}\right)$$

$$\text{Ohm's Law : } V = I \bullet R,\ I = g \bullet V \text{ , } R = \rho \bullet \left({l}/{A}\right)\text{, and } J = I/(zF)$$

$$\text{Radial Diffusion: } C(r) = C_\infty \bullet \left(1 - \frac{a}{r}\right)\text{, and } J(r) = -D \bullet C_\infty \bullet \left(\frac{a}{r^2}\right)$$

$$\text{Radial Currents: } I_m = 4 \bullet \pi \bullet a^2 \bullet \beta\text{, and } I_d = 4 \bullet \pi \bullet a \bullet D \bullet C_\infty$$

$$\text{Dimensionless relations } P_e = \frac{2 \bullet a \bullet v}{D} \text{ and } R_e = \frac{\rho \bullet v \bullet l}{\eta}$$

Goldman - Hodgkin - Katz (GHK) equation

$$\Psi = \frac{RT}{F} \ln\left(\frac{P_H c_H^o + P_{Na} c_{Na}^o + P_K c_K^o + P_{Cl} c_{Cl}^i}{P_H c_H^i + P_{Na} c_{Na}^i + P_K c_K^i + P_{Cl} c_{Cl}^o}\right)$$

Equations relevant to Membrane Transport: Water Fluxes

Volume Flow: $J_V \propto \frac{\partial P}{\partial x}$

Flow through a Pipe: $J_V = -\frac{r^2}{8 \bullet \eta} \bullet \frac{\partial P}{\partial x}$

Flow into / out of a cell:

$$J_V = -\frac{1}{A} \bullet \frac{\partial V}{\partial t}$$

$$J_V = L_p \bullet [P - RT(c_i - c_o)]$$

$$J_V = L_p \bullet \Delta\Psi$$

where $RT(c_i - c_o) = \pi_i - \pi_o$

when $\mathrm{J_V} = 0$: $P = RT(c_i - c_o)$

Cell volume, pressure and osmotic relations

$$\frac{\partial P}{\partial V} = \frac{\varepsilon}{V} \approx \frac{\Delta P}{\Delta V} = \frac{P - P_0}{V - V_0}$$

$$\frac{\partial \pi_i}{\partial V} \approx \frac{\Delta \pi_i}{\Delta V} = \frac{\pi_i - \pi_{i,0}}{V - V_0}$$

$$P(t) = \left(P - P_e\right) \bullet e^{\left(-L_p \bullet A \bullet \frac{\varepsilon + \pi_i}{V} \bullet t\right)}$$

Equations relevant to Bioenergetics

For the vectorial chemical reaction:

$$\text{ATP} + \text{n} \bullet \text{ion}^{\text{i}} \leftrightarrow ADP + P_i + \text{n} \bullet \text{ion}^{\text{o}}$$

(n is the stoichiometry)

$$\text{At equilibrium: } \Delta \text{G}_{\text{total}} = n \sum \Delta \mu_{ion} + \Delta G_{ATP}$$

$$\Delta G_{ATP} = \Delta G^{o}_{ATP} + RT \ln \frac{[ADP][P_i]}{[ATP]}$$

$$\Delta \mu_{ion} = RT \ln \frac{c^{o}_{ion}}{c^{i}_{ion}} + zF\Delta\Psi$$

Note that ΔG^{o}_{ATP} varies with pH and [Mg^{2+}]. For our purposes, specifying 10 kcal $mole^{-1}$ is a reasonable estimate.

Equations relevant to membrane capacitance

$$Q = C \cdot \Delta E \ \text{(coulombs)} = \text{(coulombs/volt) (volt)}$$

Charge (Q) for a spherical cell of radius r:

$$Q = \tfrac{4}{3} \cdot \pi \cdot r^3 \cdot c \cdot F$$

c is the concentration of net charge.

Capacitance of a spherical cell of radius r:

$$C = 4 \cdot \pi \cdot r^2 \cdot \text{C}' \quad C' \text{ is the capacitance per unit area}$$

(about 1 microFarad per square centimeter for cells).

Symbol	Value	Units	Comments
GAS CONSTANT			
R	8.314	J mol^{-1} K^{-1}	R is the Boltzmann constant times Avogadro's Number ($6.023 \bullet 10^{23}$)
	1.987	cal mol^{-1} K^{-1}	
	8.314	m^{-3} Pa mol^{-1} K^{-1}	
RT	$2.437 \bullet 10^{3}$	J mol^{-1}	At 20 ºC (293 ºK)
	$5.833 \bullet 10^{2}$	cal mol^{-1}	At 20 ºC (293 ºK)
	$2.437 \bullet 10^{-3}$	liter MPa mol^{-1}	At 20 ºC (293 ºK)
RT/F	25.3	mV	At 20 ºC (293 ºK)
2.303 • RT	5.612	kJ mol^{-1}	At 20 ºC (293 ºK)
	1.342	kcal mol^{-1}	At 20 ºC (293 ºK)
FARADAY CONSTANT			
F	$9.649 \bullet 10^{4}$	coulombs mol^{-1}	F is the electric charge times Avogadro's Number
	$9.649 \bullet 10^{4}$	J mol^{-1} V^{-1}	
	23.06	kcal mol^{-1} V^{-1}	
CONVERSIONS			
kcal	4.187	kJ (kiloJoules)	Joules is an energy unit (equal to 1 Newton•meter)
Watt	1	J sec^{-1}	
Volt	1	J $coulomb^{-1}$	
Amperes	1	coulomb sec^{-1}	
Pascal (Pa)	1	Newton $meter^{-2}$	Pascal is a pressure unit (equal to 10^{-5} bars)
Siemens	1	Ohm^{-1}	Siemens (S) is conductance, the inverse of resistance (Ohm)
PHYSICAL PROPERTIES			
η_w	$1.004 \bullet 10^{-3}$	Pa sec	viscosity of water at 20 ºC
ν_w	$1.004 \bullet 10^{-6}$	m^2 sec^{-1}	kinematic viscosity of water at 20 ºC (viscosity/density)

Source: Nobel, Park S (1991) Physicochemical and Environmental Physiology