Financial Calculations on the Texas Instruments BAII Plus

This is a first draft, and may contain errors. Feedback is appreciated

Compounding Assumptions

- The TI BAII Plus has built-in preset assumptions about compounding and payment frequencies.
- Compounding and Payment frequencies are controlled with the [P/Y] key

Compounding Assumptions

- Press the [P/Y] key ([2nd][I/Y])
- Unless the settings have been changed, you will see the default, preset payment frequency: P/Y = 12.00 - 12 payments/year
- Using the down arrow [^] or up arrow [v] will scroll you to the next window, the number of times per year the interest is compounded: C/Y = 12.00 - 12 times/year

Compounding Assumptions

- For the first part of the Time Value of Money slides, we are dealing with annual compounding and annual payments, so these values need to be changed:
- [P/Y] = 1 [ENTER]
  - [C/Y] will automatically be changed to 1
- [^] [C/Y] Display = 1
- To return to the calculator mode press [QUIT] or [2nd][CPT]

An Alternative

- One way to make the BAII Plus work very much like the Sharp EL-733A is to set the [P/Y] and [C/Y] to 1 and leave it there all the time.
- If you do this, some of the directions that follow will not work if the values of [P/Y] and [C/Y] are changed

Clearing

- It is also very important to clear the Time Value worksheet before doing a new set of calculations
- [CLR][TVM]

A Word on Rounding

- I set my BA II Plus to an artificially large number of decimals - usually 7 - which will rarely all be displayed.
- The BA II Plus will display the answer rounded correctly to the number of decimals available or as set by you, whichever is less.
- In these notes, 1/7 = 0.142857... may be written as 0.1428…, where the "…" simply means that I have stopped writing down the decimals, but I have not rounded.

Future Values

$$\begin{aligned} FV_5 &= PV_0(1+k)^n \\ &= \$44{,}651.06(1.06)^5 \\ &= \$44{,}651.06(1.33822...) \\ &= \$59{,}753.19 \end{aligned}$$

On the TI BAII Plus

- 44651.06 [PV]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -59,753.19

- To get the $FV_{k,n}$, simply use PV = 1
- 1 [PV]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -1.338225...

Present Values

- A contract that promised to pay you v59,753.19 in 5 years would be worth today, at 6% interest:

$$\begin{aligned} PV_0 &= (FV_5)(PVIF_{6\%,5}) \\ &= \$59{,}753.19(1.06)^{-5} \\ &= \$59{,}753.19(0.74725...) \\ &= \$44{,}651.06 \end{aligned}$$

On the TI BAII Plus

- 59753.19 [FV]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -44,651.06

- To get the $PV_{k,n}$, simply use FV = 1
- 1 [FV]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -0.747258...

Perpetuities

- Perpetuities, growing perpetuities and growing finite annuities must be done using the formulae as financial calculators do not have special functions for these cash flows

PV of Annuity Example

$$PV_0 = \$10{,}600\left[\frac{1-(1+k)^{-n}}{k}\right]$$

$$= \$10{,}600\left[\frac{1-(1.06)^{-5}}{.06}\right]$$

$$= \$10{,}600(4.212363...)$$

$$= \$44{,}651.06$$

On the TI BAII Plus

- 10,600 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -44,651.06

- To get the $PVA_{k,n}$, simply use PMT = 1
- 1 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -4.21236...

FV of Annuity Example

$$FV_5 = \$10{,}600\left[\frac{(1+k)^{n}-1}{k}\right]$$

$$= \$10{,}600\left[\frac{(1.06)^{5}-1}{.06}\right]$$

$$= \$10{,}600(5.637092...)$$

$$= \$59{,}753.19$$

On the TI BAII Plus

- 10,600 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -59,753.19

- To get the $FVA_{k,n}$, simply use PMT = 1
- 1 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -5.63709...

Annuities Due

- To access the toggle that switches the annuity payments between regular (END) and due (BGN) you use the [BGN] key ([2nd][PMT])
- To toggle between the BGN and END setting, use [SET] ([2nd][ENTER]) and [QUIT] to return to the calculator mode
- If set for annuities due, you will see BGN in the display

PV of an Annuity Due

$$PV_0 = \$10{,}000\left(\frac{1-(1+k)^{-n}}{k}\right)(1+k)$$

$$= \$10{,}000\left(\frac{1-(1.06)^{-5}}{.06}\right)(1.06)$$

$$= \$10{,}000(4.4651056...)$$

$$= \$44{,}651.06$$

On the TI BAII Plus

- [BGN][SET] to set to BGN
- 10,000 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -44,651.06

- To get the $PVA_{k,n}$, simply use PMT = 1
- 1 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -4.4651056...

FV of an Annuity Due

$$FVA_{n,k}(\text{Due}) = PMT\left(\frac{(1+k)^n - 1}{k}\right)(1+k) = PMT(FV_{k,n})(1+k)$$

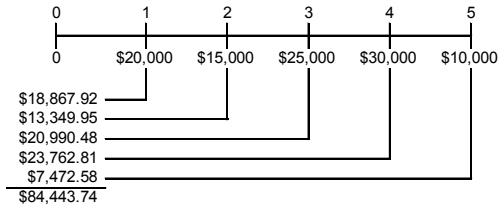
On the TI BAII Plus

- [BGN][SET] to set to BGN
- 10,000 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -59,753.19

- To get the $FVA_{k,n}$, simply use PMT = 1
- 1 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -5.9753185...

Example, Uneven Cash Flows

- Valued at 6%



On the TI BAII Plus

- We use the [CF] key,
- Initially, we see the Display: Cf0 = 0.00
- The down arrow [v] and up arrow [^] allow us to scroll through the displays
- Each Cnn is followed by Fnn to allow the user to enter multiple occurrences of a value

On the TI BAII Plus

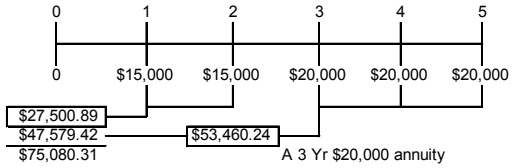
- After the cash flows are entered, we use the [NPV] key
- The first display is I = and is asking us to enter the interest or discount rate.
- After entering the rate the [v] gives us the NPV = display. [CPT] will give us the net present value of the cash flows.

Example on the TI BAII Plus

- CF0 = 0.00 [v]
- C01 = 20000 [ENTER] [v] F1 = 1 [v]
- C02 = 20000 [ENTER] [v] F2 = 1 [v]
- C03 = 20000 [ENTER] [v] F3 = 1 [v]
- C04 = 20000 [ENTER] [v] F4 = 1 [v]
- C05 = 20000 [ENTER] [v] F5 = 1 [v]
- [NPV] Display: I = 6 [ENTER] [v]
- Display: NPV = [CPT]
- Display: NPV = 84,443.74

Look for hidden annuities

- Sometimes there will be annuities to simplify your calculations that are not so obvious



Example on the TI BAII Plus

- CF0 = 0.00 [v]
- C01 = 15000 [ENTER] [v] F1 = 2 [ENTER] [v]
- C02 = 20000 [ENTER] [v] F2 = 3 [ENTER] [v]
- [NPV] Display: I = 6 [ENTER] [v]
- Display: NPV = [CPT]
- Display: NPV = 75,080.31

Example

- Suppose that Consolidated Moose Pasture (CMP) borrowed $466,500 and promised to repay $1,000,000 eight years from now. There will be no intermediate interest payments. What is the implied rate of interest?

On the TI BAII Plus

- 466500 [PV]; 1000000 [+/-] [FV]; 8 [N]
- [CPT][I/Y] Display = 10.00

Example - Annuities

- Suppose you have the choice to receive $100,000 now or $15,000 per year at the start of each of the next 10 years.
- [BGN][SET] (to toggle to BGN)
- 15000 [PMT]; 100000 [+/-][PV], 10 [N]
- [CPT][I/Y] Display: 10.409

Converting from APR to EAR

- Consider $1 for 1 year 6% compounded
  - quarterly: 1.5% every quarter for 4 quarters
  - monthly: 0.5% every month for 12 months
  - daily: (6/365)% every day for 365 days

Effective Annual Rate

Quarterly $FV = \$1 * (1.015)^4 = \$1.06136$

$EAR = 6.136\%$

Monthly $FV = \$1 * (1.005)^{12} = \$1.061678$

$EAR = 6.1678\%$

Daily $FV = \$1 * (1 + (6/365))^{365} = \$1.061831...$

$EAR = 6.1831\%$

On the TI BAII Plus

To convert from a nominal (APR) to EAR

- You can do it by using the formulaic approach from the previous slide, or
- You can use the [ICONV] worksheet (above the numeral [2])
- The first screen is NOM =
- The second screen is EFF =
- The third screen is C/Y =

On the TI BAII Plus

- Using the [ICONV] worksheet
- NOM = 6 [ENTER], [^] C/Y = 4 [ENTER], [^] EFF = [CPT] Display = 6.136355…
- NOM = 6 [ENTER], [^] C/Y = 12 [ENTER], [^] EFF = [CPT] Display = 6.167781…
- NOM = 6 [ENTER], [^] C/Y = 365 [ENTER], [^] EFF = [CPT] Display = 6.183131...

Converting from EAR to APR

The account earns an EAR of 6%

- If the account compounds interest quarterly, what is the APR?
- If the account compounds interest monthly, what is the APR?
- If the account compounds interest daily, what is the APR?

Example

$$q = \left((1 + EAR)^{(1/m)} - 1\right)m$$

Quarterly $q = \left((1.06)^{(1/4)} - 1\right)4$

$= 5.8695\%$

Monthly $q = \left((1.06)^{(1/12)} - 1\right)12$

$= 5.841...\%$

Daily $q = \left((1.06)^{(1/365)} - 1\right)365$

$= 5.8273...\%$

On the TI BAII Plus

To convert from EAR to APR

- You can do it by using the formulaic approach from the previous slide, or
- You can use the same [ICONV] worksheet with the nominal being the value to be computed

On the TI BAII Plus

- Using the [ICONV] worksheet
- EFF = 6 [ENTER], [^] C/Y = 4 [ENTER], [^] NOM = [CPT] Display = 5.86953…
- EFF = 6 [ENTER], [^] C/Y = 12 [ENTER], [^] NOM = [CPT] Display = 5.84106…
- EFF = 6 [ENTER], [^] C/Y = 365 [ENTER], [^] NOM = [CPT] Display = 5.8273559...

Example

- Your older sister just had a baby. If she opens an RESP and puts $125/month into it for 18 years, how much will be available for the child if the rate of return is 8% per annum, 2/3% per month?
- [P/Y] = 12 [ENTER] [QUIT]
- 8 [I/Y]; 125 [PMT]; 216 [N] (or 18[2nd][N])
- [CPT][FV] Display: -60,010.77

Continuous Compounding

- With continuous compounding, you must solve using the formula and the [e^x] key (or [2nd][ln])
- Suppose you want to have $1,000,000 in your retirement account when you reach 65, 44 years from now. If a financial institution is offering you 7% compounded continuously, how much would you have to deposit now, while you're 21?
- 0.07 [x] 44[+/-][=] Display: -3.08[e^x] Display: 0.045959… [x] 1000000 [=] Display: 45,959.26

Mortgage Example

- $120,000 principal (=PV)
- 25 year amortization (n=300 months)
- 8% five year term
  - EAR=8.16%
  - APR=7.87%
  - monthly=0.655819…%

Solution

$$PV = C(PVA_{k_{mon},n})$$

$$120{,}000 = C(PVA_{0.6558119\%,300})$$

$$C = \frac{120{,}000}{PVA_{0.6558119\%,300}}$$

$$= \frac{120{,}000}{131.024343...} = \$915.86$$

On the TI BAII Plus

1. Enter the the payment frequency, [P/Y] 12 [ENTER] and compounding frequency, [^] [C/Y] 2 [ENTER]
2. Enter the mortgage parameters: The principal: 120000 [PV], nominal rate 8 [I/Y] and amortization term 300 [N]
3. Compute the payment [CPT][PMT]
   Display: PMT = -915.86

MORE TO COME, DO NOT CLEAR

Renewal Balance

- The principal of a mortgage is always the PV of the payments that remain on the amortization
- After 5 years:

$$\begin{aligned} BAL_{60} &= \$915.86(PVA_{0.6558119\%,240}) \\ &= \$915.86(120.720826...) \\ &= \$110{,}563.38 \end{aligned}$$

Other Questions

Principal	$120,000.00
At Renewal	110,563.38
Principal Paid	9,436.62
Interest Paid	45,514.98
Total Paid	54,951.60

On the TI BAII Plus

- The "AMORT" key gives us access to the amortization worksheet
- Once you have accessed the AMORT worksheet, the display should say P1 = 1
  - This is the first payment in the range
- Pressing the down arrow will give you P2 = something and you can specify the last payment in the range
- If you want to see each payment sequentially, use P1 = 1, P2 = 1; then P1 = 2, P2 = 2; and so on.

On the TI BAII Plus

Using the [AMORT] worksheet:

- First Payment
  P1 = 1 [ENTER], [v] P2 = 1 [ENTER],
  [v] BAL = 119,987.13, [v] PRN = -128.87,[v] INT = -786.98
- Second Payment
  P1 = 2 [ENTER], [v] P2 = 2 [ENTER],
  [v] BAL = 119,741.41, [v] PRN = -129.72,[v] INT = -786.14

On the TI BAII Plus

- We can jump to any payment
- This is one of the situations where the calculator takes its time - and appears to die - to do the calculation

On the TI BAII Plus

- Sixtieth Payment (just before the five-year renewal)
  P1 = 60 [ENTER], [v]
  P2 = 60 [ENTER], [v]
  BAL = 110,562.91, [v]
  PRN = -189.52,[v]
  INT = -726.34

Accumulated Values

- The [AMORT] worksheet will also allow us to determine how much principal and interest has been paid over a range of periods by specifying ranges for P1 = and P2 =

On the TI BAII Plus

- Over the first five years:
  P1 = 1 [ENTER], [v]
  P2 = 60 [ENTER], [v]
  BAL = 110,562.91, [v]
  PRN = -9,437.09,[v]
  INT = -45.514.28

Car Buying or Leasing

- Suppose you have decided on a new Bolero from National Motors. Its total cost before sales taxes (15%) is $23,500. You plan to put $3,500 down regardless whether you lease or buy. The buyback at the end of the 48 month lease is $9,000. The dealer is offering 4.8% APR financing and lease rates, both compounded monthly.

Buying - Loan

- $23,500 + 15%(v23,500) - 3,500 = $23,525
- 4.8% APR, r_{MON} = 0.4%
- Assume a 48 month loan
- $PVIFA_{0.4,48}$ = 43,9542...

$$PMT = \frac{\text{Principal}}{PVIFA_{r_{MON},N}} = \frac{23{,}525}{43.5942...} = \$539.64$$

On the TI BAII Plus

- The [P/Y] and [C/Y] should both be set to 12 - this is what you get when you [CLR_WORK] the [P/Y] key
- 48 [N], 23525 [PV], 4.8 [I/Y]
  [CPT] [PMT] Display: -539.635...

Lease

- On the lease, the sales tax does not get financed, but the payments are subject to sales taxes
- The present value of the lease payments, plus the present value of the buyback on the car must equal the cash price of the car
- Lease payments are made in advance, or at the beginning of each month

Lease

$$\$20{,}000 = \text{PMT}\left(\text{PVIFAD}_{0.4\%,48}\right) + \frac{9{,}000}{(1.004)^{48}}$$

$$= \text{PMT}(43.7686...) + \frac{9{,}000}{1.2112...}$$

$$= \text{PMT}(43.7686...) + 7{,}430.61$$

$$12{,}569.39 = \text{PMT}(43.7686...)$$

$$\text{PMT} = \frac{12{,}569.39}{43.7686...} = 287.18$$

On the TI BAII Plus

- The [P/Y] and [C/Y] should both be set to 12 - this is what you get when you [CLR WORK] the [P/Y] key
- You also need to set the calculator to BGN: [BGN][SET]; BGN will display
- 48 [N], 20000 [PV], 4.8 [I/Y], -9000 [FV] [CPT] [PMT] Display: -287.178...

Bonds

- With Bonds, you can approach them two ways:
- Using the Time Value functions, OR
- Using the [BOND] worksheet

Regular Fixed Coupon Bond

$$\text{PV} \equiv B_0 = I\left(\text{PVA}_{k_b,n}\right) + \frac{M}{(1+k_b)^n}$$

Consider a 9%, 12 yr bond @7%

$$B_0 = 45(16.058...) + \frac{1000}{(1.035)^{24}}$$

$$B_0 = 722.627 + 437.957 = \$1{,}160.58$$

Using the Time Value Functions

- [P/Y] = 2 [ENTER]
- [N] = 24; [FV] = 1000; [PMT] = 45; [I/Y] = 7 [CPT][PV] = -1,160.58

Using the [BOND] Worksheet

- The [BOND] worksheet requires starting and maturity dates
  - If none are given, make them up
  - dates are entered in the form mm.ddyy
  - Jan 1, 2001 would be 1.0101
- The maturity value [RV] defaults to 100 and is fine if you keep the coupon as a percent
- You will also get a price that is a percent of par

Using the [BOND] Worksheet

Using the Default Value of RV = 100

- SDT = 1.0101 [ENTER] [v]
- CPN = 9 [ENTER] [v]
- RTD = 1.0113 [ENTER] [v] (Maturity Date)
- RV = 100 [ENTER] [v] (Maturity Value)
- ACT [v] (Actual day count)
- 2/Y [v] (2 coupons/yr)
- YLD = 7 [ENTER] [v]
- PRI = [CPT] Display: 116.058...

Using the [BOND] Worksheet

Using the Value of RV = 1000

- SDT = 1.0101 [ENTER] [v]
- CPN = 90 [ENTER] [v]
- RTD = 1.0113 [ENTER] [v] (Maturity Date)
- RV = 1000 [ENTER] [v] (Maturity Value)
- ACT [v] (Actual day count)
- 2/Y [v] (2 coupons/yr)
- YLD = 7 [ENTER] [v]
- PRI = [CPT] Display: 1,160.58...