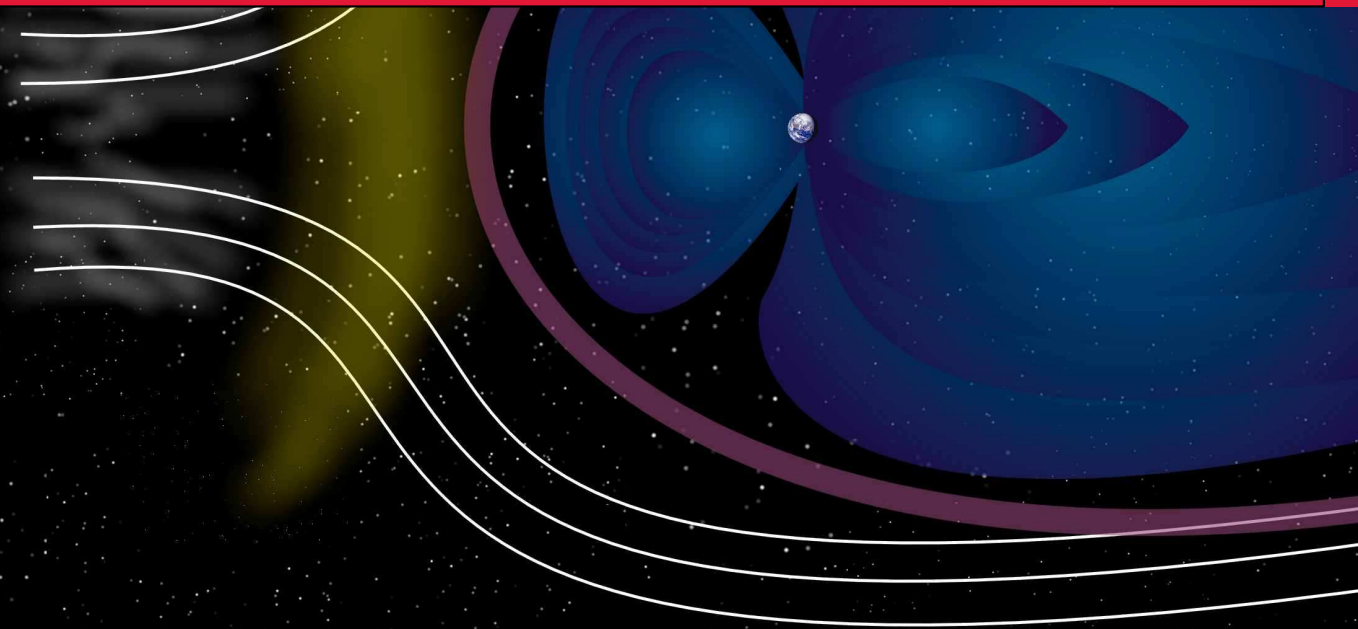
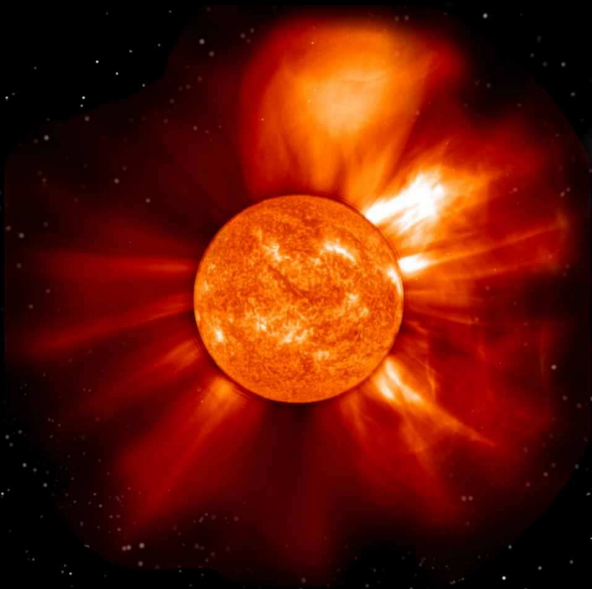
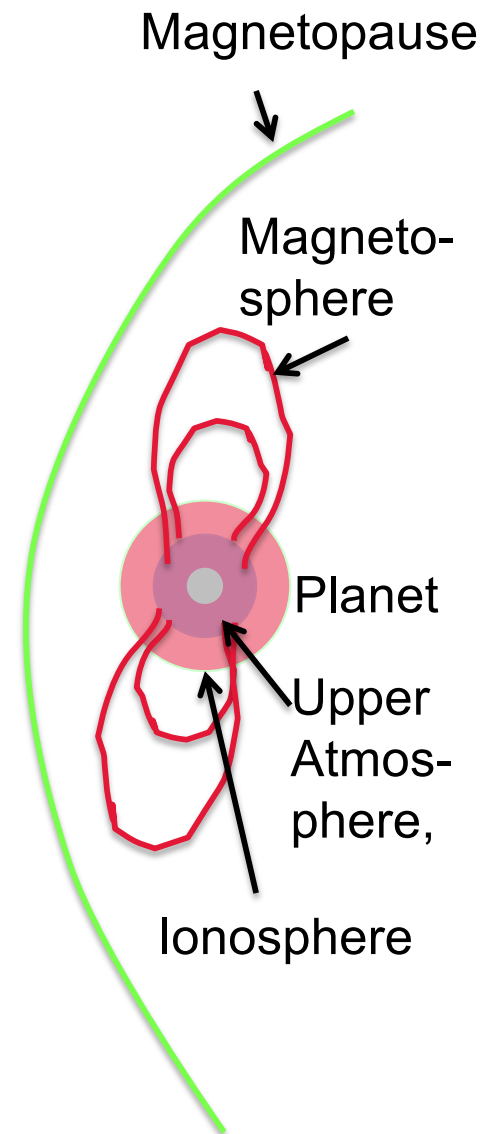
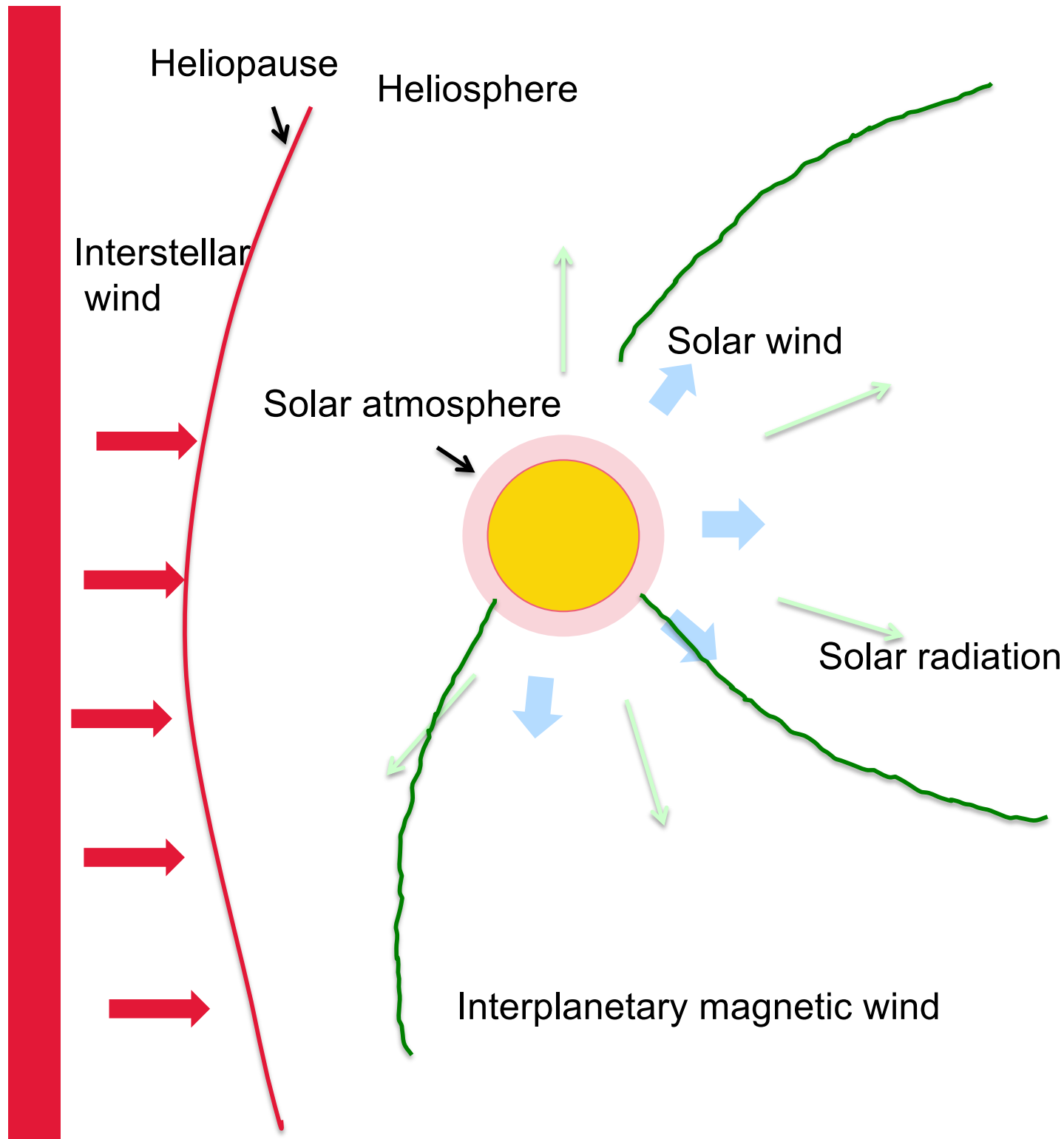


PHYS 3280

Physics of the space environment

2. Neutral upper atmosphere





Scope

- The physics of gases
 - Gas “macroscopic” physical properties
 - Pressure
 - Temperature
 - Mass density, etc.
 - How these relate to “microscopic” properties
 - Particle radius
 - Particle mass etc.
- How the atmosphere can be divided up
 - Based on temperature
 - Based on pressure
- Textbook sections 2.1 – 2.3

Why?

- Determine temperature, density changes as a function of altitude
- Determine mean free path for objects in upper atmosphere (affects dynamics, thermal environment etc.)
- Predict atmospheric environment on other planetary bodies using point measurements and knowledge of molecular interactions

Gas macroscopic state parameters

- Chemical composition
 - Composition at Earth surface?
 - Composition at 300 km above?
- Temperature
 - Range of temperatures varies widely with altitude
- Pressure
 - Pressure variation with altitude is many orders of magnitude – variability at a given altitude is much smaller
- Mass density
 - Which of the above variations most closely matches density variation?
- Flow velocity
 - Magnitude - air speed. Range of possible speeds varies significantly with altitude

Macroscopic state parameters

Parameters that describe the general state of an atmosphere

Parameter	Symbol	Gas kinetic meaning	Height		Unit
			0 km	300 km	
Chemical composition		n_i / n	78% N ₂ 21% O ₂ 1% Ar	78% O 21% N ₂ 1% O ₂	
Mass density	ρ	$\sum_i m_i n_i$	1.3	$2 \cdot 10^{-11}$	kg/m ³
Flow velocity	\vec{u}	$\langle \vec{v} \rangle$	0 – 50	0 - 1000	m/s
Temperature	T	$\frac{2}{3} k \cdot \frac{1}{2} m \overline{c^2}$	200 – 320 2500	600 -	K
Pressure	p	$\frac{1}{3} n m \overline{c^2}$	10^5	10^{-5}	Pa

1 Pa = 1 pascal = 1 N/m²
1 atm = 101325 Pa = 760 torr

Gas kinetics

- Macroscopic state parameters are sufficient to describe the general macroscopic characteristics of an atmosphere.
- Gas kinetics are necessary to give physical interpretation of the macroscopic state parameters.
- Gas kinetics are necessary to describe other parameters like diffusion, viscosity and heat conduction.
- Gas kinetics describe the behaviour of gas particles with *atomic parameters, dynamical variables and boundary conditions*.

Atomic parameters

- Particles to consider:
 - H He O Ar N₂ O₂
- Particle radius
 - Not radically different for any of the constituent particles
 - $1 - 3 \times 10^{-10}$ m (If the particle radius would be scaled up to 1 cm, then 1m would have to be scaled up to the distance from here to the moon.)
- Mass number (atomic mass number – sum of protons and neutrons – vs. Atomic number – number of protons only) – and particle mass (mass number times mass of 1 amu - 1.66×10^{-27} kg)
- Degrees of freedom
 - 3 for atoms, usually 5 for molecules:
 - rotation about the axis joining the two atoms has no impact on kinetic energy – negligible moment of inertia about this axis.
 - Vibration in the joining axis generally negligible as well

Periodic Table of Elements

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H Hydrogen 1.00794	2 He Helium 4.002602																
3 Li Lithium 6.941	4 Be Beryllium 9.012182																
11 Na Sodium 22.98976928	12 Mg Magnesium 24.3050																
19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.955912	22 Ti Titanium 47.887	23 V Vanadium 50.9415	24 Cr Chromium 51.9961	25 Mn Manganese 54.938045	26 Fe Iron 55.845	27 Co Cobalt 58.933195	28 Ni Nickel 58.6934	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.64	33 As Arsenic 74.92160	34 Se Selenium 78.96	35 Br Bromine 79.904	36 Kr Krypton 83.798
37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.90585	40 Zr Zirconium 91.224	41 Nb Niobium 92.90638	42 Mo Molybdenum 95.96	43 Tc Technetium (97.9072)	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.90550	46 Pd Palladium 106.42	47 Ag Silver 107.8682	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.710	51 Sb Antimony 121.760	52 Te Tellurium 127.80	53 I Iodine 126.90447	54 Xe Xenon 131.293
55 Cs Caesium 132.9054519	56 Ba Barium 137.327																
87 Fr Francium (223)	88 Ra Radium (226)																
72 Hf Hafnium 178.49	73 Ta Tantalum 180.94788	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.217	78 Pt Platinum 195.084	79 Au Gold 196.966569	80 Hg Mercury 200.59	81 Tl Thallium 204.3833	82 Pb Lead 207.2	83 Bi Bismuth 208.98040	84 Po Polonium (209.9824)	85 At Astatine (209.9871)	86 Rn Radon (222.0176)			
104 Rf Rutherfordium (261)	105 Db Dubnium (262)	106 Sg Seaborgium (266)	107 Bh Bohrium (264)	108 Hs Hassium (277)	109 Mt Meitnerium (268)	110 Ds Darmstadtium (271)	111 Rg Roentgenium (272)	112 Uub Ununbium (285)	113 Uut Ununtrium (284)	114 Uuq Ununquadium (289)	115 Uup Ununpentium (288)	116 Uuh Ununhexium (292)	117 Uus Ununseptium	118 Uuo Ununoctium (294)			

C Solid
Hg Liquid
H Gas
Rf Unknown

Metals
Alkali metals
Alkaline earth metals
Lanthanoids
Actinoids
Transition metals
Poor metals
Nonmetals
Other nonmetals
Noble gases

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

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57 La Lanthanum 138.90547	58 Ce Cerium 140.116	59 Pr Praseodymium 140.90768	60 Nd Neodymium 144.242	61 Pm Promethium (145)	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.92535	66 Dy Dysprosium 162.500	67 Ho Holmium 164.93032	68 Er Erbium 167.259	69 Tm Thulium 168.93421	70 Yb Ytterbium 173.054	71 Lu Lutetium 174.9668
89 Ac Actinium (227)	90 Th Thorium 232.03806	91 Pa Protactinium 231.03688	92 U Uranium 238.02891	93 Np Neptunium (237)	94 Pu Plutonium (244)	95 Am Americium (243)	96 Cm Curium (247)	97 Bk Berkelium (247)	98 Cf Californium (251)	99 Es Einsteinium (252)	100 Fm Fermium (257)	101 Md Mendelevium (258)	102 No Nobelium (259)	103 Lr Lawrencium (262)

For Space Scientists

	1									18
1	1.00794 H 1									4.0026 He 2
2	6.941 Li 3	9.0122 Be 4								
3	22.9898 Na 11	24.3050 Mg 12								
			3	12						
					13	14	15	16	17	
					10.811 B 5	12.011 C 6	14.0067 N 7	15.9994 O 8	18.9984 F 9	20.1797 Ne 10
					26.9815 Al 13	28.0855 Si 14	30.9738 P 15	32.066 S 16	35.4527 Cl 17	39.948 Ar 18

For Astronomers



Metals

Atomic parameters

Parameter	Symbol	Gas type						Unit
		H	He	O	N ₂	O ₂	Ar	
Particle radius	r	$1 - 3 \times 10^{-10}$						m
Mass number	\mathcal{M}	1	4	16	28	32	40	
Particle mass	m	$= m_u \mathcal{M} = 1.66 \times 10^{-27} \mathcal{M}$						kg
Degrees of freedom	f	3	3	3	5	5	3	

$m_u = \text{amu} = \text{atomic mass unit}$

Gas kinetic parameters

At 0 km, assume 300 K, use primary composition N_2

At 1000 km, assume 1000 K, use primary composition O

- Particle density
- Random velocity
- Collision frequency
- Mean free path

Important gas kinetic parameters

can be derived if statistical behaviour of gas particles is known

Parameter	Symbol	Height		Unit
		0 km (300 K, N ₂)	300 km (T _∞ = 1000 K; O)	
Particle density	n	2x10 ²⁵ *	1x10 ¹⁵	1/m ³
Random velocity Mean random speed	\vec{c} \overline{c}	470	1100	m/s
Collision frequency	ν	6x10 ⁹	0.4	1/s
Mean free path	l	8x10 ⁻⁸	3000	m

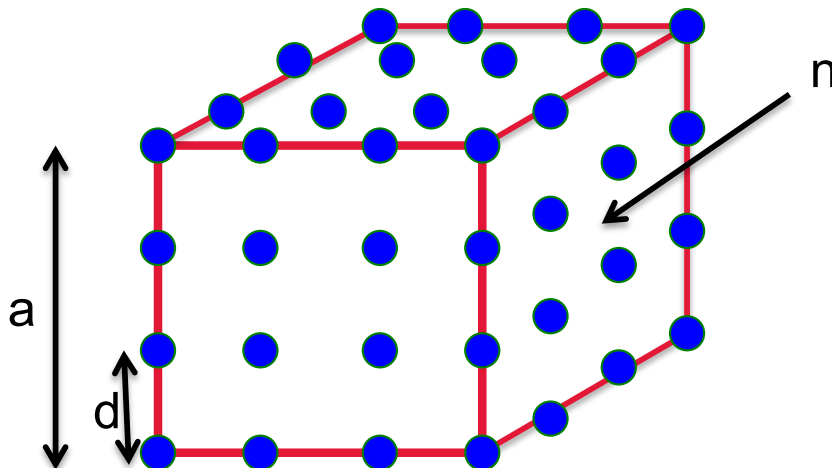
* For T=0 C, n=2.69 x 10²⁵ m⁻³

Particle number density

- Particle density, n , of a gas is a function of
 - position within the gas and
 - time
- It can be expressed as

$$n(\vec{r}, t) = \lim_{\Delta V \rightarrow dV} \left(\frac{\Delta N}{\Delta V} \right)_{\vec{r}, t}$$

where dV is large enough to contain enough particles but small enough so that the density does not vary.



If the edge of dV is of size a and the distance between particles is d , and we assume particles are on average equidistant from each other and located on a volumetric grid

- Number of particles on an edge of length $a = \frac{a}{d} + 1$
- Number of particles N in the volume $dV = a^3$ is $\left(\frac{a}{d} + 1\right)^3$

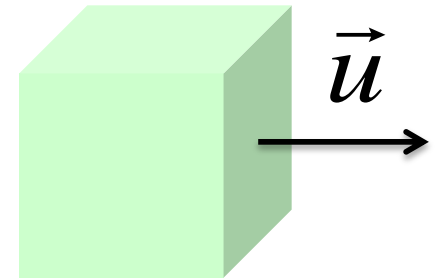
Knowing $d \ll a$, $N \approx (a/d)^3$

Remembering n is the density of particles, the number of particles N in a volume dV is $a^3 \times n$, so the mean distance between particles, $d \approx 1/\sqrt[3]{n}$

The number of particles in a volume dV will typically fluctuate by $1/\sqrt{N}$, so if we want to constrain the particle density variation to $< 0.1\%$ N must be $> 1,000,000$. If we have a particle density of 1×10^6 particles/m³, we would need a volume of at least 1 m

Three types of gas velocities

- Particle velocity \vec{v}
- Flow (bulk or wind) velocity \vec{u}
- Random (thermal) velocity \vec{c}



$$\vec{u}(\vec{r}, t) = \langle \vec{v} \rangle_{\vec{r}, t} = \lim_{\Delta V \rightarrow dV} \left(\frac{1}{n\Delta V} \sum_{i=1}^{n\Delta V} \vec{v}_i \right)_{\vec{r}, t}$$

Random velocity is the deviation from the ordered mean flow velocity.

$$\langle \vec{c} \rangle = \langle \vec{v} - \vec{u} \rangle = \langle \vec{v} \rangle - \vec{u} = 0 \quad : \text{Mean random velocity}$$

$$\bar{c} = \langle |\vec{c}| \rangle : \text{Mean random speed}$$

Boltzmann energy distribution

Derived for identical but distinguishable particles

The probability that a particle will have energy E

With increasing energy E, it is progressively less likely that any given particle will attain that energy, so more particles will be found with lower energies. It is assumed that an unlimited number of particles can occupy any energy state.

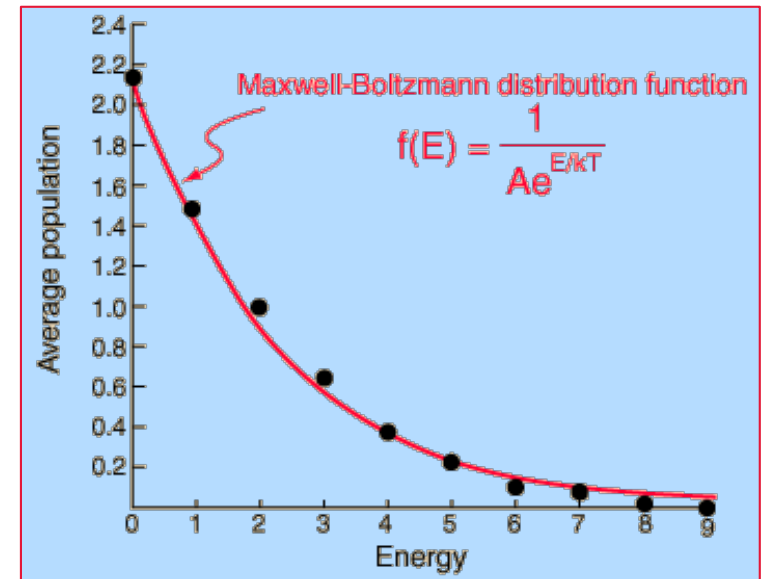
$$f(E) = \frac{1}{Ae^{E/kT}}$$

Normalization constant A

The probability for occupying a given energy state decreases exponentially with energy

Boltzmann's constant k times the absolute temperature T. The implication of this term is that for a higher temperature, it is more probable that a given particle can be found with energy E.

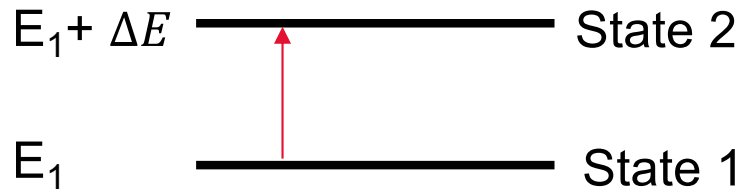
Boltzmann's constant
 $k = 1.38 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$



Example 2-1

Consider a system that consists of only 2 possible energy states, E_1 and $E_2 = E_1 + \Delta E$

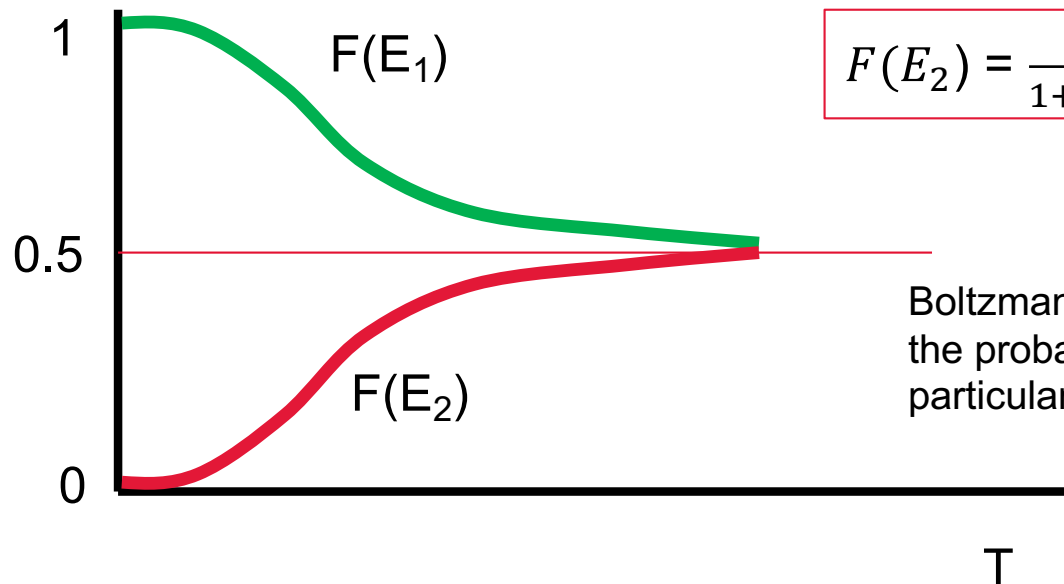
$$F(E_1) + F(E_2) = 1$$



$$\frac{F(E_2)}{F(E_1)} = \frac{e^{-(E_1 + \Delta E)/kT}}{e^{-E_1/kT}} = e^{-\Delta E/kT}$$

$$F(E_1) = \frac{1}{1 + e^{-\Delta E/kT}}$$

$$F(E_2) = \frac{1}{1 + e^{+\Delta E/kT}}$$



Boltzmann energy distribution gives the probability of observing a system in any particular microstate.

- If the energy in the Boltzmann energy distribution is just 1-dimensional kinetic energy, then

$$f(c_z) = Ae^{-\left(\frac{mc_z^2}{2kT}\right)}$$

- After normalization which gives us an expression for A, we get:

$$\int_{-\infty}^{+\infty} f(c_z) dc_z = 1$$

$$f(c_z) = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-\left(\frac{mc_z^2}{2kT}\right)}$$

Maxwell-Boltzmann speed distribution

The Maxwell-Boltzmann speed distribution (for idealized gases) is obtained by converting the 1-dim energy distribution to a 3-dim energy distribution and then integrating over all velocity directions.

$$f(c_x, c_y, c_z) = \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\left(\frac{m(c_x^2 + c_y^2 + c_z^2)}{2kT} \right)}$$

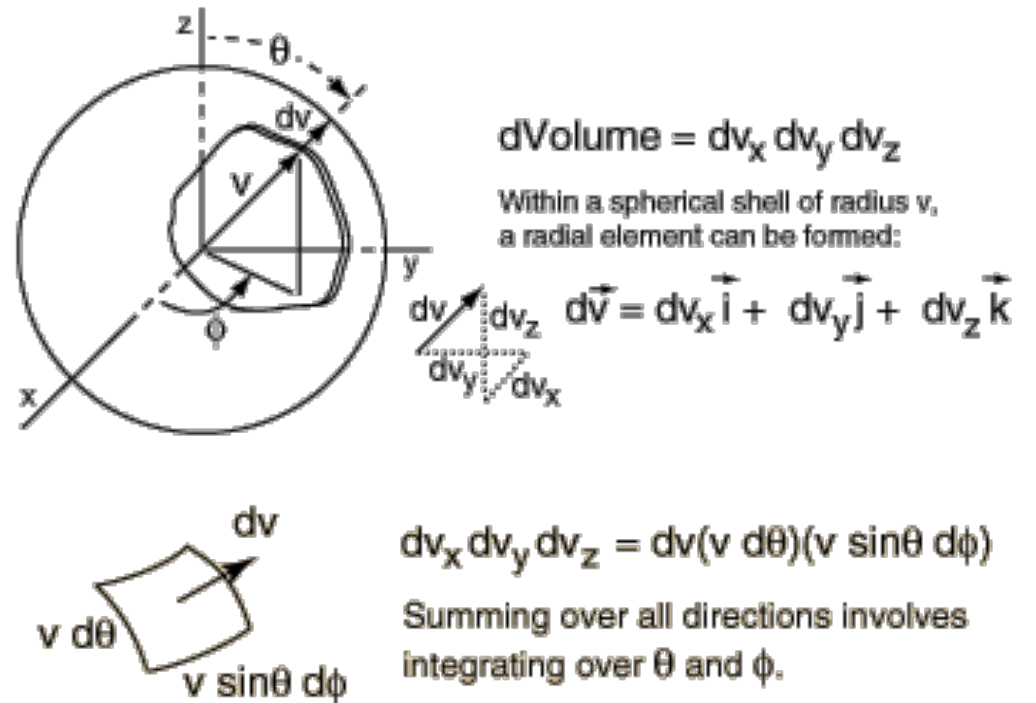
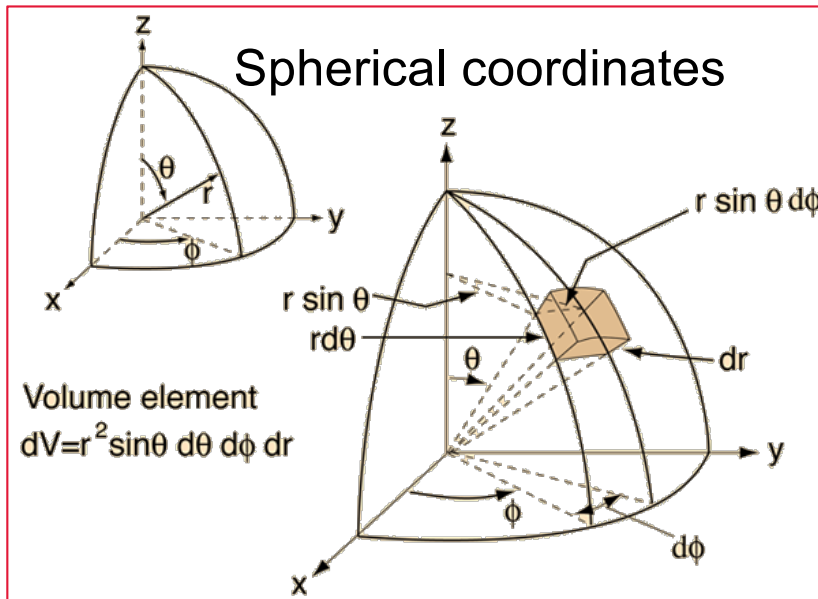
$$f(\vec{c}) = \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\left(\frac{mc^2}{2kT} \right)}$$

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

For the integration over all directions it is best to use spherical coordinates and consider a shell in velocity space.

Attention: in these diagrams, the random velocity \vec{c} is \vec{v}

Shell of thickness dv in velocity space



This gives us the Maxwell-Boltzmann speed distribution

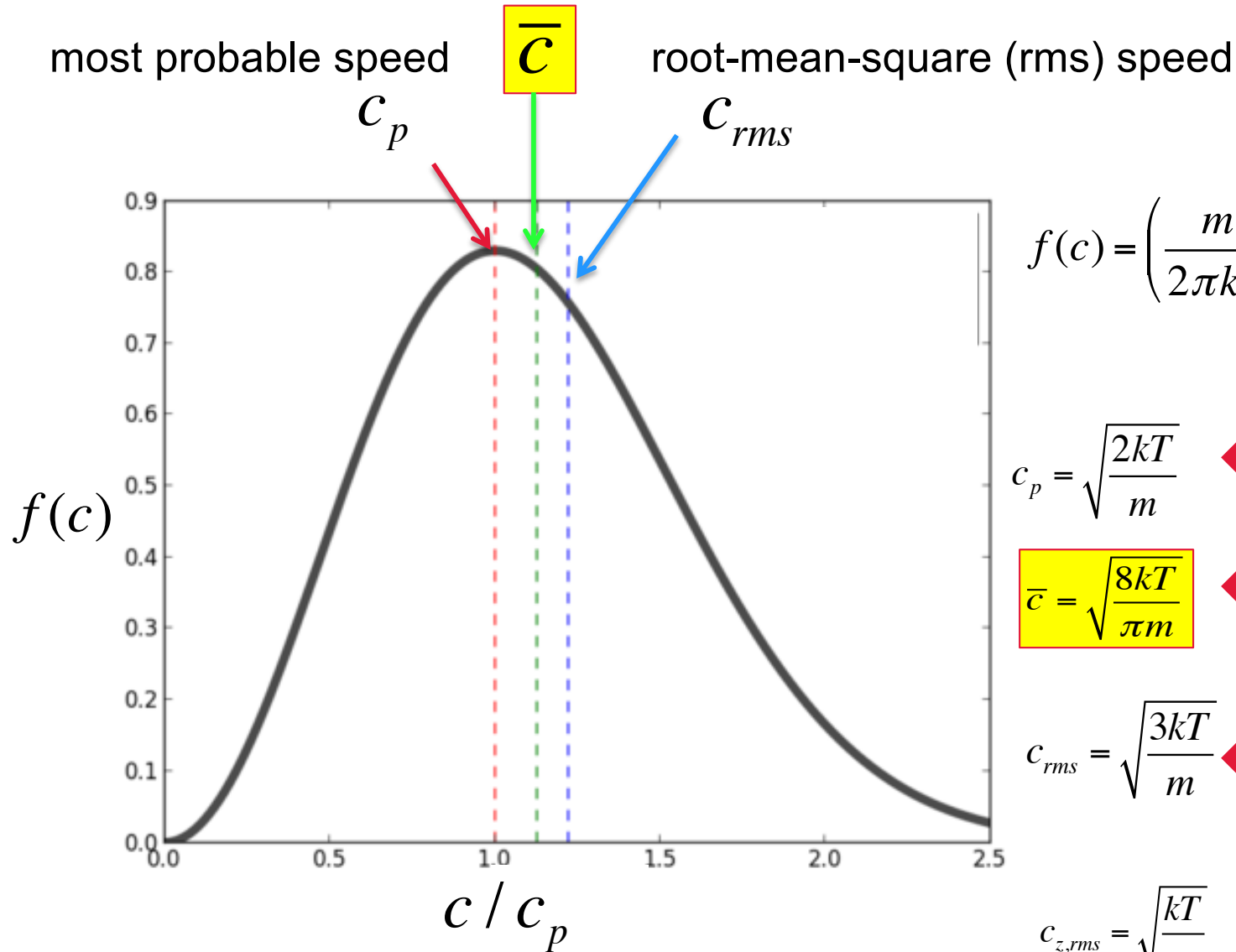
$$f(c) = \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} 4\pi c^2 e^{-\left(\frac{mc^2}{2kT} \right)}$$

$$f(v) = \left[\frac{m}{2\pi kT} \right]^{3/2} e^{-mv^2/2kT} \underbrace{\int_0^\pi \int_0^{2\pi} v^2 \sin \theta \, d\phi \, d\theta}_{= 4\pi v^2}$$

<http://hyperphysics.phy-astr.gsu.edu/hbase/Kinetic/maxspe.html>

It takes out the directionality of the velocity and replaces it with the “radial” (scalar) speed.

average speed (mean random speed)



$$f(c) = \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} 4\pi c^2 e^{-\left(\frac{mc^2}{2kT} \right)}$$

$$c_p = \sqrt{\frac{2kT}{m}} \quad \leftarrow \quad \frac{df(c)}{dc} = 0$$

$$\bar{c} = \sqrt{\frac{8kT}{\pi m}} \quad \leftarrow \quad \bar{c} = \int_0^{\infty} cf(c)dc$$

$$c_{rms} = \sqrt{\frac{3kT}{m}} \quad \leftarrow \quad c_{rms} = \left(\int_0^{\infty} c^2 f(c)dc \right)^{\frac{1}{2}}$$

$$c_{z,rms} = \sqrt{\frac{kT}{m}}$$

from 1-dim distribution

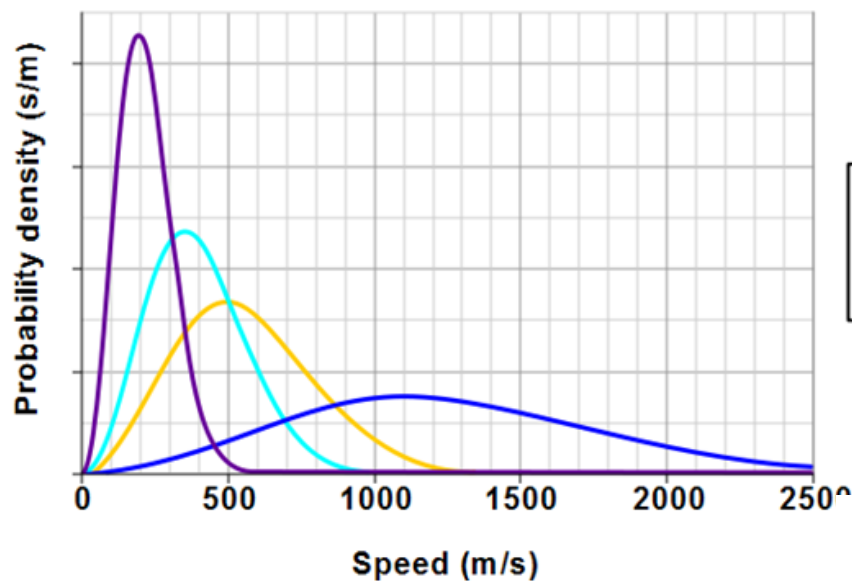
$$0.886\bar{c} = c_p < \bar{c} < c_{rms} = 1.085\bar{c}$$

Maxwell speed distribution or Maxwell-Boltzmann speed distribution

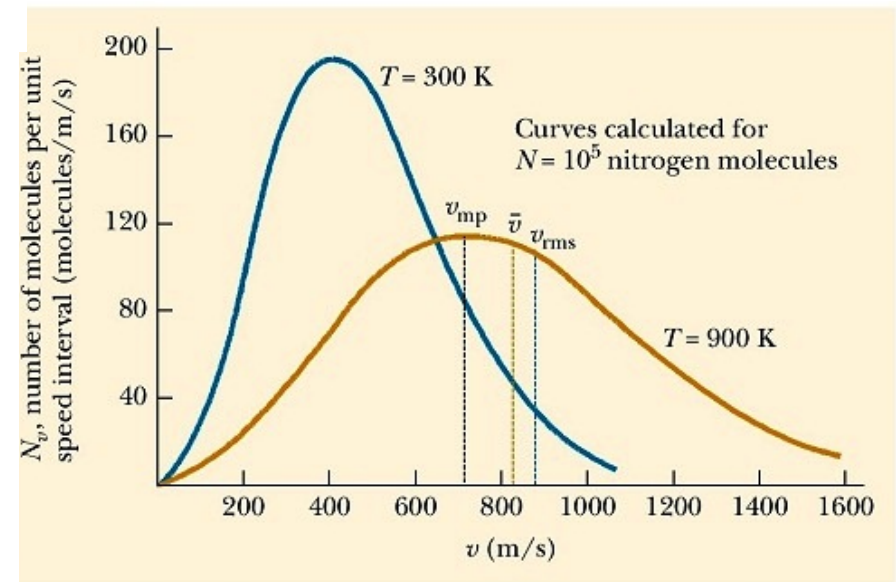
Example 2-2

$$f(c) = \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} 4\pi c^2 e^{-\left(\frac{mc^2}{2kT} \right)}$$

m varied
T constant



m constant
T varied



Collision frequency

mean number of collisions of a particle in a gas per unit time

Real scenario: A particle of radius, r_1 , and gas type 1 moves with its random velocity, $\vec{C}_{1,i}$, in a set of particles with radius, r_2 , in a gas of type 2 with flow velocity = zero but with random velocities, $\vec{C}_{2,j}$. The particles collide frequently with each other. Each collision changes the magnitude and the direction of each colliding particle.

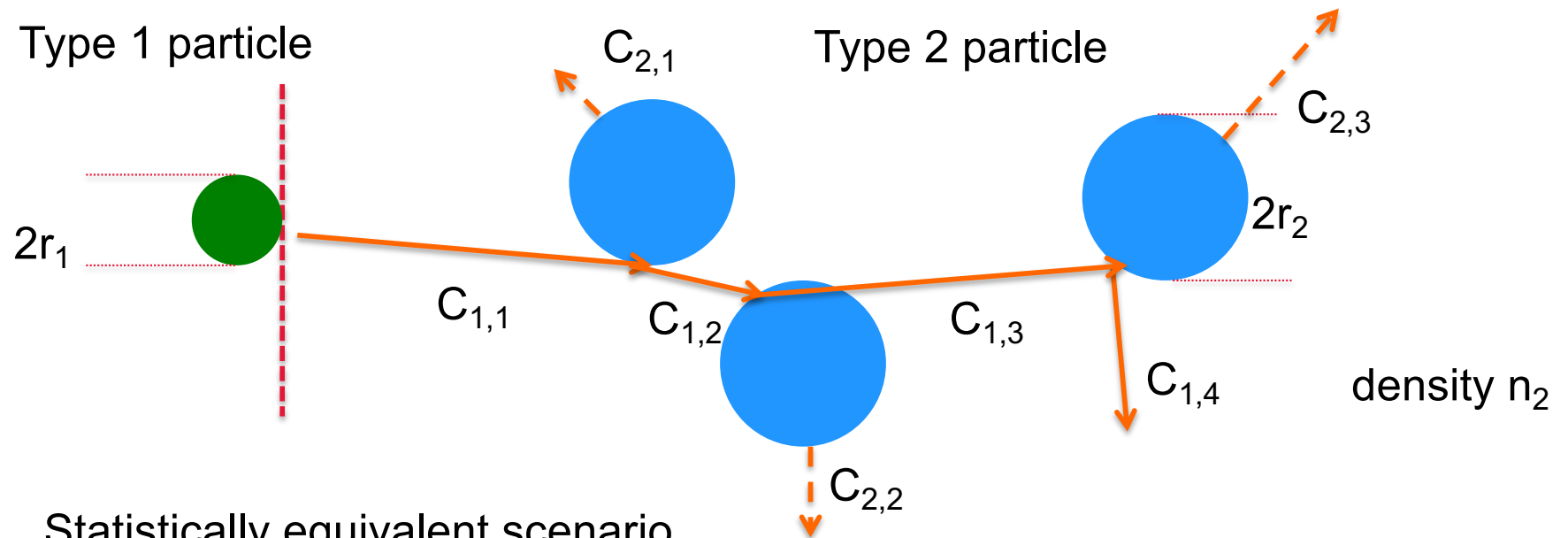
Statistically equivalent scenario: Collisions occur statistically at the same rate as between particles of radius $r_1 + r_2$ of gas type 1 with point particles of gas type 2. Furthermore, we can model this scenario as a test particle with the mean relative speed $\bar{c}_{1,2} = \langle |\vec{c}_1 - \vec{c}_2| \rangle$ and a stationary point particle. The zig-zag course can also be replaced, namely by a straight trajectory along the axis of a cylinder with a base area of radius, $r_1 + r_2$. During time, Δt , the test particle flies a distance, $\bar{c}_{1,2} \Delta t$, colliding with all point particles in the cylinder. The number of collisions is then: $\pi(r_1 + r_2)^2 \bar{c}_{1,2} \Delta t n_2$, and

$$v_{1,2} = \frac{\pi(r_1 + r_2)^2 \bar{c}_{1,2} \Delta t n_2}{\Delta t} = \sigma_{1,2} n_2 \bar{c}_{1,2} \quad : \text{Collision frequency}$$

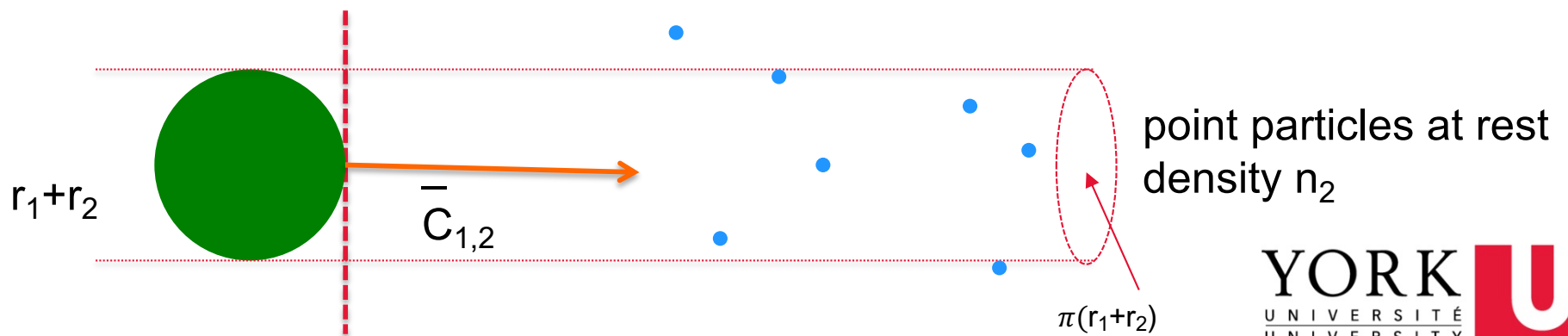
$$\sigma_{1,2} = \pi(r_1 + r_2)^2 \quad : \text{Collision cross section}$$

Consideration for calculating the collision frequency

Real scenario



Statistically equivalent scenario



- To repeat:
- Gas particle 1 maps out volume $\pi(r_1 + r_2)^2$ times $\bar{c}_{1,2} = \langle |\vec{c}_1 - \vec{c}_2| \rangle$ times Δt . During time interval Δt it will collide with that volume times n_2 particles of particle 2.
- The collision frequency, $\nu_{1,2}$, that is the number of collisions per unit time, Δt , is:

$$\nu_{1,2} = \pi(r_1 + r_2)^2 n_2 \bar{c}_{1,2}$$

$$\nu_{1,2} = \sigma_{1,2} n_2 \bar{c}_{1,2}$$

where $\sigma_{1,2}$ is the collision cross-section. But we still need an explicit number for the collision frequency in terms of temperature of each of the gases and the mass of the particles in each of the two gases.

Now we want to consider a mix of two gases with particles 1 and particles 2 and temperature 1 and temperature 2. We have then the two Maxwell-Boltzmann speed distributions:

$$f(c_1) = \left(\frac{m_1}{2\pi kT_1} \right)^{\frac{3}{2}} 4\pi c_1^2 e^{-\frac{m_1 c_1^2}{2kT_1}}$$

$$f(c_2) = \left(\frac{m_2}{2\pi kT_2} \right)^{\frac{3}{2}} 4\pi c_2^2 e^{-\frac{m_2 c_2^2}{2kT_2}}$$

It can be shown that the mean relative velocity is

$$\bar{c}_{1,2} = 4\pi \left(\frac{m_{1,2}}{2\pi kT_{1,2}} \right)^{\frac{3}{2}} \int c_{1,2}^2 e^{-\frac{m_{1,2} c_{1,2}^2}{2kT_{1,2}}} dc_{1,2}$$

$$\bar{c}_{1,2} = \sqrt{\frac{8k}{\pi} \left(\frac{T_1}{m_1} + \frac{T_2}{m_2} \right)} = \sqrt{\frac{8kT_{1,2}}{\pi m_{1,2}}}$$

where $T_{1,2}$ and $m_{1,2}$ are the reduced temperature and the reduced mass, respectively:

$$T_{1,2} = \frac{m_2 T_1 + m_1 T_2}{m_1 + m_2}$$

$$m_{1,2} = \frac{m_1 m_2}{(m_1 + m_2)}$$

The Collision frequency is then given as:

$$\nu_{1,2} = \sigma_{1,2} n_2 \sqrt{\frac{8kT_{1,2}}{\pi m_{1,2}}}$$

Mean free path

- The mean free path is the mean distance, $\bar{c}_1 \Delta t$, travelled by particle 1 in this mixed gas in the time interval, Δt , divided by the number of collisions, $\nu_{1,2} \Delta t$, during that same time interval.

$$l_{1,2} = \frac{\bar{c}_1}{\nu_{1,2}}$$

$$l_{1,2} = \frac{1}{\sigma_{1,2} n_2 \sqrt{1 + \frac{m_1 T_2}{m_2 T_1}}}$$

When all the particles are the same, we get:

$$\sigma_{1,1} = 4\pi r^2$$

$$\nu_{1,1} = 4\sigma_{1,1} n \sqrt{\frac{kT}{\pi m}}$$

$$l_{1,1} = \frac{1}{\sqrt{2} n \sigma_{1,1}}$$

Example 2-3

Assume for Earth:

atmosphere with 100% N₂
height = 0 km above sea level,
T=300 K

\mathcal{M} : mass number =28

$\rho = 1.3 \text{ kg/m}^3$

$r = 2 \times 10^{-10} \text{ m}$

$m = \mathcal{M} \times \text{amu} = 28 \times 1.66 \times 10^{-27} \text{ kg}$

$n = \rho / (\mathcal{M} \times \text{amu}) = 1.3 / (28 \times 1.66 \times 10^{-27})$



$$\nu_{1,1} = 6.7 \times 10^9 \text{ s}^{-1}$$

$$l_{1,1} = 7 \times 10^{-8} \text{ m}$$

Gas kinetic parameters

- We have now successfully derived expressions for:
 - Particle density
 - Random velocity
 - Collision frequency
 - Mean free path


Macroscopic state parameters

Parameters that describe the general state of an atmosphere

Parameter	Symbol	Gas kinetic meaning	Height		Unit
			0 km	300 km	
Chemical composition		n_i / n	78% N ₂ 21% O ₂ 1% Ar	78% O 21% N ₂ 1% O ₂	
Mass density	ρ	$\sum_i m_i n_i$	1.3	$2 \cdot 10^{-11}$	kg/m ³
Flow velocity	\vec{u}	$\langle \vec{v} \rangle$	0 – 50	0 - 1000	m/s
Temperature	T	$\frac{2}{3}k \cdot \frac{1}{2}m\overline{c^2}$	200 – 320 2500	600 -	K
Pressure	p	$\frac{1}{3}n\overline{mc^2}$	10^5	10^{-5}	Pa

1 Pa = 1 pascal = 1 N/m²
1 atm = 101325 Pa = 760 torr

Gas kinetics

- 
- Macroscopic state parameters are sufficient to describe the general macroscopic characteristics of an atmosphere.
 - Gas kinetics are necessary to give a physical interpretation of the macroscopic state parameters.
 - Gas kinetics are necessary to describe other parameters like diffusion, viscosity and heat conduction.
 - Gas kinetics describe the behaviour of gas particles with *atomic parameters, dynamical variables and boundary conditions*.

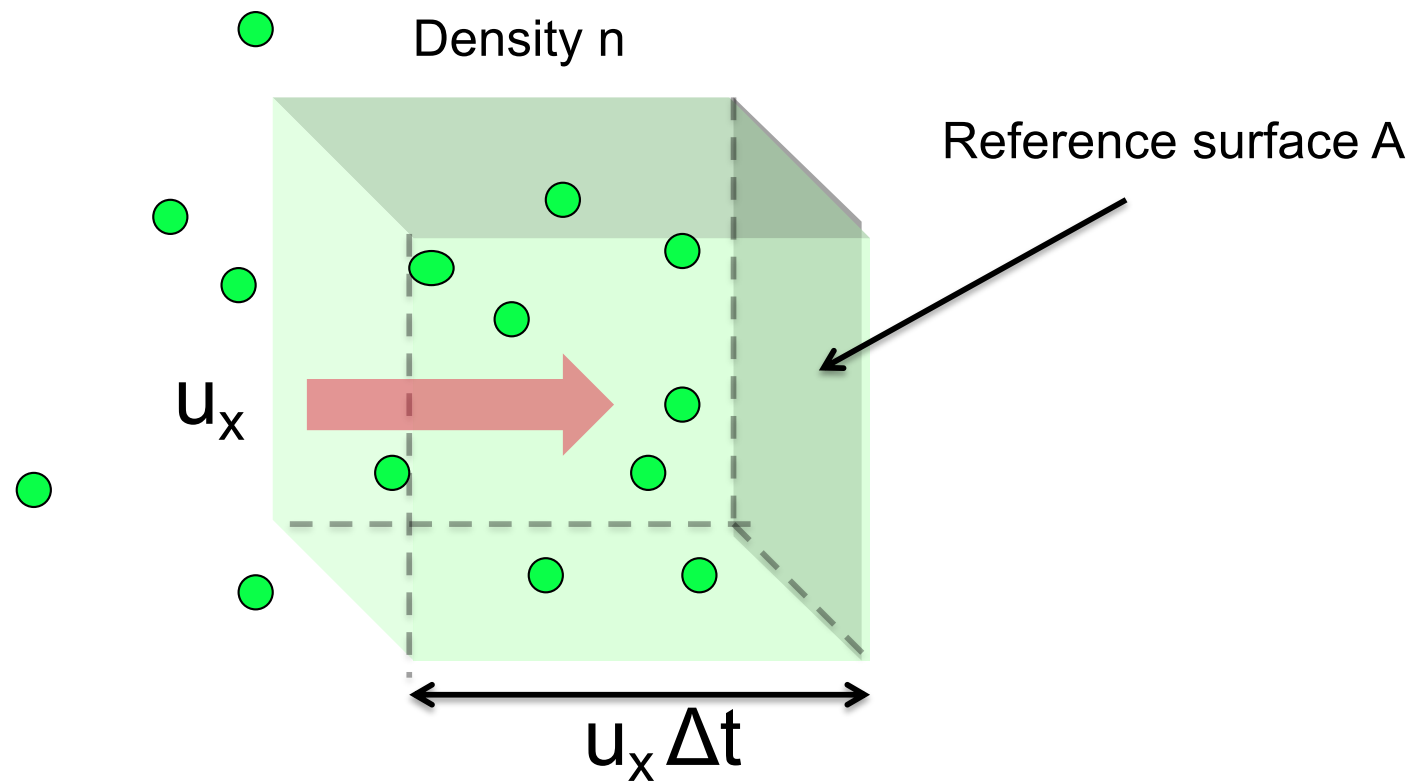
Macroscopic properties

- Pressure
- Temperature
- Specific heat capacity

We will introduce the concept of “flux”:

- Net amount of a scalar quantity transported per unit area per unit time through a reference surface normal to the flow.
 - Scalar quantity could be number of particles, heat, charge, even a component of a vector

Example: particle flux



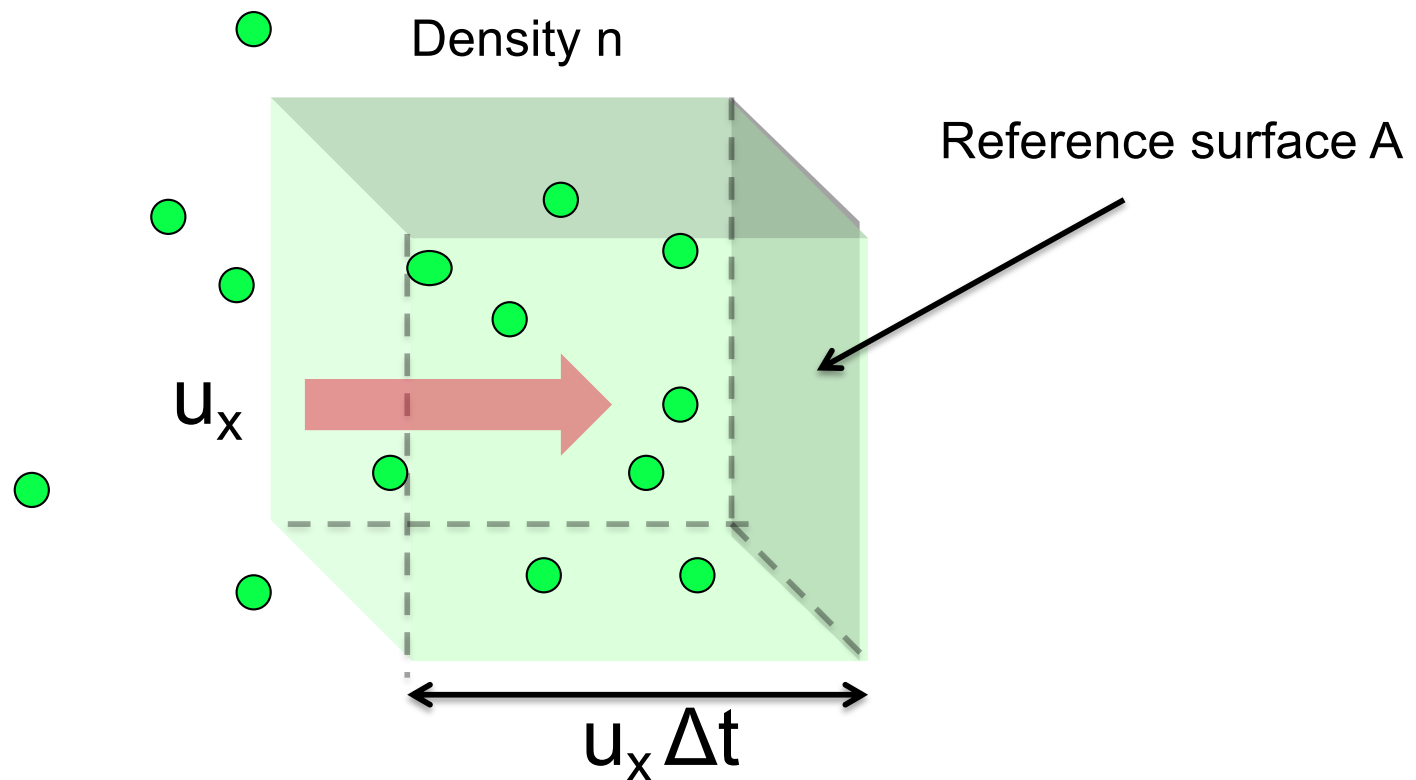
$$\phi_x = \frac{u_x \Delta t A n}{A \Delta t} = n u_x$$

Particle flux

$$\vec{\phi} = n \vec{u}$$

Momentum flux

$I_x(u) = mu_x$: x-component of kinetic momentum carried by each particle travelling with velocity component, u_x

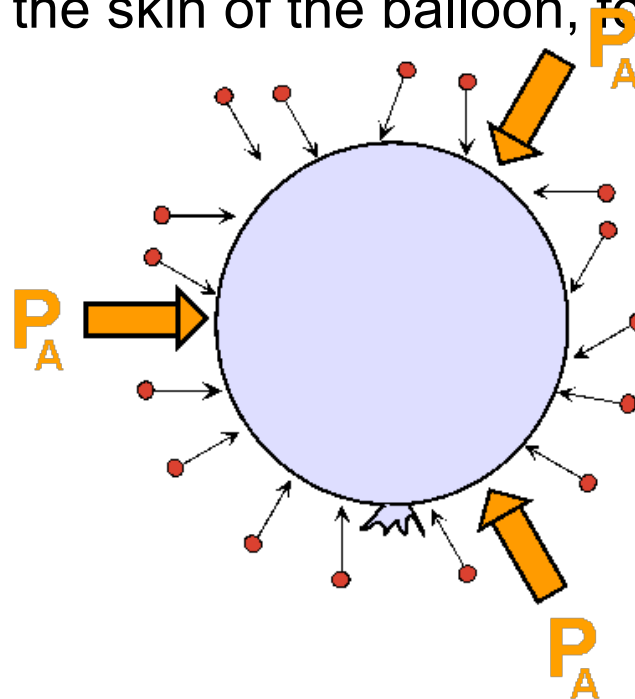
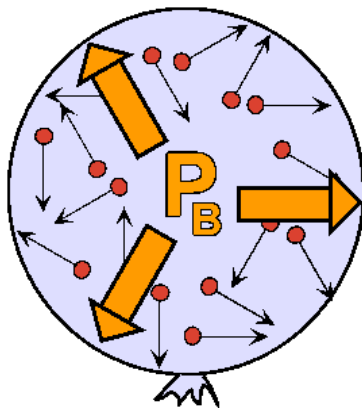


$$\phi_{x,x}^{I(u)} = I_x(u)\phi_x = mnu_x^2$$

Momentum flux

Now to pressure

- Particles (atoms or molecules) of a gas inside the balloon move around randomly.
- During this motion they frequently collide with the each other and with the skin of the balloon.
- The result is a force exerted on the inside of the skin of the balloon.
- Pressure is the force per surface area.
- This inside gas pressure, P_B , is balanced by the gas pressure, P_A , exerted on the outside of the skin of the balloon, for instance, the atmosphere.



Pressure

- The **thermodynamic pressure** is the mean net transport of momentum flux produced by the thermal motion of the particles with random velocity c . Only the components parallel to the surface normal are considered and averaging is done over all three (x, y, z) directions of a Cartesian coordinate system.

$$p = \frac{1}{3} \left(\phi_{x,x}^{I(c)} + \phi_{y,y}^{I(c)} + \phi_{z,z}^{I(c)} \right)$$



(Net) momentum flux of \bar{c} in the direction x

An explicit expression is obtained from extending the one box of gases to two boxes, 1 and 2, separated by an imaginary surface A where all particles have the same speed and 1/6 of them move in each of the 6 directions, $\pm x, \pm y, \pm z$.

Then the momentum transport from 1 to 2 per unit area and unit time is, $\phi_1^{I(c)}$, and the momentum transport from 2 to 1 per unit area and unit time is, $\phi_2^{I(c)}$.

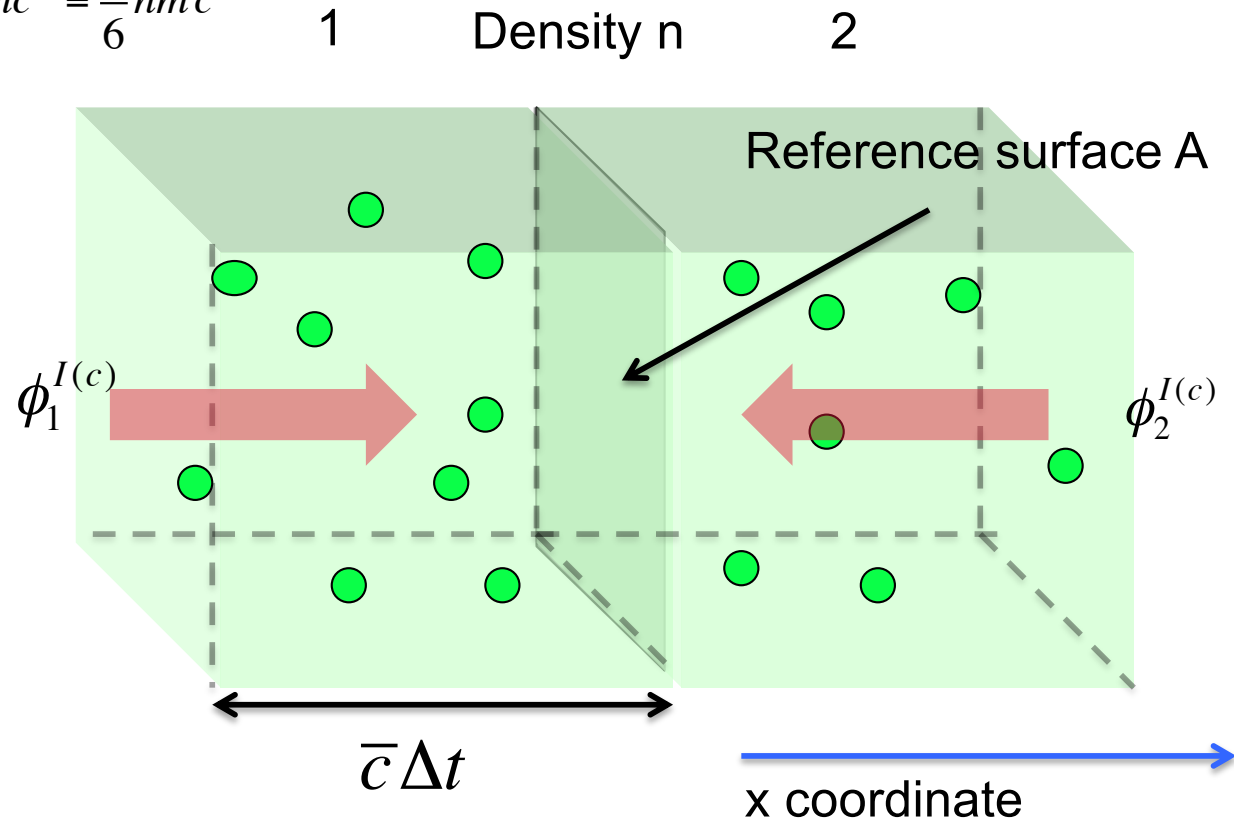
$$\phi_1^{I(c)} = \frac{1}{6} \frac{\bar{c} \Delta t A n}{A \Delta t} \bar{c} m = \frac{1}{6} n m \bar{c}^2 = \frac{1}{6} n m \bar{c}^2$$

$$\phi_2^{I(c)} = -\phi_1^{I(c)}$$

Net momentum flux:

$$\phi_{x,x}^{I(c)} = \phi_1^{I(c)} - \phi_2^{I(c)}$$

$$\phi_{x,x}^{I(c)} = \phi_{y,y}^{I(c)} = \phi_{z,z}^{I(c)}$$



$$p = \frac{1}{3} n m \bar{c}^2$$

Thermodynamic pressure

Conventional derivation of pressure

$$\Delta t = \frac{2L}{c_x}$$

$$F = m \frac{dc}{dt} = ma$$

$$m\Delta c = mc_x - (-mc_x) = 2mc_x$$

$$\bar{F} = \frac{m\Delta c}{\Delta t} = \frac{mc_x^2}{L}$$

$$p_{1particle} = \frac{\bar{F}}{A} = \frac{mc_x^2}{LA} = \frac{mc_x^2}{V}$$

$$p_{Nparticles} = \sum_1^N \frac{m}{V} c_{x,i}^2$$

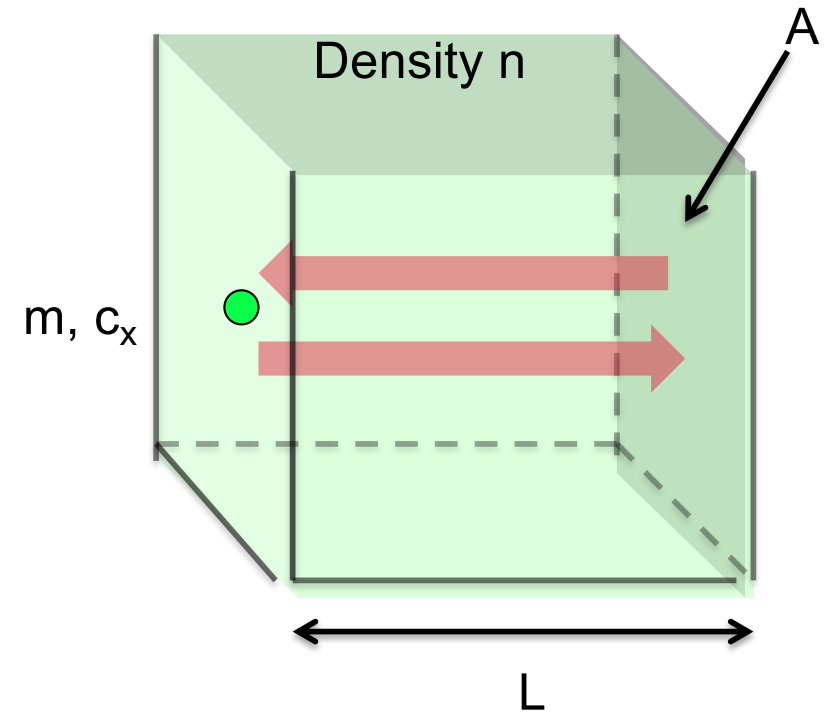
$$p_{Nparticles} = \frac{Nm\bar{c}_x^2}{V}$$

$$\bar{c}^2 = \bar{c}_x^2 + \bar{c}_y^2 + \bar{c}_z^2$$

$$\bar{c}_x^2 = \bar{c}_y^2 = \bar{c}_z^2$$

$$n = \frac{N}{V}$$

$$p = \frac{1}{3} nm \bar{c}^2$$



Dynamic pressure

- In addition to the random velocity, any flow velocity will also transport momentum
- For a surface perpendicular to the flow velocity, the mean net transport of “kinetic momentum” per unit area per unit time, or dynamic pressure, $p_d = (m\vec{u})n\vec{u} = mnu^2$
- The dynamic pressure can also be written as:

$$p_d = \phi_{x,x}^{I(u)} + \phi_{y,y}^{I(u)} + \phi_{z,z}^{I(u)}$$

Flow velocity

Temperature

- Temperature is a measure of the kinetic energy due to the random motion of the particles of the gas.
- It is now important to consider the number of degrees of freedom of a gas as a number of distinct and independent modes of energy storage.
- For a monoatomic gas we have 3 (translational) degrees of freedom. For a diatomic molecule we usually have two additional (rotational) degrees of freedom.
- Temperature is then a measure of the mean energy, or average amount of heat, \bar{U}_f , stored per degree of freedom, with

$$T = \left(\frac{2}{k}\right)\bar{U}_f$$

Recall 1-dim distribution:

$$c_{z,rms} = \sqrt{\frac{kT}{m}}$$

Ideal gas equation

- When heat is equally distributed over all, say, 3 degrees of freedom, meaning that the gas is in thermal equilibrium, then:

$$T = \frac{2}{3k} \left(\frac{1}{2} m \bar{c}^2 \right)$$



Note: $\bar{U}_f = \frac{1}{2} m \bar{c}_z^2$

$$c^2 = c_x^2 + c_y^2 + c_z^2$$
$$\frac{1}{3} \bar{c}^2 = \bar{c}_x^2 = \bar{c}_y^2 = \bar{c}_z^2$$

- Compare that with what we know from the Maxwell-Boltzmann speed distribution

$$c_{z,rms} = \sqrt{\frac{kT}{m}} \quad c_{z,rms} = \sqrt{\bar{c}_z^2}$$
$$c_{rms} = \sqrt{\frac{3kT}{m}} \quad c_{rms} = \sqrt{\bar{c}^2}$$

- From this follows the equation of state for an ideal gas

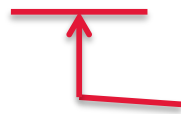
$$p = nkT \quad \text{ideal gas law}$$

Specific heat capacity

- Temperature is a measure of a body's heat content. To raise the temperature of a body, heat must be supplied.
- Consider a gas of mass M in a fixed enclosure. How much heat (energy), ΔQ , is needed to raise the temperature of N particles with f degrees of freedom by ΔT ?

$$\Delta T = \frac{2}{k} \Delta \bar{U}_f = \frac{2}{k} \frac{\Delta Q}{Nf} = \frac{2\Delta Q}{k \left(\frac{M}{m} \right) f}$$

$$\Delta Q (= \Delta U) = \frac{M}{m} f \frac{k}{2} \Delta T$$

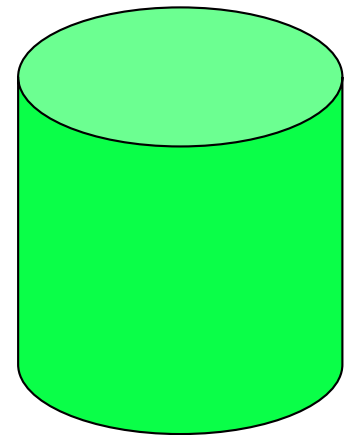


Gas heat capacity at constant volume

$$\Delta Q' (= \Delta U') = \frac{\Delta Q}{M} = c_v \Delta T \quad \text{Specific heat needed to raise temperature by } \Delta T \text{ with } V = \text{const}$$

$$c_v = \frac{kf}{2m}$$

Specific heat capacity at constant volume



Example 2-4

1) Specific heat capacity at constant volume for N₂

$$c_V = \frac{kf}{2m} = \frac{1.38 \cdot 10^{-23} \cdot 5}{2 \cdot 28 \cdot 1.66 \cdot 10^{-27}} = 742 \text{ J K}^{-1} \text{ kg}^{-1}$$

2) How much energy is needed to heat up 10 kg of N₂ by 2 C?

$$\Delta Q (= \Delta U) = \frac{M}{m} f \frac{k}{2} \Delta T = \frac{10 \cdot 1.38 \cdot 10^{-23} \cdot 5}{2 \cdot 28 \cdot 1.66 \cdot 10^{-27}} \cdot 2 = 14.48 \text{ kJ}$$

3) How large is the pressure of a N₂ gas with density $\rho = 1.3 \text{ kg/m}^3$ at a temperature $T = 300 \text{ K}$?

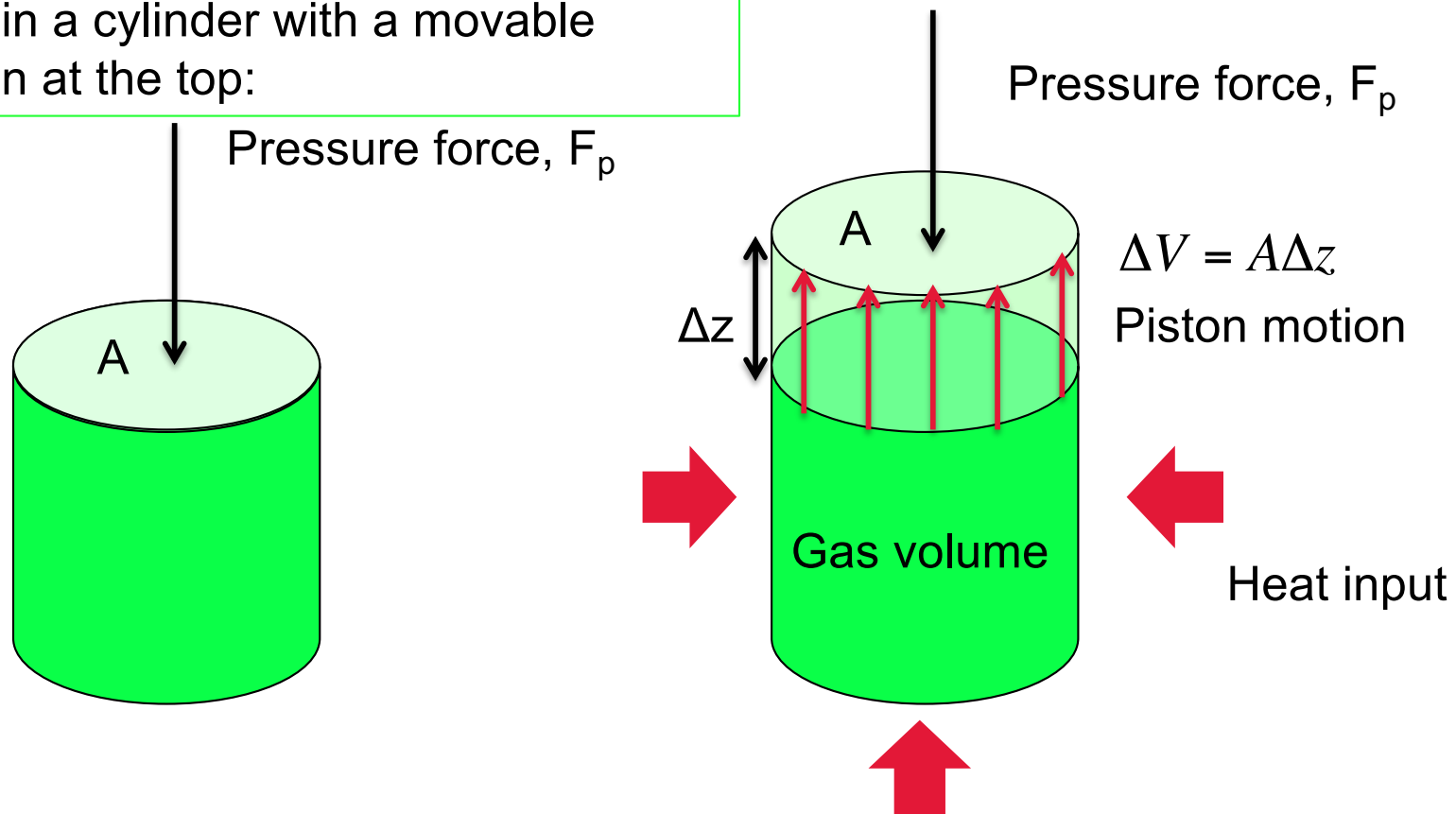
$$p = nkT = \frac{\rho}{m} kT = \frac{1.3}{28 \cdot 1.66 \cdot 10^{-27}} 1.38 \cdot 10^{-23} \cdot 300 = 115,800 \text{ P}$$

That is indeed approximately the atmospheric pressure at sea level.

Heat capacity at constant pressure

At constant p but variable V , we need more energy to raise T of gas than if V were constant, since we have to increase internal energy and do work to expand the gas.

Gas in a cylinder with a movable piston at the top:



The gas has performed work of expansion, ΔW ,

$$\Delta W = -F_p \Delta z = -p \Delta V$$

Negative sign: energy is lost to the gas

- Using the ideal gas law with pressure = constant, we get:

$$p = nkT = \frac{N}{V}kT = \text{const} \Rightarrow \frac{V}{T} = \text{const} \Rightarrow \Delta V = V \frac{\Delta T}{T}$$

$$\Delta W = -\frac{pV}{T} \Delta T = -kN \Delta T$$



$$\Delta W' = -\frac{k}{m} \Delta T$$

Work performed per unit mass

Remember: $N = \frac{M}{m}$

- To raise temperature in the cylinder by ΔT we need to supply amount of heat per unit mass of $\Delta Q'$, with

$$\Delta Q' = c_v \Delta T + \frac{k}{m} \Delta T = c_p \Delta T \quad \text{with} \quad \Delta Q' = \Delta U' - \Delta W'$$

$$c_p = \frac{k}{m} \left(\frac{f}{2} + 1 \right) \quad \text{heat capacity at constant pressure}$$

$$\gamma = \frac{c_p}{c_v} = \frac{f+2}{f} \quad \text{adiabatic exponent (or heat capacity ratio)}$$

$$\Rightarrow f = \frac{2}{\gamma - 1}$$

$\gamma = \text{const}$ for a given gas, indicates how much more work is needed for gas expansion

- Assume change of state of a gas proceeds with no heat exchange with the environment, i. e. $\Delta Q=0$
- Work during expansion done at the expense of own internal energy

$$\Delta Q = \Delta U - \Delta W = 0$$

$$\Delta W = -p\Delta V = \Delta U = Nf\left(\frac{k}{2}\Delta T\right)$$

$$dW = -pdV = dU = Nf\left(\frac{k}{2}dT\right)$$

Divide by TV
and with $N = nV$
 $p = nkT$

$$\frac{dT}{T} = -\frac{2}{f} \frac{dV}{V}$$

Integration $\rightarrow \ln\left(\frac{T}{T_0}\right) = -\frac{2}{f} \ln\left(\frac{V}{V_0}\right)$

Adiabatic law

$$T = T_0 \left(\frac{V}{V_0}\right)^{-\frac{2}{f}}$$

$$TV^{\frac{2}{f}} = \text{const}$$

$$TV^{\gamma-1} = \text{const}$$

Alternative forms if $N=\text{const.}$

$$n = \text{const} \cdot p^{\frac{1}{\gamma}}$$

$$p \rho^{-\gamma} = \text{const}$$

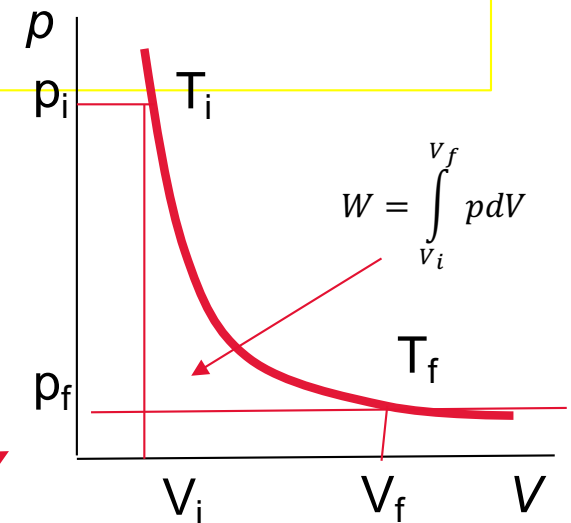
$$pV^{\gamma} = \text{const}$$

$$p^{1-\gamma} T^{\gamma} = \text{const}$$

Remember: $\rho = \sum_i m_i n_i$

i: index for a type of gas

For one type of gas: $\rho = m n$



Macroscopic properties

- We have now successfully derived expressions for:
 - Pressure
 - Temperature
 - Specific heat capacity
- In addition we have derived the adiabatic law.

Atmospheric divisions

- Temperature
- Composition
- Vertical transport
- Gravitational binding
- Thermal plasma

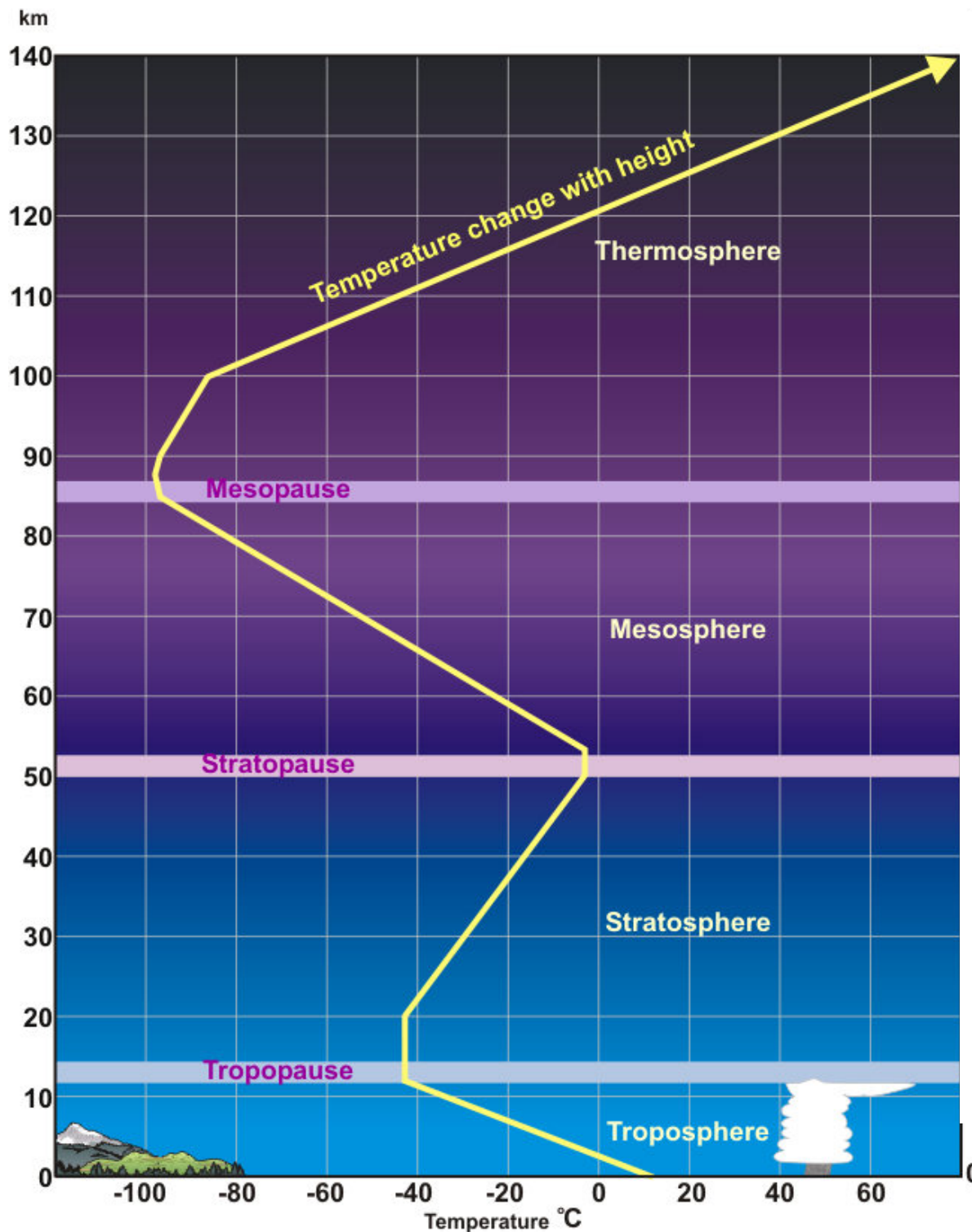
Atmospheric divisions

		Inter-	planetary	space	
100,000 km					
10,000					
1000					
100					
10					
0					
Quantity	T	Compositi on	Vertical transport	Grav. binding	Thermal plasma



Atmospheric divisions

		Inter-	planetary	space	
100,000 km	Thermosphere	H-sphere	Effusosphere	Exosphere	Plasma sphere
10,000		Hetero-sphere	Diffusosphere	Barosphere	
1000					
100	Mesosphere	Homosphere			F-region E-region D-region
10	Stratosphere				
0	Troposphere				
Quantity	T	Composition	Vertical transport	Grav. binding	Thermal plasma



<http://www.srh.noaa.gov/srh/jetstream/atmos/atmprofile.htm>

3 maxima:

- Earth surface,
- Stratopause,
- Thermopause

2 minima:

- Tropopause
- Mesopause

Troposphere:

- Sun absorption,
- IR radiation reabsorption
- 10 km deep, T_{down} : rad. cooling

Stratosphere:

- UV light absorption (100%) by ozone, O₃ T up: energy absorption

Mesosphere

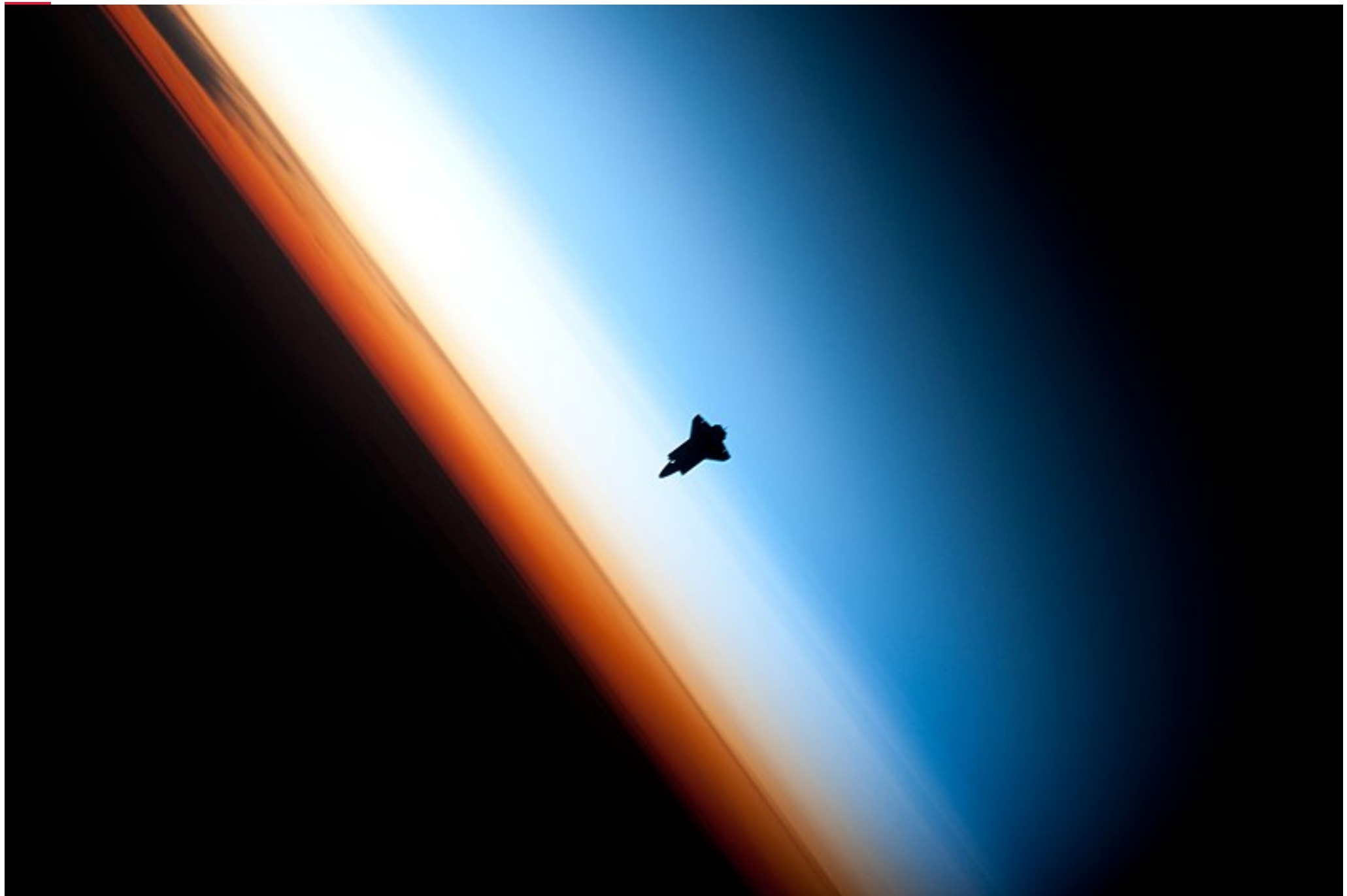
- Radiative cooling (CO₂ emission) outpaces solar absorption

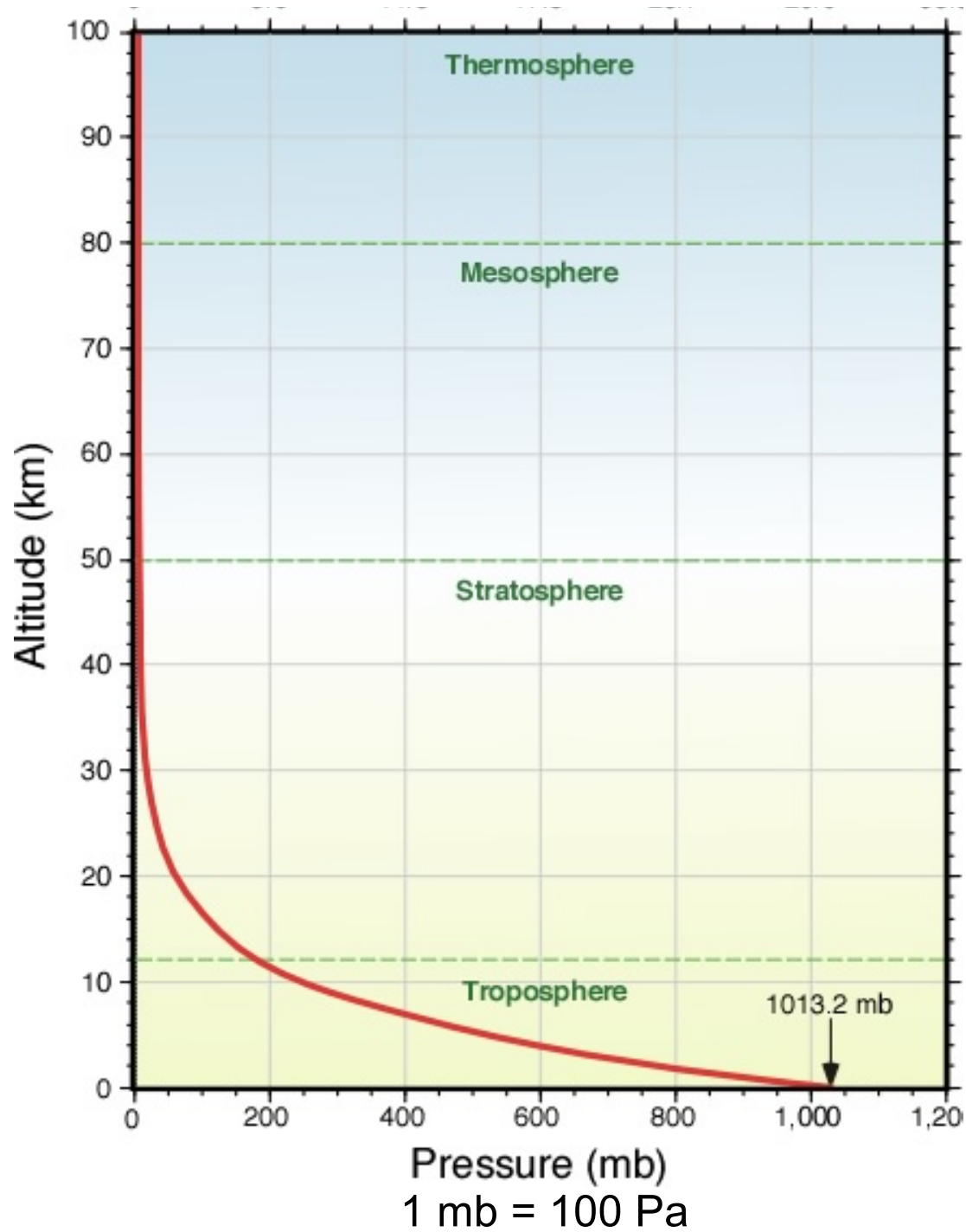
Thermosphere

- Rapid temperature increase from solar absorption, no effective heat loss

Thermopause

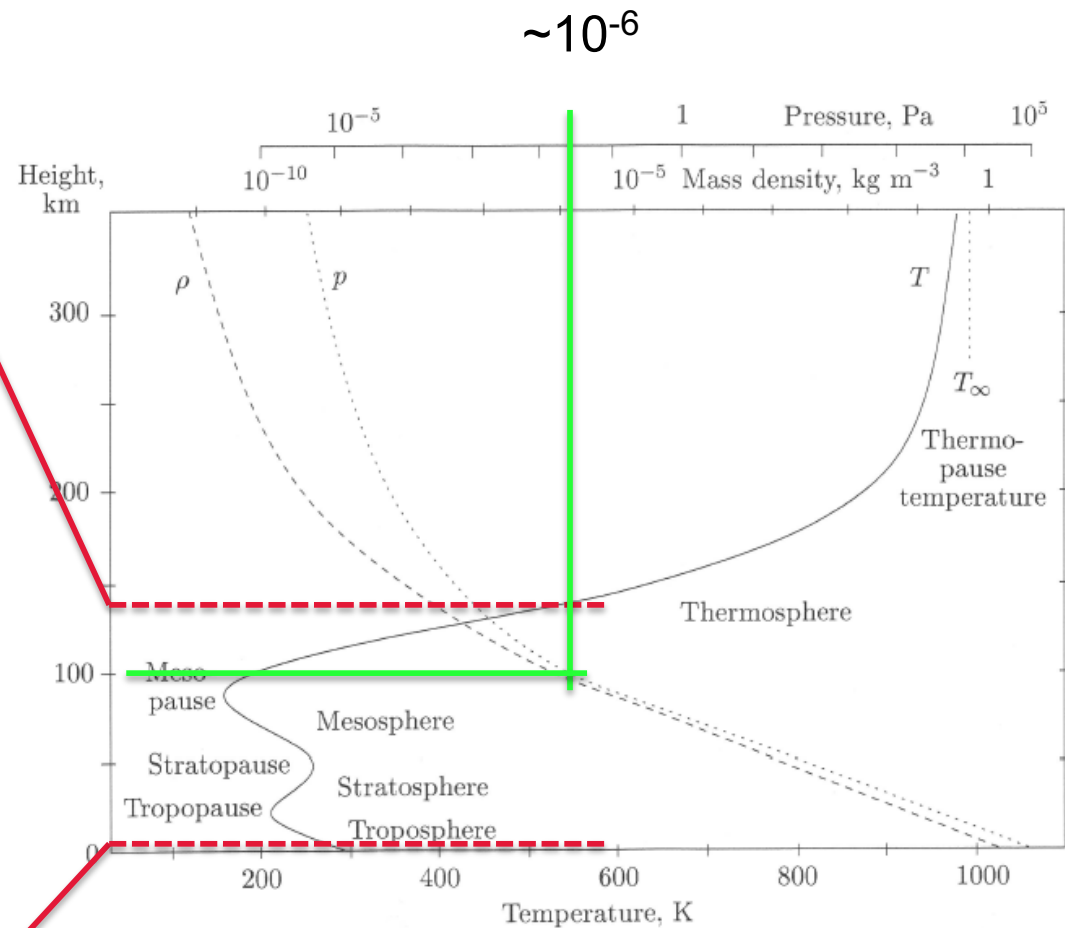
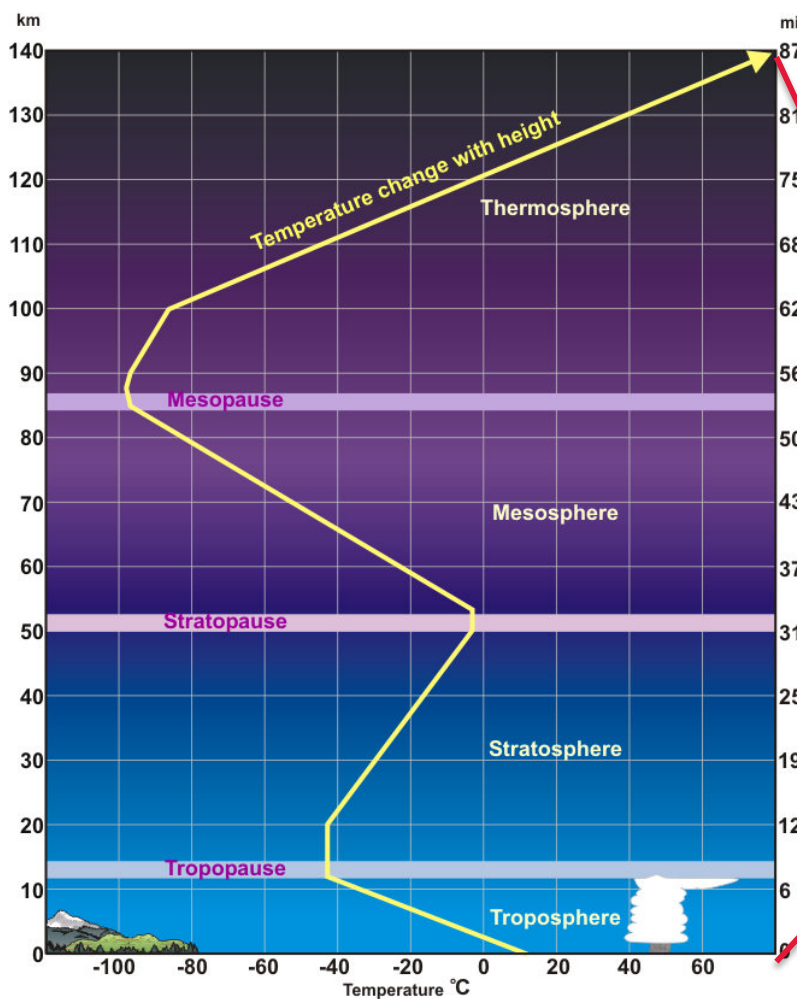
- Thermosphere asymptotically approaches thermopause temperature (nominally 1000 K – generally between 600 and 2500K)



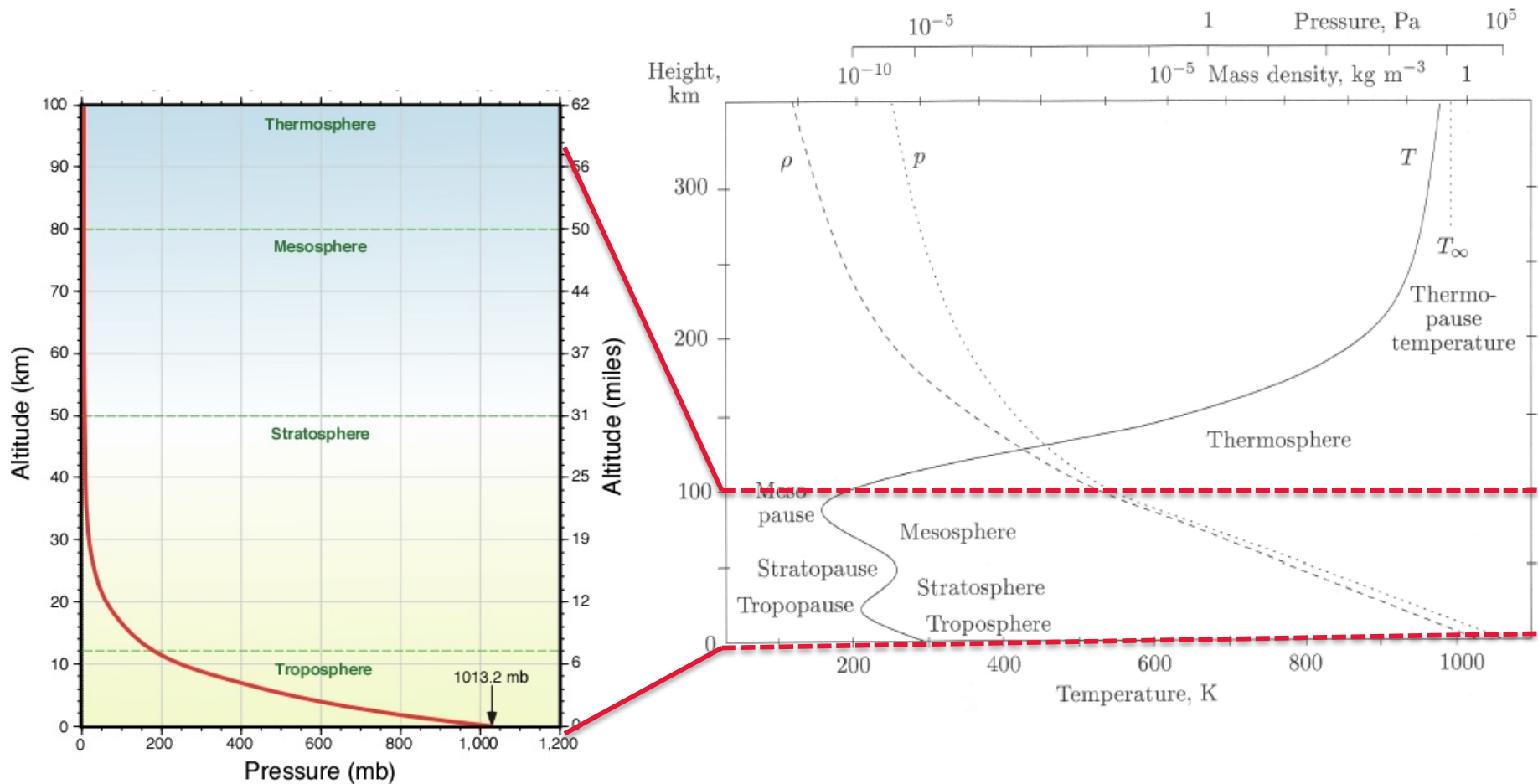


<http://www.physicalgeography.net/fundamentals/7d.html>

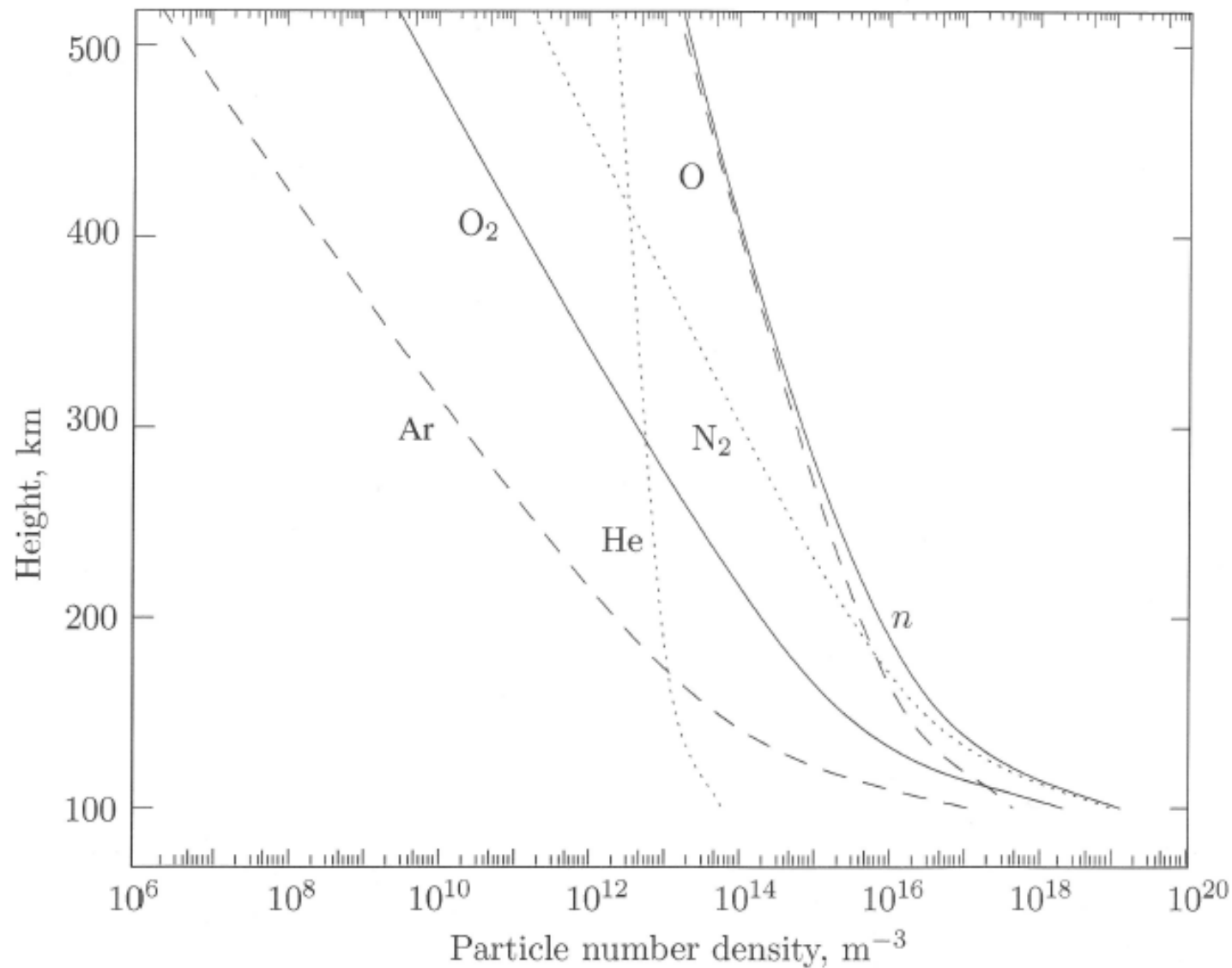
Temperature, pressure, mass density



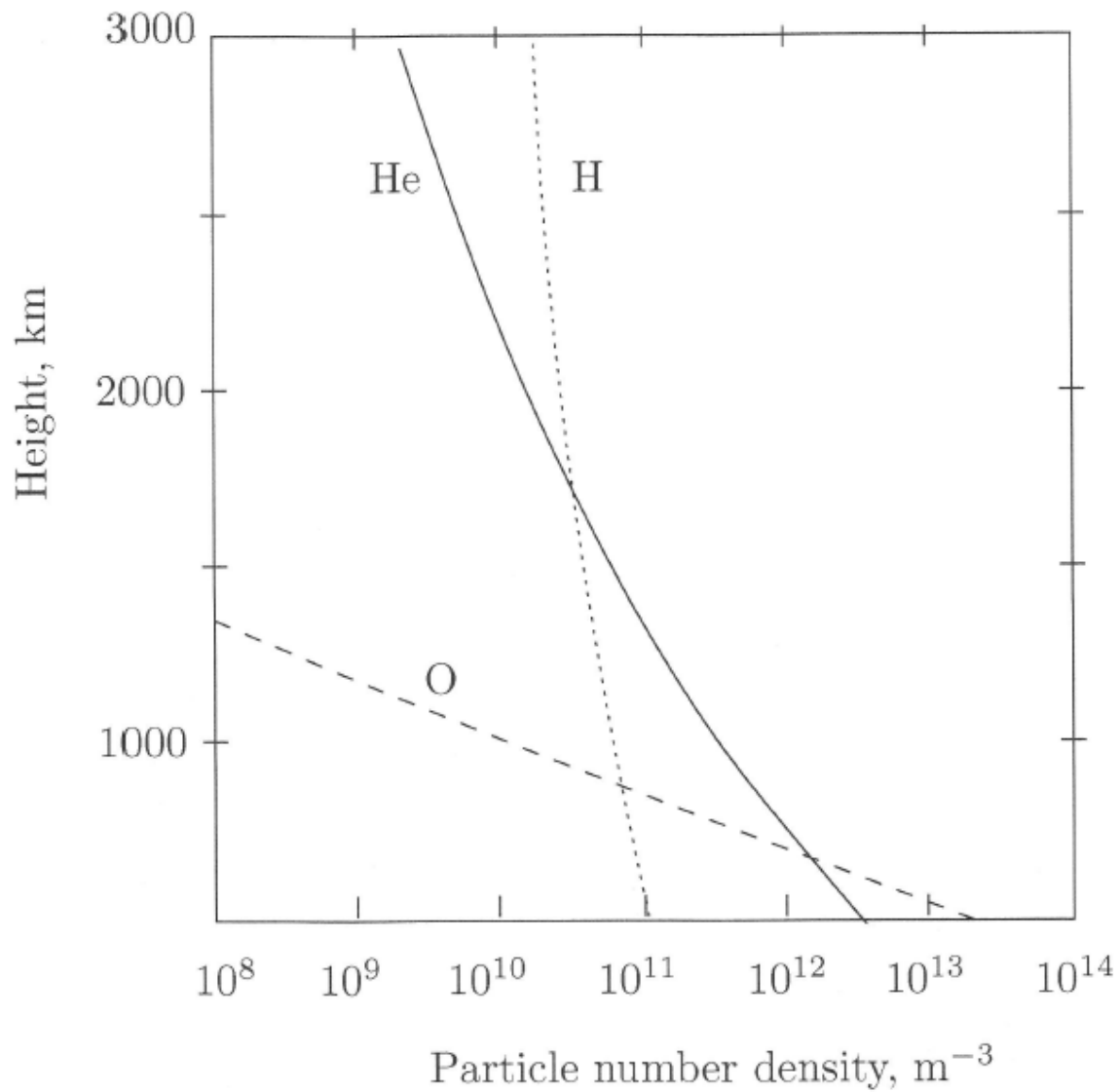
Temperature, pressure, mass density



Particle number density



Particle number density



Collision frequency, mean free path, mean mass number

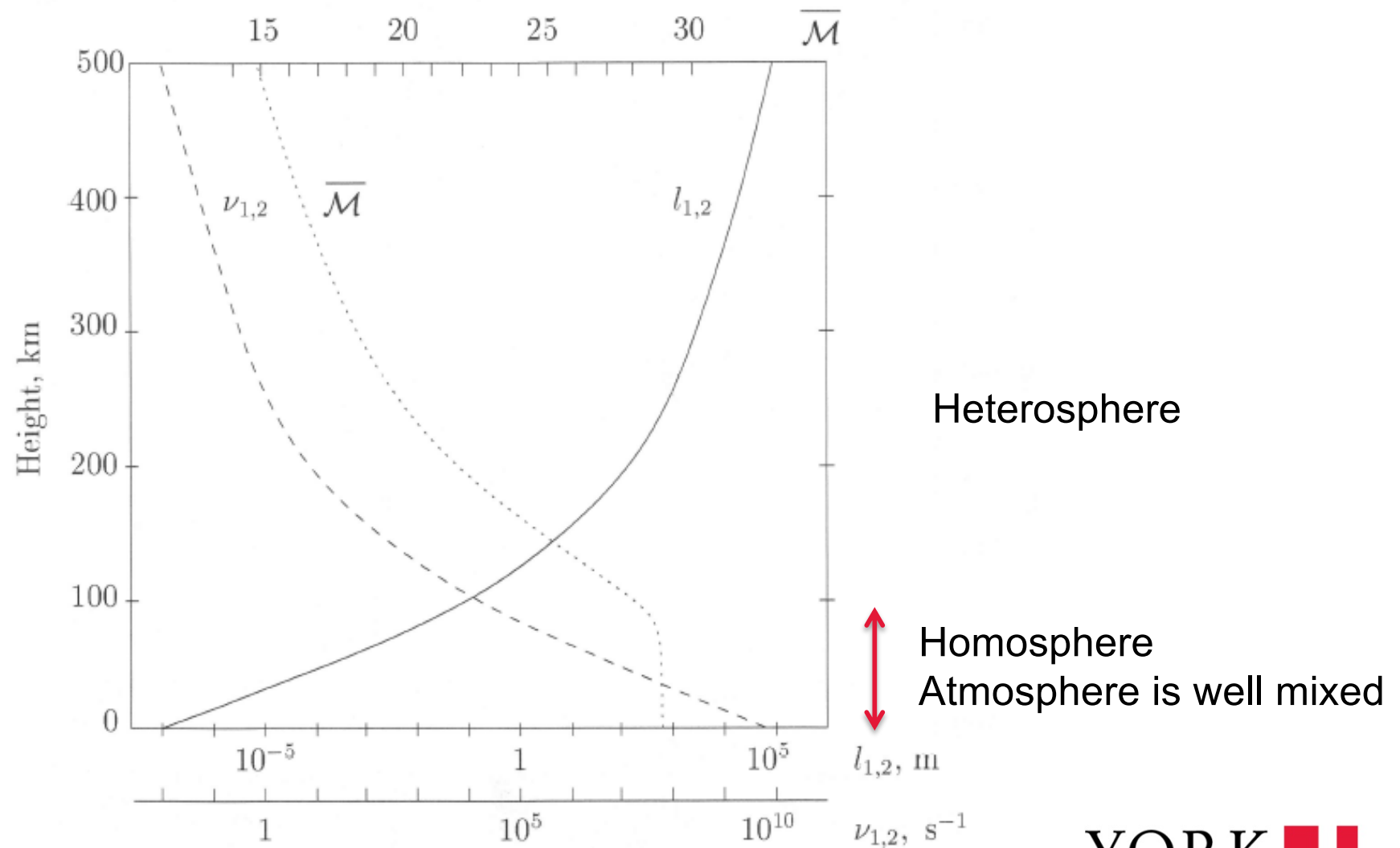


Fig. 2.10. Height profiles of the collision frequency ($\nu_{1,2}$), the mean free path ($l_{1,2}$) and the mean mass number ($\overline{\mathcal{M}}$) in the terrestrial atmosphere

Aerostatics

As opposed to *aerodynamics*, aerostatics deals with density allocation at a macroscopic level, e.g. air density variation within the atmosphere as a function of height.

It is assumed that the gases are not in motion with respect to the coordinate system they are considered within.

That leads us to the aerostatic equation.

Aerostatic equation

- We want to consider a massless membrane at height z and area A that experiences pressure from the gas below and the weight of the gas above.

Pressure force:

$$F_p(z) = Ap(z)$$

Weight of the gas:

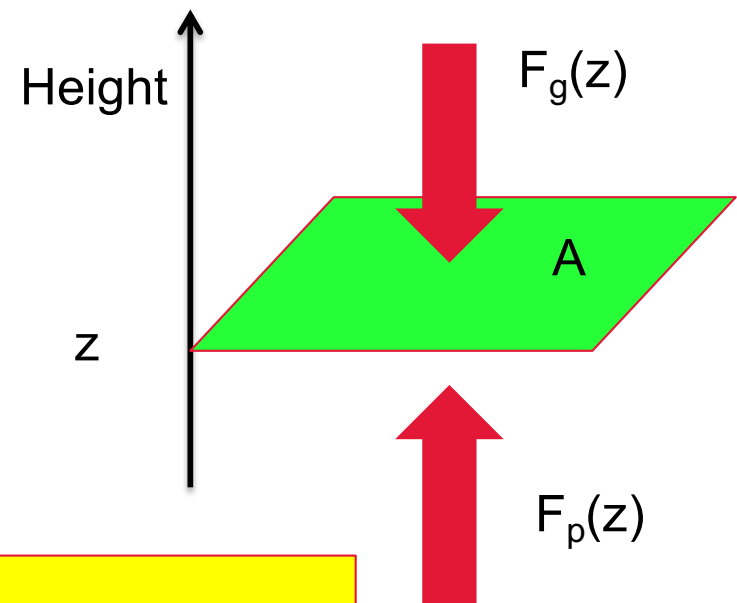
$$F_g(z) = A \int_z^{\infty} \rho(z')g(z')dz'$$

- For static equilibrium, $F_p = F_g$

$$p(z) = \int_{z'=z}^{\infty} \rho(z')g(z')dz'$$

$$\frac{dp}{dz} = -\rho(z)g(z)$$

Aerostatic equation



Example 2-5

You are a scuba diver and know that the pressure under water increases the deeper you dive. What is the pressure at a depth of 50 m? Assume that the change of the gravitational acceleration and the density of water with depth can be ignored. That is assume that $\rho = \text{const}$, $g = \text{const}$.

$$\int_{P_0}^{P_z} dP = -\rho g \int_0^z dz'$$

$$p_z - p_0 = -\rho g z$$


$$\begin{aligned} p_z - p_0 &= -1000 \times 9.81 \times (-50) \\ &= 490.5 \text{ kPa} \end{aligned}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$g = 9.81 \text{ m/s}^2$$

$$z = -50 \text{ m}$$

$$p_0 = 1 \text{ atm} = 101.3 \text{ kPa}$$


$$P_{z=50} = 490.5 + 101.3 = 591.8 \text{ kPa}$$

In general, however, ρ and g should be considered to depend on z .

The aerostatic equation describes the change of pressure, p , with height, z , as a function of mass density, ρ , and the gravitational acceleration g .

It ignores the centrifugal force resulting from the celestial object's rotation. This force is usually small.

It ignores any thermodynamic pressure at the top. But that is OK because in thermodynamic equilibrium it is maintained by the gravitational acceleration. For Earth, the gravitational acceleration is given as

$$g(z) \approx \frac{GM_E}{r^2} = \frac{g_0}{\left(1 + \frac{h}{R_E}\right)^2}$$

with $g_0 = 9.81 \text{ m s}^{-2}$
 $R_E = 6371.0 \text{ km}$
 $M_E = 5.97 \times 10^{24} \text{ kg}$

With M_E and R_E as the mass and radius of the Earth and g_0 the gravitational acceleration at the Earth surface.

For other celestial objects, M_E , R_E and g_0 need to be replaced with the corresponding values for that object.

Barometric law

- The aerostatic law gives a relation between pressure as a function of height and mass density as a function of height. With the ideal gas law we can express the law in terms of T as a function of height.

$$p = nkT$$

$$\rho = \bar{m} \frac{p}{kT}$$

$$\frac{dp}{dz} = -\frac{\bar{m}(z)g(z)}{kT(z)} p(z) = -\frac{p}{H}$$

$$\frac{dp}{dz} = -\frac{1}{H(z)} p(z)$$

$$\int_{p(h_0)}^{p(h)} \frac{dp}{p} = -\int_{h_0}^h \frac{\bar{m}(z)g(z)}{kT(z)} dz$$

with

$$H(z) = \frac{kT(z)}{\bar{m}(z)g(z)}$$

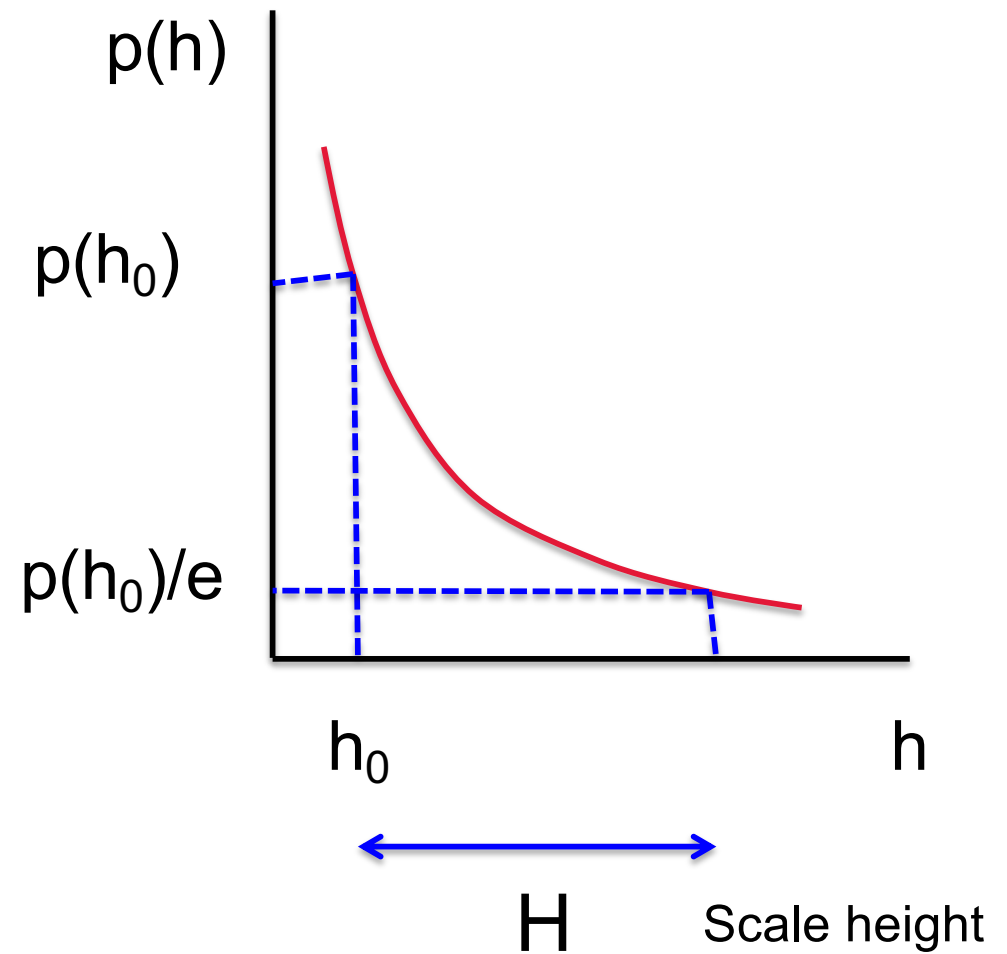
Pressure scale height

$$\ln \frac{p(h)}{p(h_0)} = -\int_{h_0}^h \frac{dz}{H(z)}$$

$$p(h) = p(h_0) \exp \left\{ -\int_{h_0}^h \frac{dz}{H(z)} \right\}$$

Barometric law for the vertical pressure profile in an atmosphere

- For $H(z) \approx \text{const}$



- We can also derive a similar expression for the number density profile:

$$n(h)kT(h) = n(h_0)kT(h_0)\exp\left\{-\int_{h_0}^h \frac{dz}{H(z)}\right\}$$

$$n(h) = n(h_0) \frac{T(h_0)}{T(h)} \exp\left\{-\int_{h_0}^h \frac{dz}{H(z)}\right\}$$

Barometric law for the vertical density profile in an atmosphere

or

$$n(h) = n(h_0) \exp\left\{-\int_{h_0}^h \frac{dz}{H_n(z)}\right\}$$

with

$$\frac{1}{H_n} = \left(\frac{1}{H} + \frac{1}{T} \frac{dT}{dH} \right)$$

- For the pressure or the density, the scale height describes a vertical distance over which the pressure or the density changes significantly, namely approximately by 1/e or e.

- Derivation for pundits:

Looking at the pressure law we have

$$\ln \frac{n(h)}{n(h_0)} = - \int_{h_0}^h \frac{dz}{H_n(z)}$$

$$\frac{dn}{n} = \frac{dz}{H_n}$$

$$H_n = \left(\left| \frac{dn}{dz} \right| \frac{1}{n} \right)^{-1}$$

$$\frac{dn}{dz} = -n(h_0)T(h_0)T(z)^{-2} \frac{dT}{dz} e^{-(z-h_0)/H} - n(h_0) \frac{T(h_0)}{T(z)} \frac{1}{H} e^{-(z-h_0)/H}$$

$$\frac{dn}{dz} \frac{1}{n} = \frac{1}{T} \frac{dT}{dh} + \frac{1}{H} = \frac{1}{H_n}$$

The pressure and density scale heights are not the same. But since particularly for heights > 200 km the temperature term is relatively small, $H \approx H_n$

Applications

- If we know the atmospheric characteristics of a planet at some height, we can predict the variation of these characteristics with height.

Example 2-6

Atomic Oxygen is the primary constituent in the thermosphere of Mars at heights above ca. 200 km. The gas temperature there has almost reached an equilibrium value of 200 K, which is approximately constant up to 1000 km.

Calculate the relative decrease in particle density in this region for an increase in altitude of 100km.

Assume the gravitational acceleration to be constant and equal to the value at 200 km

$$n(h) = n_{h_0} \frac{T(h_0)}{T(h)} e^{-(h-h_0)/H}$$

- We want the change in density over a height change of 100 km
- We need to determine the scale height H for the Mars atmosphere in this region

$$H(z) = \frac{kT(z)}{\overline{m}(z)g(z)}$$

$$g = G \frac{M}{r^2}$$

$$g_0 = G \frac{M_{mars}}{R_{mars}^2}$$

$$g(z) = \frac{g_0}{\left(1 + \frac{h}{R_{mars}}\right)^2}$$

$$g_0 = 6.67 * 10^{-11} \frac{6.42 * 10^{23}}{(3.397 * 10^6)^2} = 3.71 ms^{-2}$$

$$g(200km) = \frac{3.71}{\left(1 + \frac{200}{3397}\right)^2} = 3.31 ms^{-2}$$

$$H(h) = \frac{kT(h)}{\bar{m}(h)g(h)} = \frac{1.38 * 10^{-23} * 200}{16 * 1.66 * 10^{-27} * 3.31} = 31.5 km$$

So the scale height at 200 km is 31.5 km.

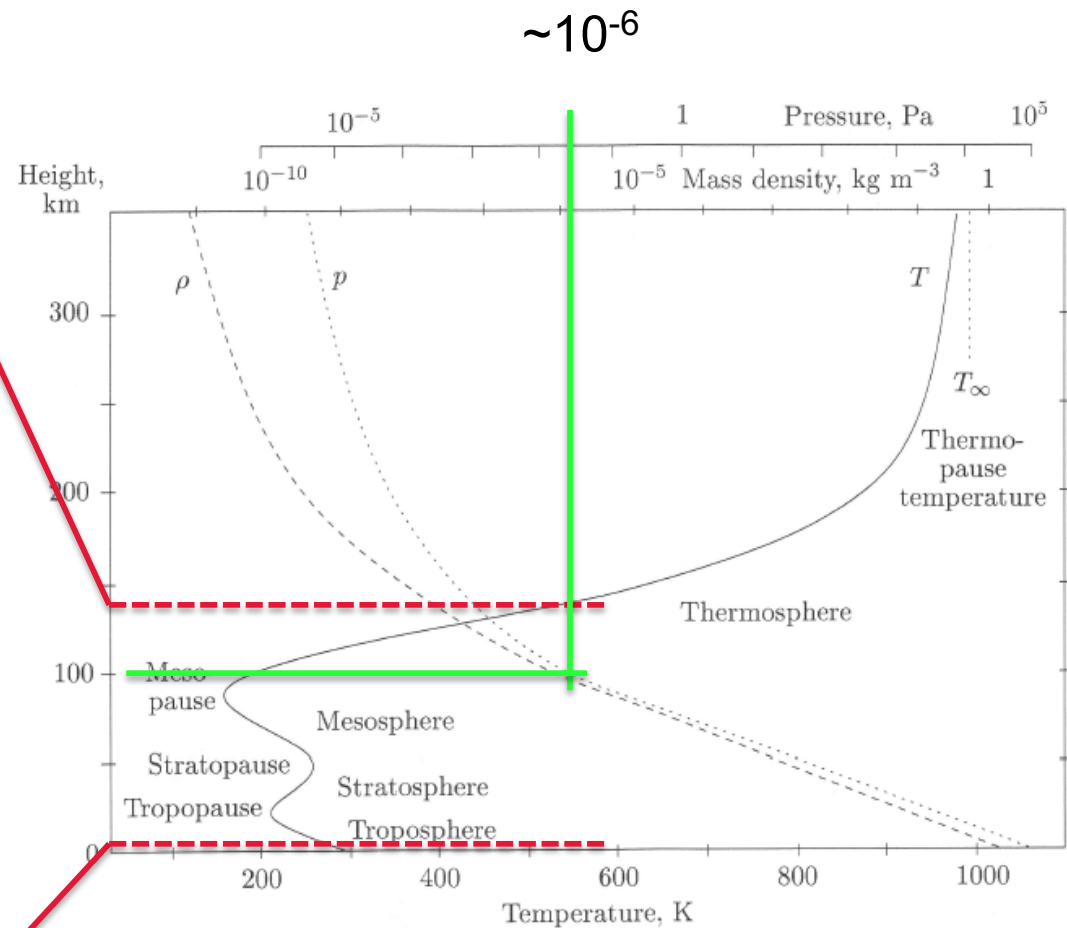
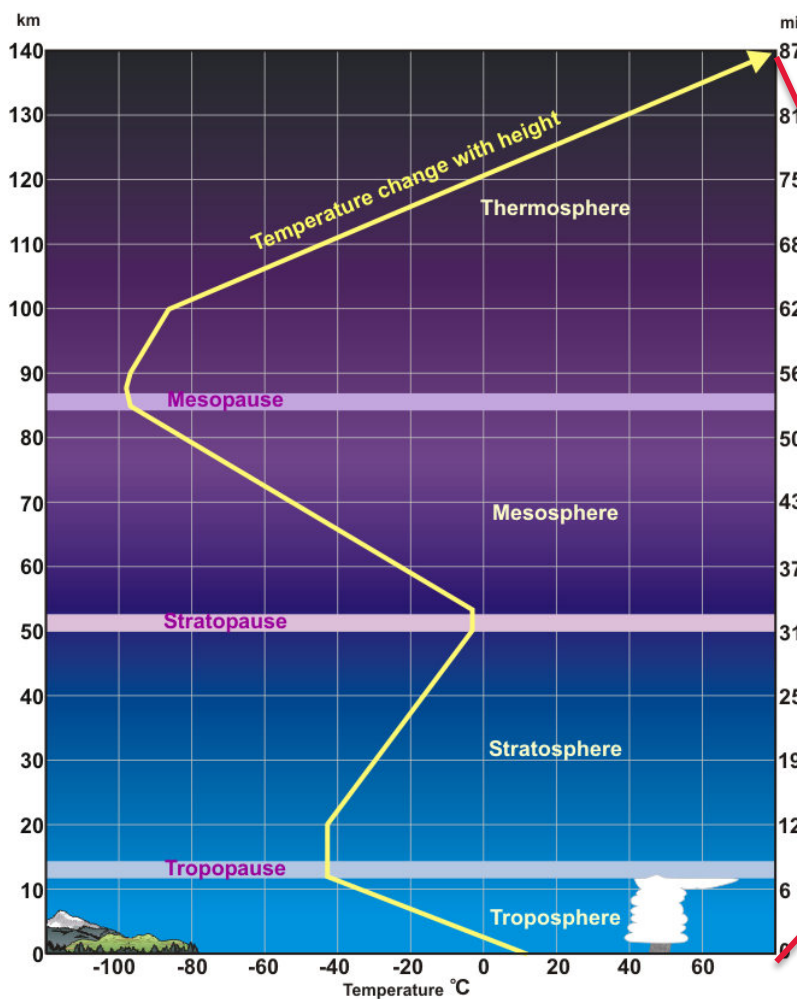
- At that height we are near the thermopause for Mars and we will assume that the temperature remains constant over the next 100 km.

$$\frac{n(h)}{n(h_0)} = \exp\left\{-\frac{100}{31.5}\right\} = 0.042$$

- From this follows that the number density at 300 km is 4.2% of the number density at 200 km.

Now back to the atmospheric divisions.

Temperature, pressure, mass density



Atmospheric divisions

		Inter-	planetary	space		
100,000 km	Thermosphere	H-sphere	Effusosphere	Exosphere	Plasma sphere	
10,000		Hetero- sphere	Diffusosphere Turbosphere	Barosphere		F-region E-region D-region
1000						
100	Mesosphere	Homosphere				
10	Stratosphere					
0	Troposphere					
Quantity	T	Composition	Vertical transport	Grav. binding	Thermal plasma	

Composition

- The **homosphere** lies between the Earth's surface and the heterosphere. Gases are nearly uniformly mixed through this layer even though density decreases with height above the surface. The only exceptions is the "ozone layer" from 19 to 50 km and near surface variations in water vapor, carbon dioxide and air pollutants.
- The **heterosphere** is the outer-most sphere where gases are distributed in distinct layers by gravity according to their atomic weight. Extending from an altitude of 80-100 km, the lightest elements (hydrogen and helium) are found at the outer margins of the atmosphere. The heavier elements (nitrogen and oxygen) are found at the base of the layer.
- The barometric law applies whether we are in an atmosphere composed of a single gas, a well-mixed homosphere or a gravitationally-separated heterosphere

Vertical transport

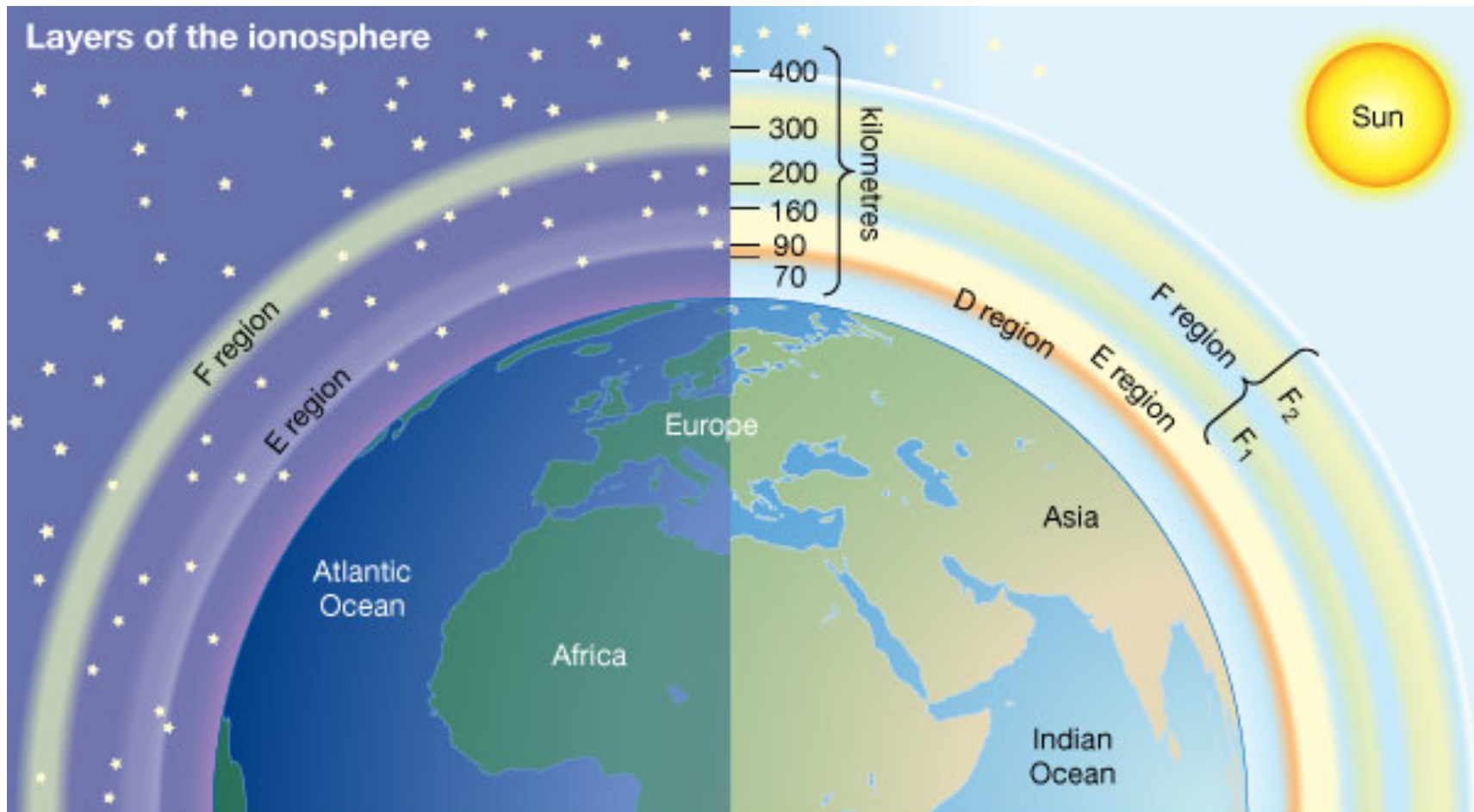
- The turbosphere, 0 to 100 km, is the region where convection effectively mixes atmospheric constituents.
- Above the turbosphere lies the diffusosphere, where diffusion is more important than turbulent mixing in distributing the constituents.
- Finally there is the effusosphere, where the atmosphere is no longer well bound to Earth and can escape under the right conditions

Gravitational binding

- The barosphere is the region of the atmosphere which is well bound to the Earth through gravitational attraction.
- The exosphere starts at about 600 km, where particle concentration is about 10^7 cm^{-3} , to 5000 km, where the concentration is 10^2 cm^{-3}
- The exosphere is the outer limit of the Earth's neutral atmosphere.
- In this region a particle travelling away from the Earth averages only one collision and pursues essentially a ballistic trajectory in the Earth's gravitational field. Particles with sufficient velocity and no electrical charge can escape from the atmosphere.

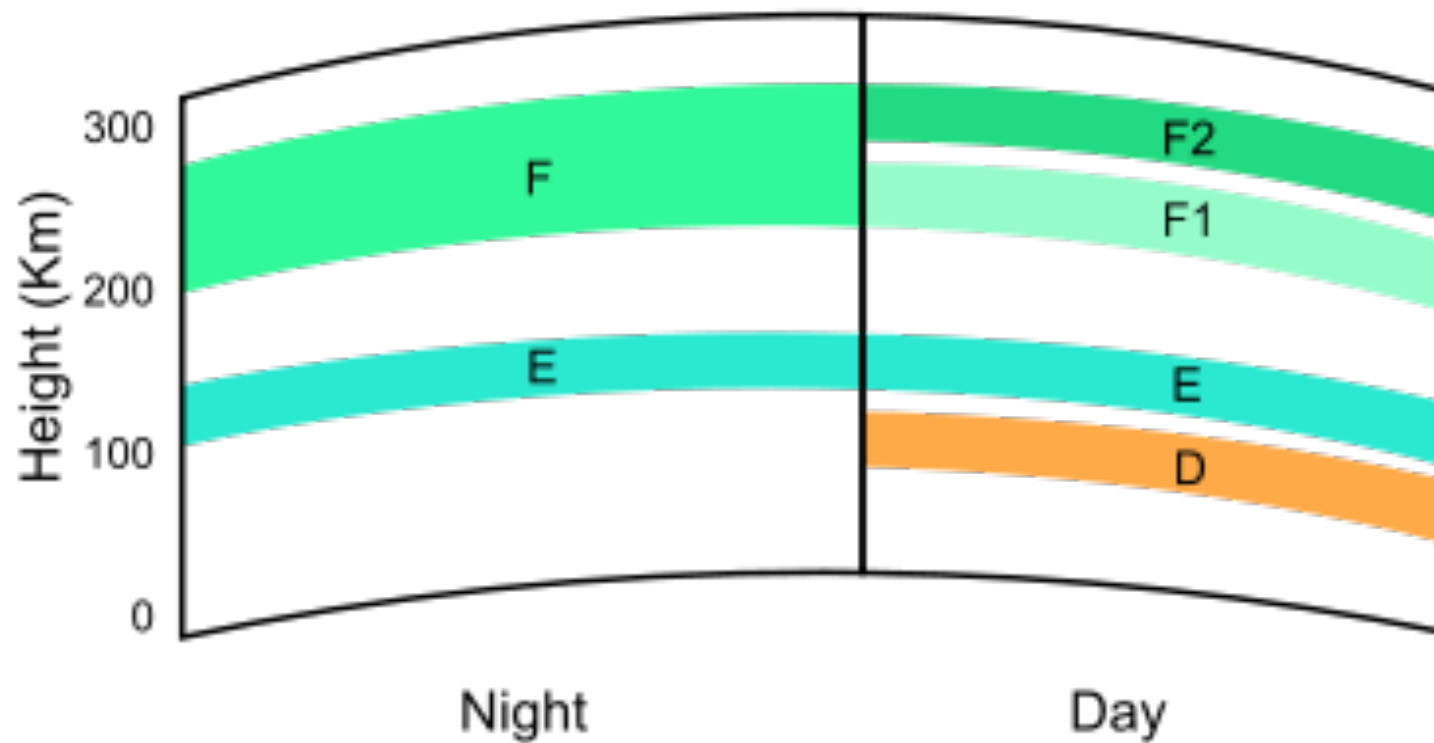
Thermal plasma - Ionosphere

- The ionosphere extends from about 60 km to 1,000 km and includes the thermosphere and parts of the mesosphere and exosphere.
- It is ionized by solar radiation, plays an important part in atmospheric electricity and forms the inner edge of the magnetosphere.
- It has practical importance because, among other functions, it influences radio propagation to distant places on the Earth.



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Ionospheric layers



- At night the F layer is the only layer of significant ionization present, while the ionization in the E and D layers is extremely low. During the day, the D and E layers become much more heavily ionized, as does the F layer, which develops an additional, weaker region of ionisation known as the F₁ layer. The F₂ layer persists by day and night and is the region mainly responsible for the refraction of radio waves.

D, E, F layers

D layer

- The D layer is 60 to 90 km above the surface of the Earth. There are many more neutral air molecules than ions in this layer. Medium frequency (MF) and lower high frequency (HF) radio waves are significantly damped within the D layer, as the passing radio waves cause electrons to move, which then collide with the neutral molecules, giving up their energy. The lower frequencies move the electrons farther, with a greater chance of collisions. This is the main reason for absorption of HF radio waves, particularly at 10 MHz and below, with progressively smaller absorption as the frequency gets higher. This effect peaks around noon and is reduced at night due to a decrease in the D layer's thickness; only a small part remains due to cosmic rays. A common example of the D layer in action is the disappearance of distant AM broadcast band stations in the daytime.

E layer

- The E layer is 90 to 150 km above the surface of the Earth. Ionization is due to soft X-ray (1–10 nm) and far ultraviolet (UV) solar radiation ionization of molecular oxygen (O_2). Normally this layer can only reflect radio waves having frequencies lower than about 10 MHz and may contribute a bit to absorption on frequencies above. At night the E layer weakens because the primary source of ionization is no longer present. After sunset an increase in the height of the E layer maximum increases the range to which radio waves can travel by reflection from the layer.
- (560 mi) to 2,500 km (1,600 mi). Double-hop reception over 3,500 km (2,200 mi) is possible.

F layer

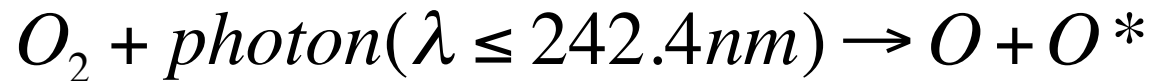
- The F layer extends from about 150 to more than 500 km above the surface of Earth. It is the densest point of the ionosphere, which implies signals penetrating this layer will escape into space. At higher altitudes, the number of oxygen ions decreases and lighter ions such as hydrogen and helium become dominant. The F layer consists of one layer at night, but during the day, a deformation often forms in the profile that is labeled F_1 . The F_2 layer remains by day and night responsible for most skywave propagation of radio waves, facilitating high frequency (HF, or shortwave) radio communications over long distances.

Atmospheric division (with more detail)

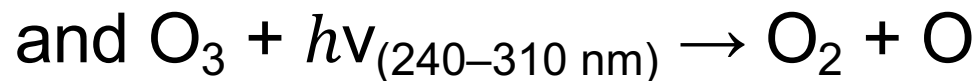
Height /km	Temperature	Composition	Vertical transport	Gravitational binding	Thermal plasma	
100,000	Thermosphere	Hydrosphere (Geocorona)	Effusosphere	Exosphere		
10,000					Heterosphere	Exobase
1,000						
		Diffusosphere		F-region		
				Mesopause	E-region	
100	Mesosphere	Homopause	Turbopause	Barosphere	D-region	
	Stratopause	Homosphere	Turbosphere			
	Stratosphere					
10	Tropopause					
0	Troposphere					

Atomic Oxygen

- Atomic oxygen does not occur in the lower atmosphere but it is abundant in the thermosphere. It is produced through *photodissociation*.



- higher frequency UV light (top end of UV-B, UV-C and above)
- Each oxygen atom then quickly combines with an oxygen molecule to form an ozone molecule:



UV light between UV-B and UV-C

- This cycle converts UV-B light into heat, helping prevent UV-B from penetrating the lower layers of the atmosphere.

Removal

- $\text{O}_3 + \text{O}\cdot \rightarrow 2 \text{O}_2$
- $2 \text{O}\cdot \rightarrow \text{O}_2$

Stratospheric ozone is then determined as a balance of production by solar radiation and removal.

The removal rate is slow due to a low concentration of O atoms

Some free radicals: Hydroxyl (OH), Nitric Oxide (NO) and Chlorine and Bromine atoms (Cl, Br) catalyze the recombination reaction, so remove ozone much more quickly than recombination alone

OH and NO are mostly naturally occurring, but Cl, Br generally a result of CFC emissions.

Escape velocity

- In order to escape Earth, a particle with mass, m , must be moving with sufficiently high velocity to overcome earth's gravity. The velocity that is just sufficiently large to do that is the escape velocity, v_{esc} .
- The escape velocity can be computed from the kinetic energy of the particle being equal to the work required to overcome the gravitational field along the trajectory

$$\frac{1}{2}mv_{esc}^2 = \int_r^{\infty} mg(r') dr' = \int_r^{\infty} mGM_E r'^{-2} dr' = -mGM_E r'^{-1} \Big|_r^{\infty} = mGM_E r^{-1} = mg(r)r$$

where $r = R_E + h$, is the distance from Earth's centre, $R_E = 6371$ km, is the mean radius of Earth and h is the height above mean sea level, and $g(r) \approx GM_E/r^2$, the gravitational acceleration.

$$v_{esc} = \sqrt{2g(r)r} = \frac{v_{esc}(h=0)}{\sqrt{1 + \frac{h}{R_E}}}$$

Escape velocity

Heterosphere

What do we know about the heterosphere?

What do we know about the heterosphere?

- Above the homosphere
- Starts around 80-100 km
- Chemical composition no longer uniform – more dense constituents lower in heterosphere than less dense ones
- Leads to the Hydrosphere (Geocorona)

Atmospheric division (with more detail)

Height /km	Temperature	Composition	Vertical transport	Gravitational binding	Thermal plasma
100,000	Thermosphere	Hydrosphere (Geocorona)	Effusosphere	Exosphere	
10,000		Heterosphere			Exobase
1,000			Diffusosphere	Plasmasphere	
				Barosphere	F-region
		Mesopause	E-region		
100	Mesosphere	Homopause	Turbopause		D-region
	Stratopause	Homosphere			Turbosphere
	Stratosphere				
10	Tropopause				
0	Troposphere				

Heterosphere

- Heterospheric density distributions
- Gas Kinetic Interpretation of the Barometric Law
- Transition from Homosphere to Heterosphere

Collision frequency, mean free path, mean mass number

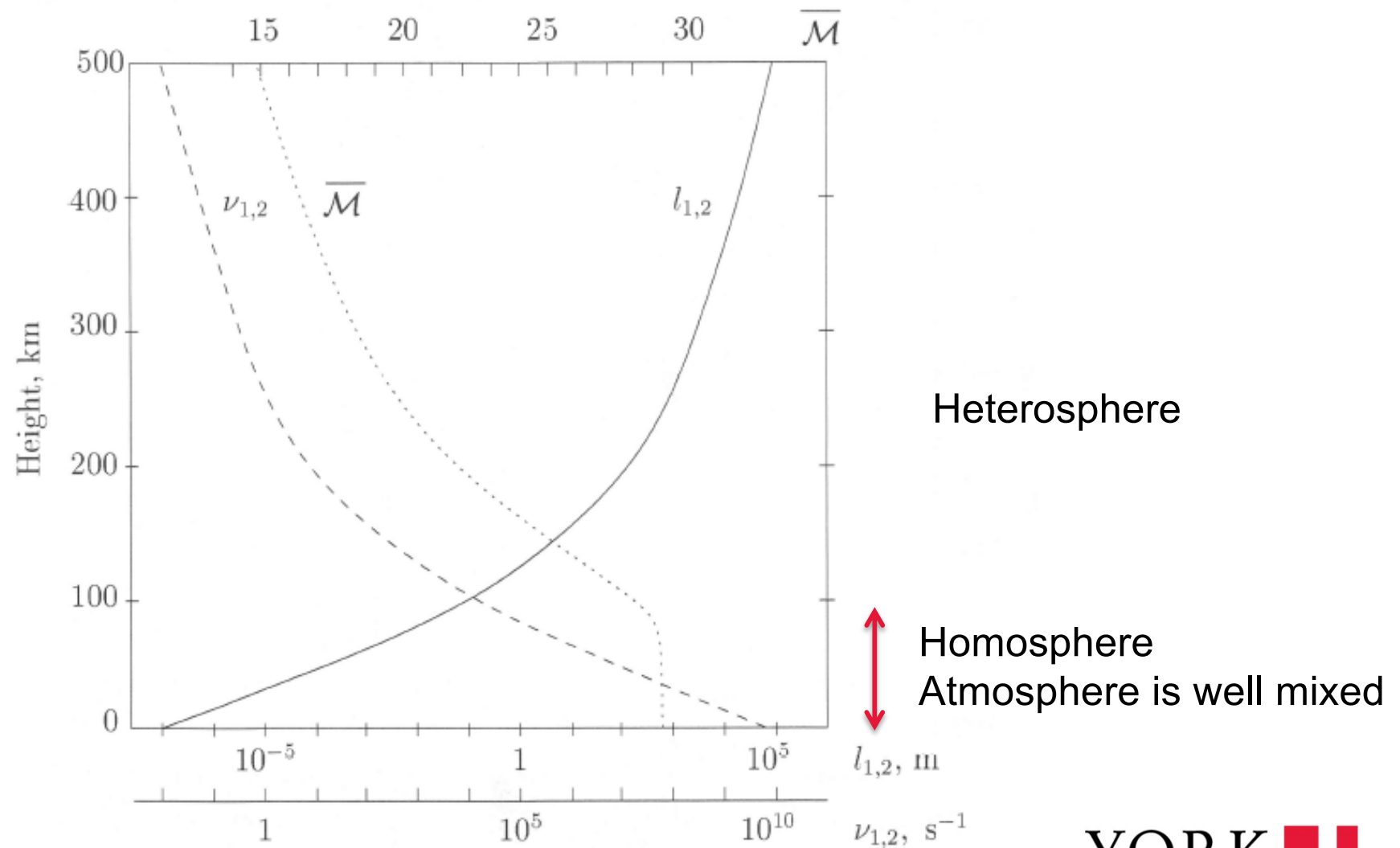


Fig. 2.10. Height profiles of the collision frequency ($\nu_{1,2}$), the mean free path ($l_{1,2}$) and the mean mass number ($\overline{\mathcal{M}}$) in the terrestrial atmosphere

Heterospheric density distributions

- The barometric law, as we derived it, can only be explicitly evaluated properly for a single gas or a well-mixed gas as in the homosphere where the mean particle mass, $\bar{m}(h)$ is known.
- In the heterosphere $\bar{m}(h)$ is not easily known, since it depends on the density profiles (density as a function of height) of the individual gases. For instance, the gases, O, O₂ and N₂ have different density profiles, and therefore the mean particle mass changes with height.
- If all the gases, i , are in thermodynamic equilibrium, we can use the Dalton-Stefan Law: the total pressure of a gas mixture is the sum of the partial pressures.

$$\sum_i p_i = \sum_i n_i kT = nkT = p$$

- Substituting into the aerostatic equation

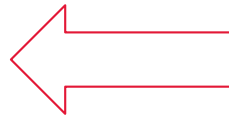
Aerostatic equation

$$p(z) = \int_{z'=z}^{\infty} \rho(z')g(z')dz'$$

$$\frac{dp}{dz} = -\rho(z)g(z)$$

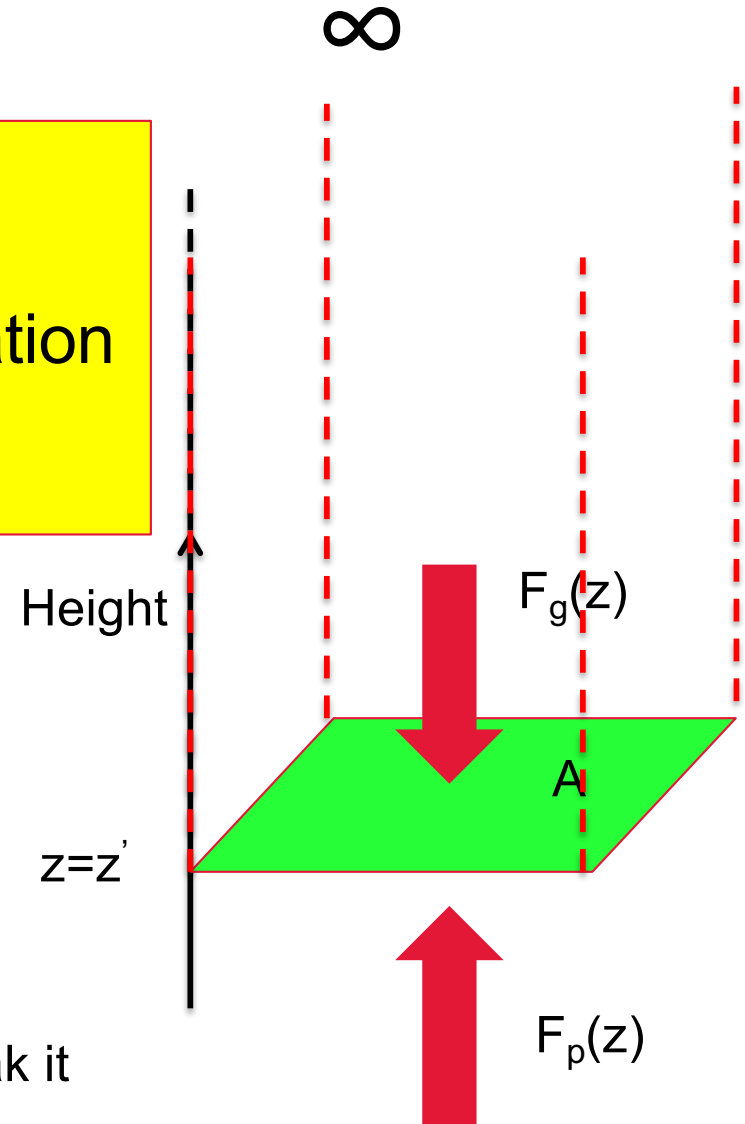
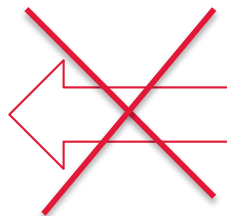
Aerostatic equation

$$\sum_i \frac{dp_i}{dz} = -\sum_i n_i m_i g$$



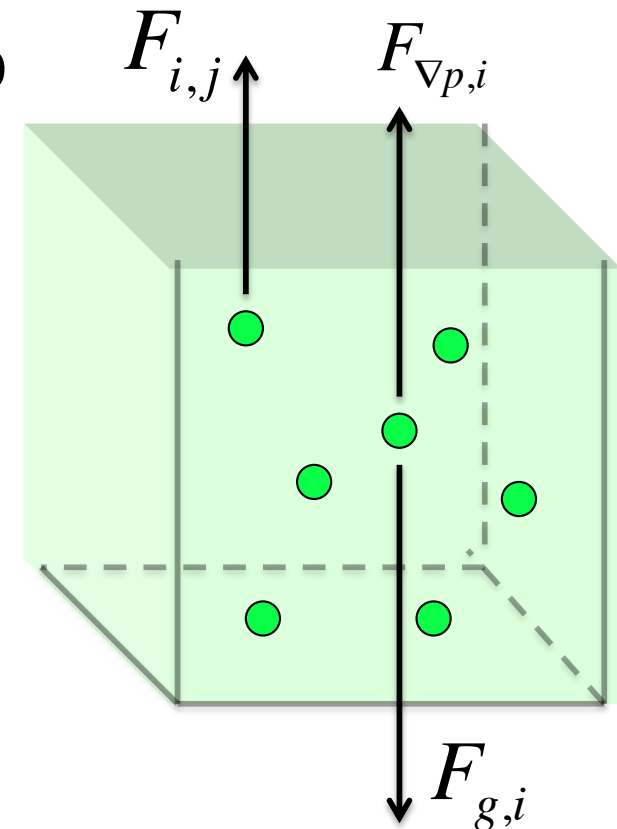
However, this equation does not allow to break it down for individual gases so that

$$\frac{dp_i}{dz} = -n_i m_i g$$



Let us consider a volume element with single gas species, i , in a mix of other gas species, j

- The gas species i is subject to
 - gravitational force,
 - pressure gradient force,
 - force from species j (and any other species) on species i ,
- The last of these could be electrostatic or frictional
- Frictional forces are due to collisions between species.
- The force from particles of species 2 acting on particles of species 1, is

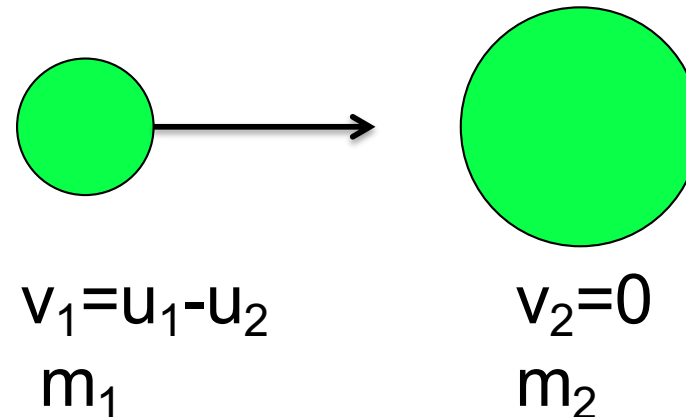


$$\vec{F}_{1,2} = \vec{F}_{fr,1} = \frac{\sum \Delta \vec{I}_1}{\Delta t} = \langle \Delta \vec{I}_1 \rangle \nu_{1,2}$$

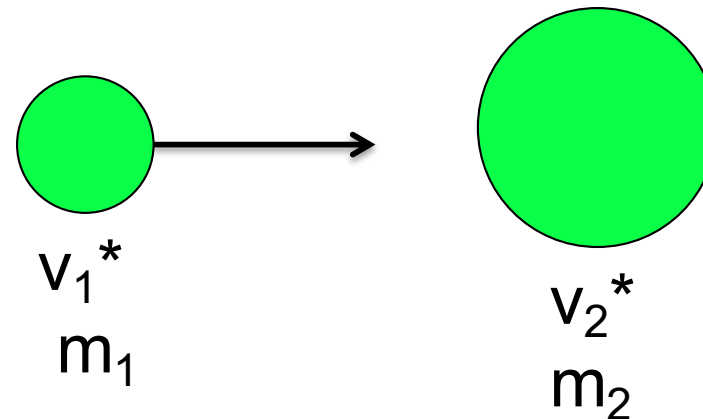
Mean change of momentum
times collision frequency

- Consider two particles of masses m_1 and m_2 in a head-on collision

Before collision



After collision



$$m_1 v_1 = m_1 v_1^* + m_2 v_2^*$$

$$m_1 v_1^2 = m_1 (v_1^*)^2 + m_2 (v_2^*)^2$$

Momentum and

energy conservation

- The combination yields

$$v_1^* = v_1 \frac{m_1 - m_2}{m_1 + m_2}$$

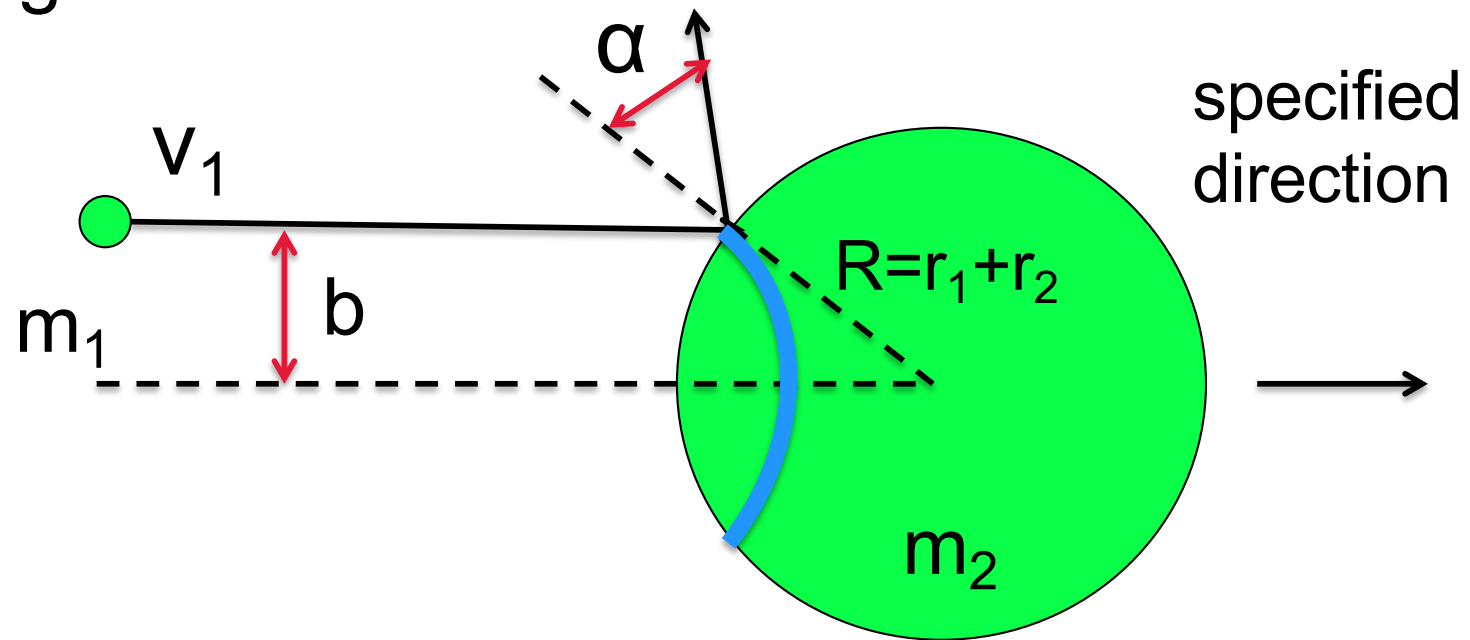
- The change in momentum of particle of type 1 gas is then

$$\Delta I_1 = (v_1^* - v_1)m_1 = v_1 m_1 \left(\frac{m_1 - m_2}{m_1 + m_2} - 1 \right) = v_1 \left(\frac{-2m_1 m_2}{m_1 + m_2} \right)$$

$$v_1 = u_1 - u_2$$

with $v_1 = u_1 - u_2$ as the difference of the average flow velocities.

- For the collision with arbitrary impact parameter, b , the determination of the mean change of momentum goes as follows:



- We again reduce the radius of particle 1 to near zero and that of particle 2 to $r_1 + r_2$. Particle 1 hits particle 2 with an impact parameter b and an impact angle α .

- The collision induced momentum change is reduced by the factor $\cos\alpha$ once because of the angle of the impact and again because we only consider the transfer of impact in one direction, for instance the height.

$$\Delta I_1(b) = \Delta I_1(b=0) \cos^2 \alpha$$

- For the mean change of momentum we have to integrate over the possible impact parameters

$$\langle \Delta I_1 \rangle = \int_{b=0}^R \Delta I_1(b=0) \cos^2 \alpha \frac{2\pi b db}{R^2 \pi}$$

- And get

$$\langle \Delta I_1 \rangle = \Delta I_1(b=0) \frac{2}{R^4} \int_{b=0}^R (R^2 - b^2) b db = \frac{\Delta I_1(b=0)}{2}$$

$$b = R \sin \alpha$$

$$\sin^2 \alpha = \frac{b^2}{R^2}$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$\cos^2 \alpha = 1 - \frac{b^2}{R^2} = \frac{R^2 - b^2}{R^2}$$



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Fraction of particles for which the change of momentum is just $\Delta I_1(b=0)$

- And get

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- The frictional force that a particle of gas type 1 experiences due to collision with a particle of gas type 2 is then

$$F_{fr,1} = \frac{\Delta I_1(b=0)}{2} v_{1,2}$$

- For an ensemble of particles of type 1 gas the frictional force per unit volume is

$$F_{fr,1}^* = n_1 F_{fr,1}$$

remember

$$F_{fr,1}^* = \frac{4}{3} n_1 \left(\frac{m_1 m_2}{m_1 + m_2} \right) v_{1,2} (u_2 - u_1) = m_1 n_1 v_{1,2}^* (u_2 - u_1)$$

$$v_{1,2} = \sigma_{1,2} n_2 \sqrt{\frac{8kT_{1,2}}{\pi m_{1,2}}}$$

Factor $\frac{3}{4}$ comes from more detailed computations

- And for three dimensions:

$$v_{1,2}^* = \frac{4}{3} \frac{m_2}{m_1 + m_2} v_{1,2}$$

$$\vec{F}_{fr,1}^* = m_1 n_1 v_{1,2}^* (\vec{u}_2 - \vec{u}_1)$$

- Frictional force is proportional to flow velocity differences.
- For stationary gases, $\vec{F}_{i,j} = \vec{F}_{fr,1} = 0$.
- For stationary gases each gas behaves as if it were alone in the mix.
- For stationary gases in a mixture the aerostatic equation for a single gas is the same as that for the gas mixture.
- We can then really write:

$$\frac{dp_i}{dz} = -n_i m_i g$$

- The density of a gas in this region under the assumption of no flow difference is then similar to what we derived before.

$$n_i(h) = n_i(h_0) \frac{T(h_0)}{T(h)} \exp \left\{ - \int_{h_0}^h \frac{dz}{H_i(z)} \right\}$$

Barometric law for a single heterospheric gas i in thermodynamic equilibrium with the mixture of gases ($T_i=T$).

$$H_i(h) = \frac{kT(h)}{m_i g(h)}$$

With the assumption of T and g constant, then

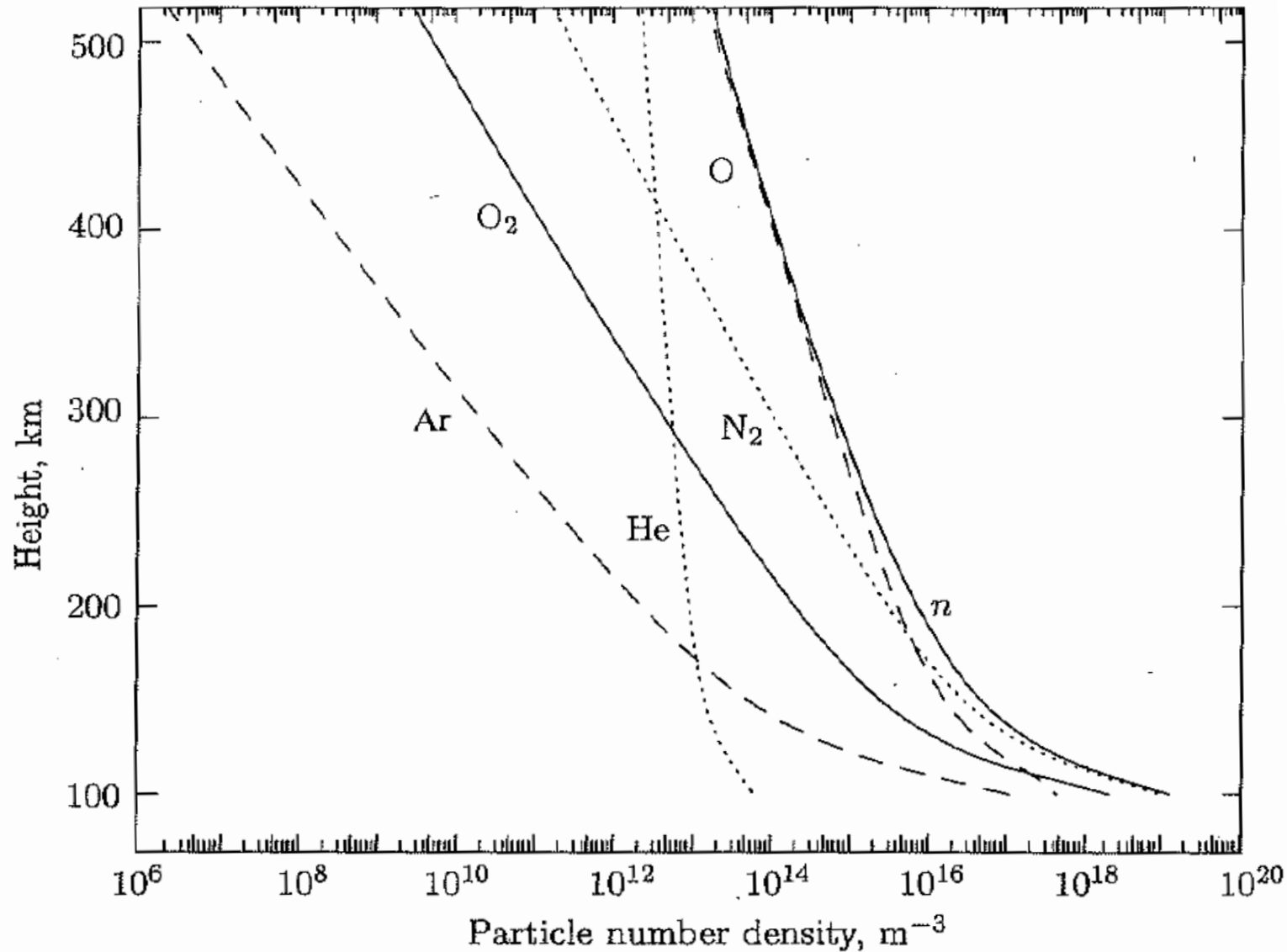
$$n_i(h) = n_i(h_0) e^{-\frac{h-h_0}{H_i}}$$

Lighter molecules have larger scale heights than heavier molecules.

In-class project

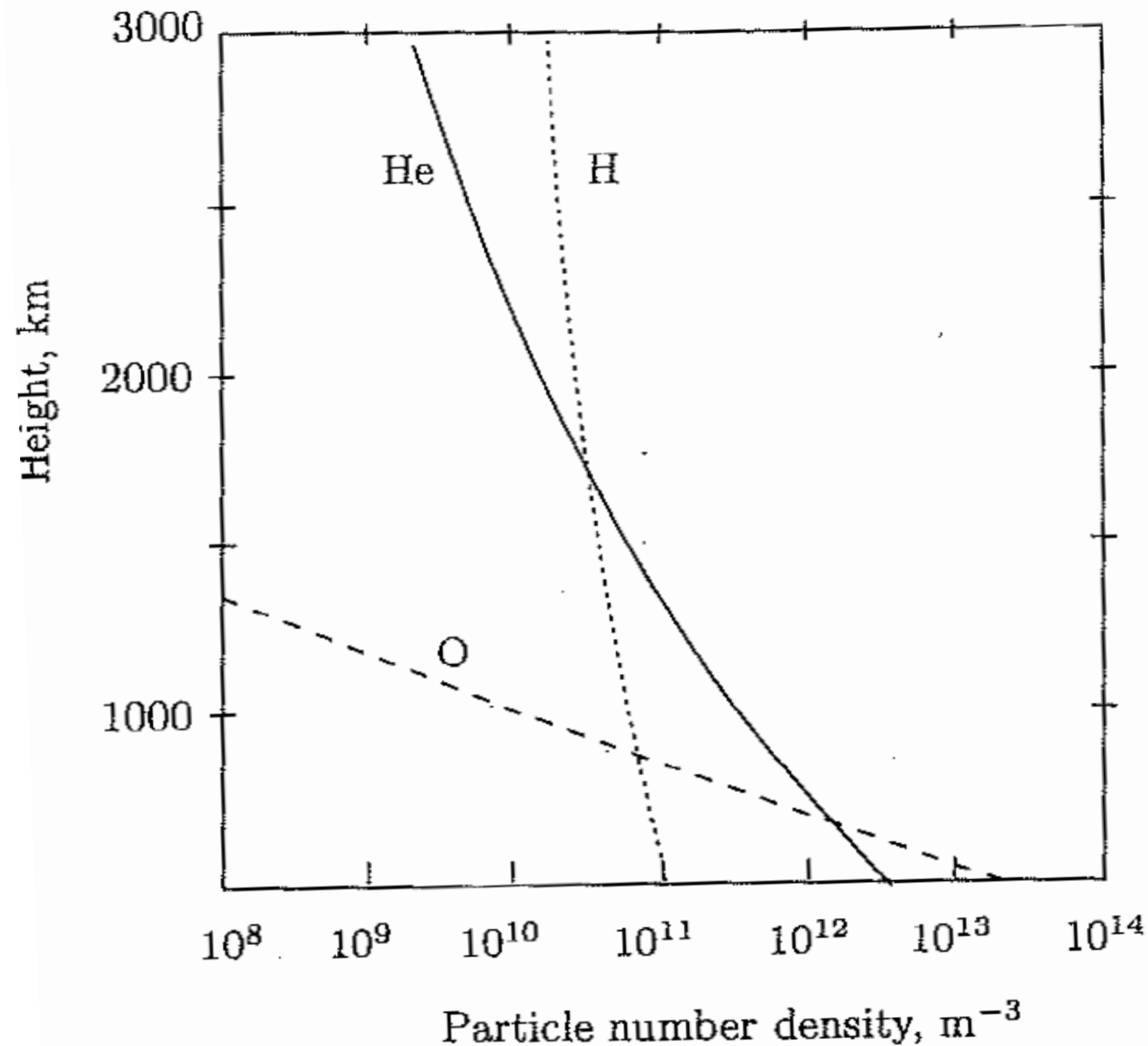
- Compute $n_i(500\text{km})/n_i(300\text{km})$ for O_2 and O for $T = T_\infty$
 - Write down equations with given parameters inserted
 - Evaluate scale heights
 - Take $n_{\text{O}_2}(300\text{km}) = 10^{13} \text{ m}^{-3}$ and $n_{\text{O}}(300\text{km}) = 10^{15} \text{ m}^{-3}$
 - Evaluate $n_{\text{O}_2}(500\text{km})$ and $n_{\text{O}}(500\text{km})$
 - Discuss how n decreases for lighter constituents of the atmosphere

Particle number density (100 to 500 km)



Proelss, 2010

Particle number density (500 to 3000 km)



Proelss, 2010

Atmospheric models

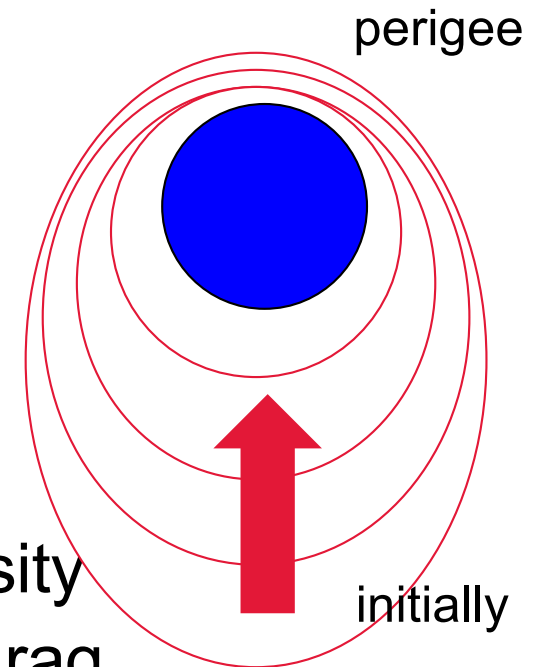
- A *Standard Atmosphere* is defined as a vertical distribution of atmospheric temperature, pressure and density, which by international agreement is taken to be representative of the Earth's atmosphere
- The first *Standard Atmospheres* established by international agreement were developed in the 1920's primarily for purposes of pressure altimeter calibrations. Aircraft performance calculations, aircraft and rocket design, ballistic tables, etc.

Why?

- A good knowledge of temperature, total density, concentrations of major and minor constituents and pressure is important for a wide range of scientific purposes and for the planning and execution of many space missions exploiting the low-earth orbit (LEO) regime below ~ 2500 km altitude.
- Aerodynamic forces on the spacecraft due to orbital motion of a satellite through a rarefied gas which itself can have variable high velocity winds, are important for planning satellite lifetime, maintenance of orbits, collision avoidance maneuvering and debris monitoring, sizing the necessary propulsion system, design of attitude control system, and estimating the peak accelerations and torques imposed on sensitive payloads.
- Surface corrosion effects due to the impact of large fluxes of atomic oxygen are assessed to predict the degradation of a wide range of sensitive coatings of spacecraft and instruments.
- The reactions of atomic oxygen around a spacecraft can also lead to intense “vehicle glow.”

Satellite atmospheric drag

- Spacecraft in LEO are affected by increased drag that will change the Keplerian orbital parameters in such a way that the altitude and the eccentricity of the orbit are lowered.
- Short-term drag effects if perigee height < 1000 km (thermosphere)
- Drag increase due to
 - solar UV output,
 - geomagnetic storms heating of thermosphere
 - tidal effects
- Drag models use 10.7 cm radio flux density (F10.7) 250 sfu, $K_p > 6 \rightarrow$ significant drag
 $1 \text{ sfu} = 10^4 \text{ Jy}$, $1 \text{ Jy} = 10^{-26} \text{ W Hz}^{-1} \text{ m}^{-2}$



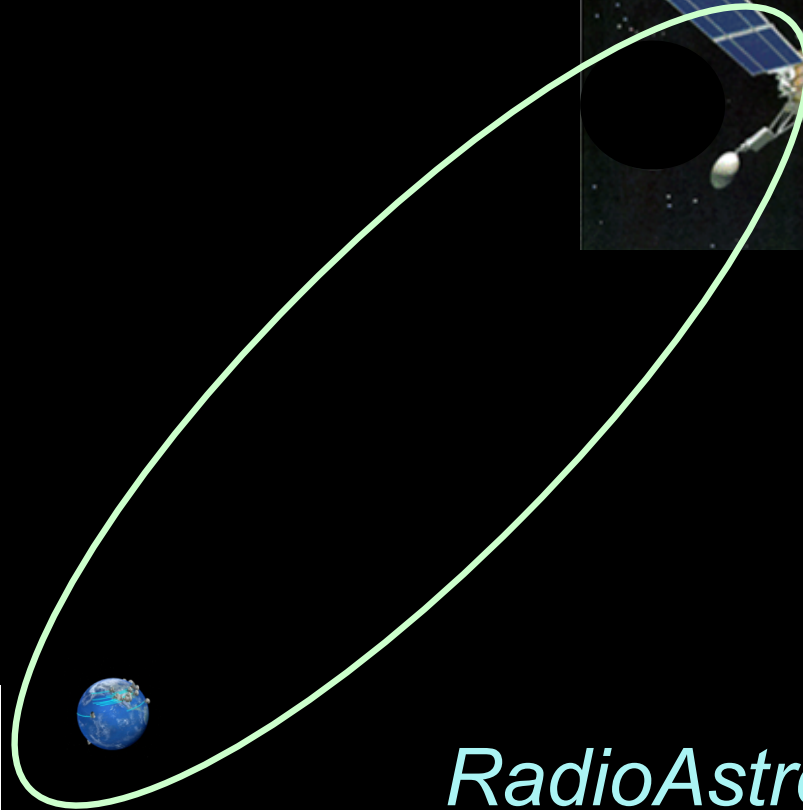
Probing gravitational redshift for
potentially most sensitive test of
general relativity (with grad student
Nelson Nunes)



H-clock-2



H-clock-1



RadioAstron
Ground-space VLBI mission

Density variations at 400 km

Type	Relative change (%)	Frequency (once per)
Diurnal effects	250	1 day
Significant Geomagnetic storm	800	3 days
Solar UV	250	27 days
Semi annual	120	12 months
Solar cycle	1600	11 years

Satellite atmospheric drag

$$\frac{dE}{dt} = F_d v = -\frac{1}{2} \rho C_d A v^3$$

$$a_d = -\frac{1}{2} \rho \frac{C_d A}{m} v^2$$

- F_d drag force
- v velocity of satellite relative to atmosphere
- ρ atmospheric mass density (calculated with 15% error)
- C_d drag coefficient (difficult to measure)
- A cross-sectional area of spacecraft perpendicular to motion
- a_d drag acceleration

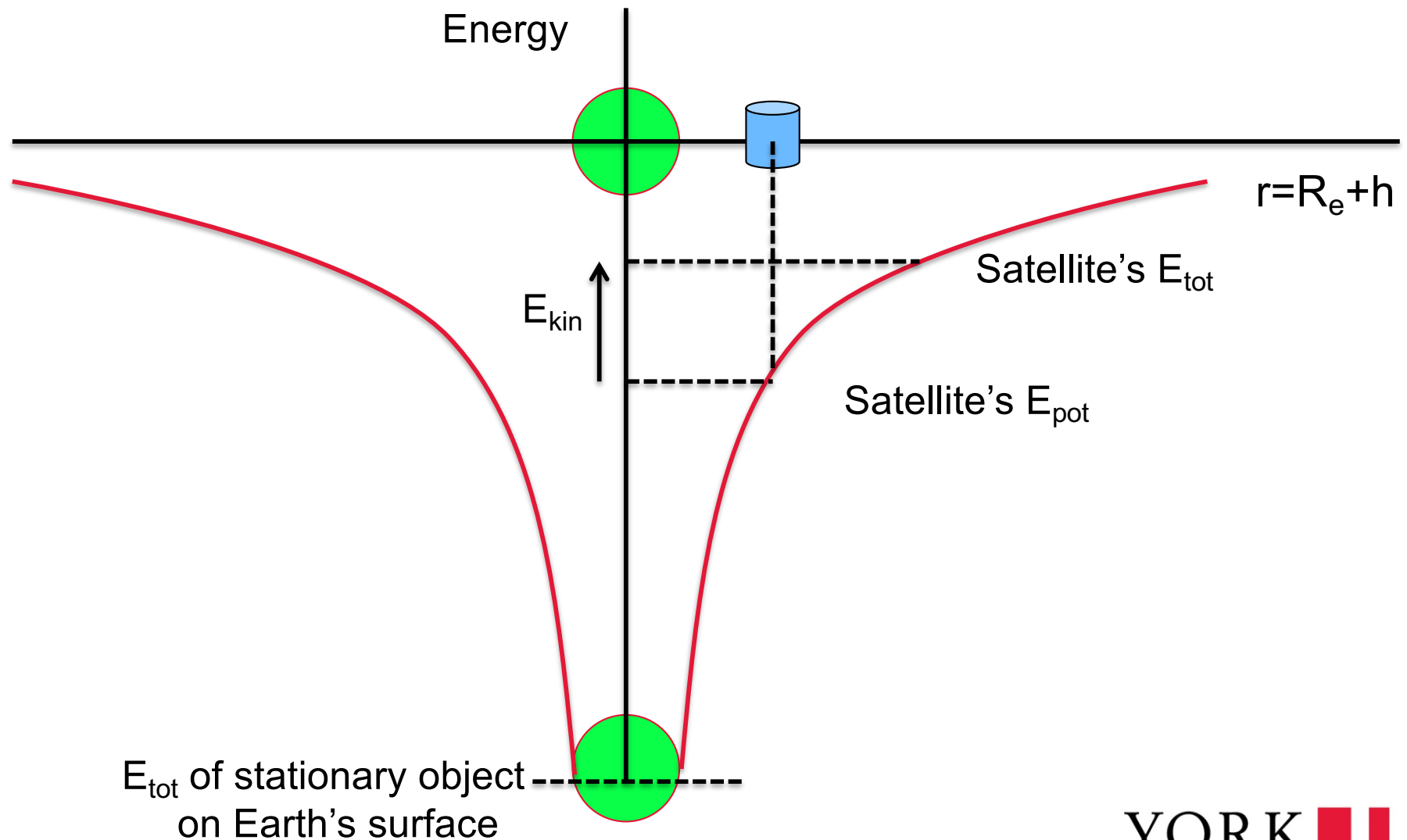
Satellite drag has an effect on orbit

- Orbit decays and circularizes due to the satellite drag.
- Energy lost by a spacecraft because of the drag is given by the drag force and the velocity of the spacecraft.

$$\frac{dE}{dt} = F_d v$$

- Energy loss can be related to loss of orbit height.

Energy for satellite in circular orbit



$$E_{tot} = E_{pot} + E_{kin} = -G \frac{Mm}{r} + \frac{1}{2}mv^2$$

$$E_{tot} = -\frac{1}{2} \frac{GMm}{r}$$

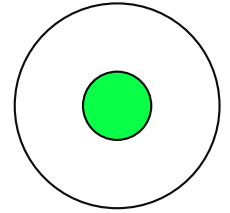
$$\frac{dE_{tot}}{dt} = \frac{1}{2} \frac{GMm}{r^2} \frac{dr}{dt} = F_D v = -\frac{1}{2} \rho C_D A v^3$$

$$\frac{dr}{dt} = -r^2 \rho \frac{C_D}{GMm} A v^3$$

With $r=R_E+h$, the drag force reduces the height.

How is the velocity affected?

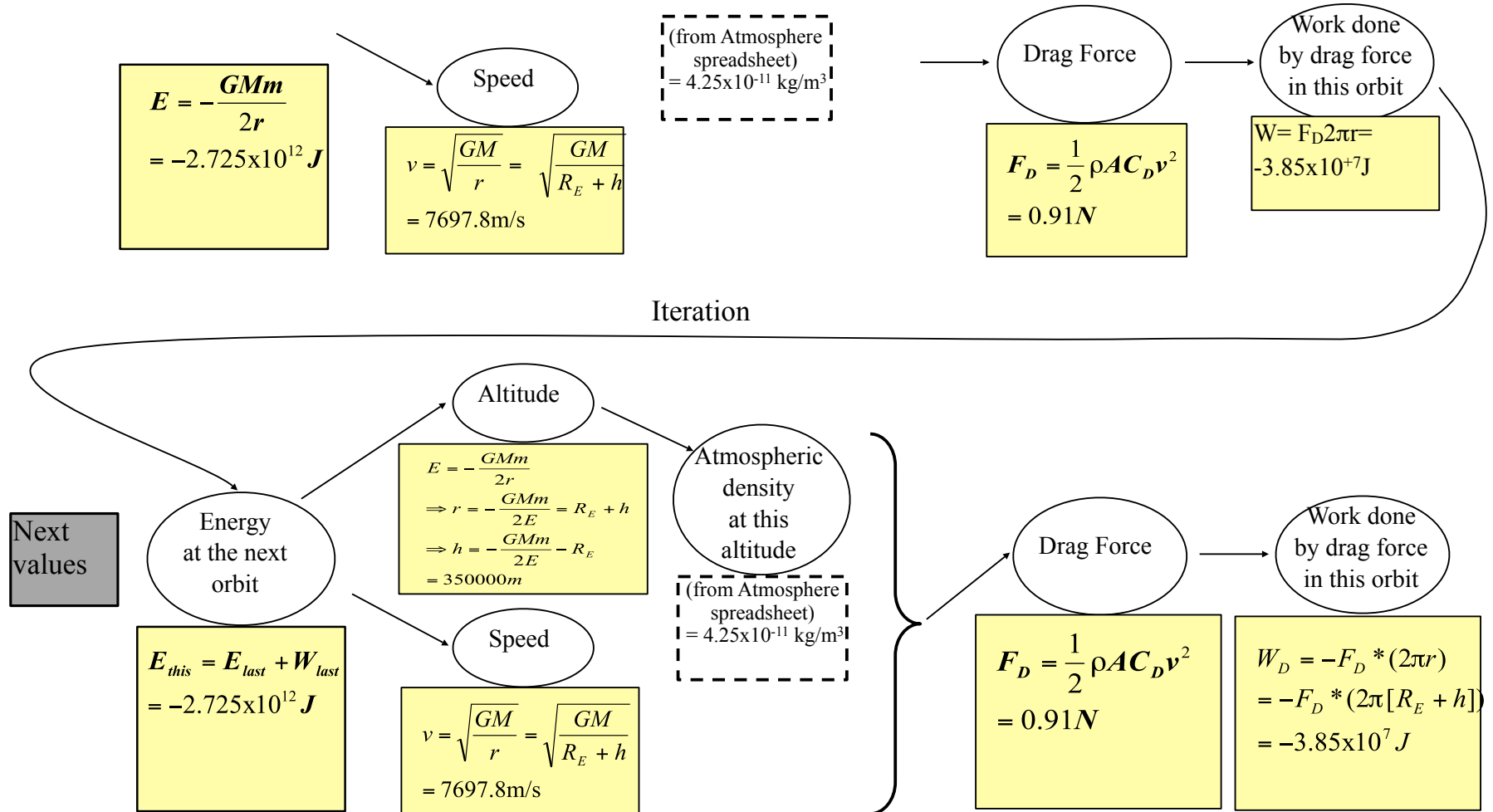
For circular orbit:



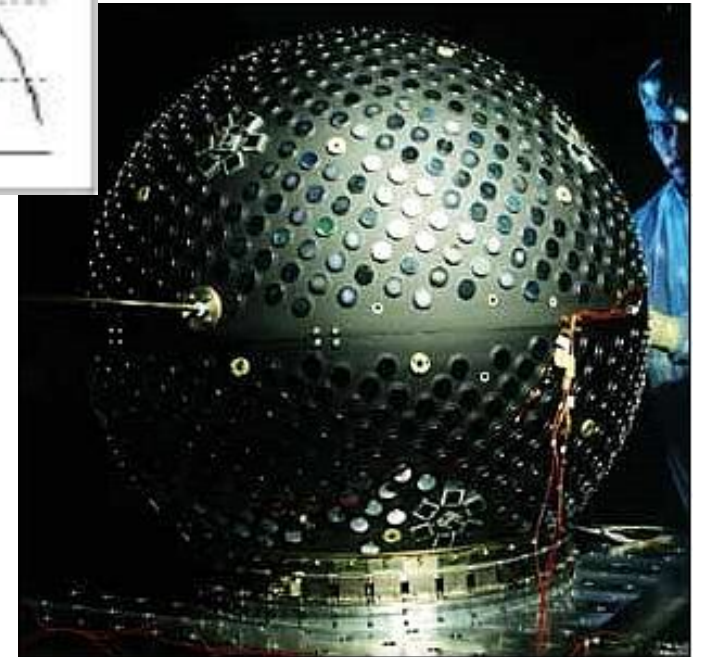
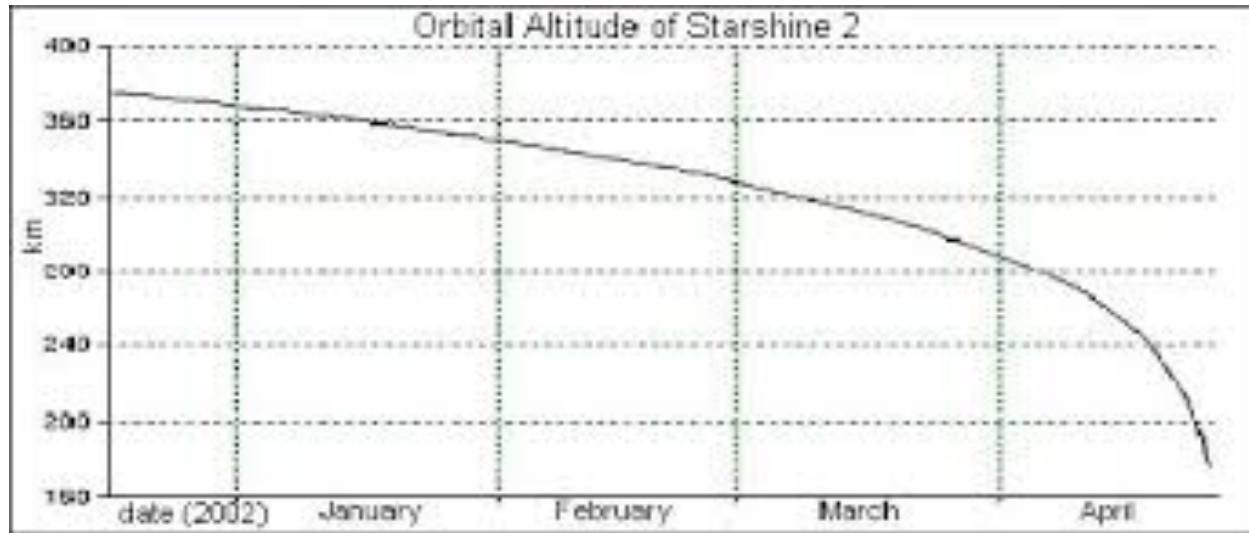
$$G \frac{Mm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

Orbital Drag Laboratory Concept Map Worksheet

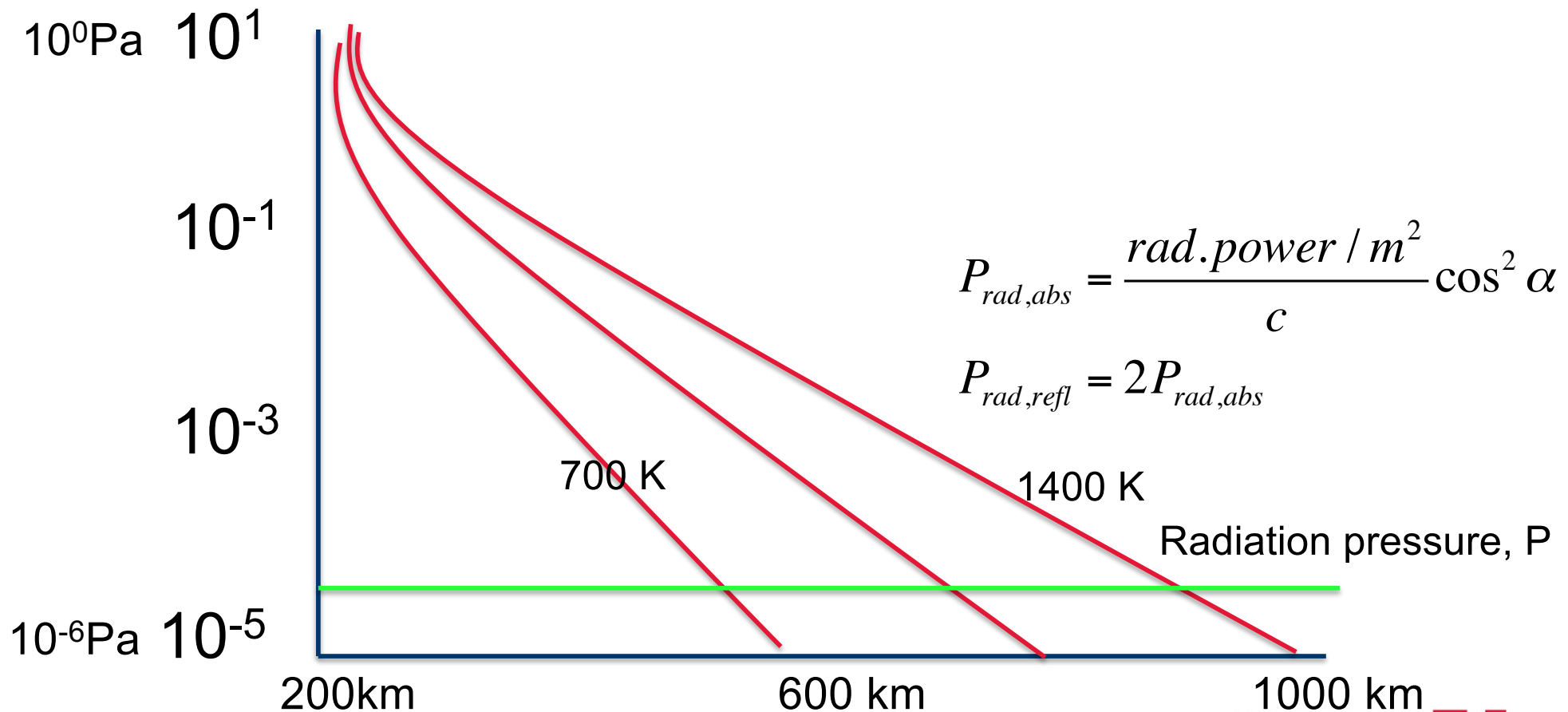


Starshine's orbital decay



Drag force as a function of T

Drag force (dyne cm⁻²)



1 dyne cm⁻² = 0.1 Pa

Radiation pressure

For single photon

$$p_{\gamma, \text{momentum}} = \frac{h}{\lambda} = \frac{E_{\gamma}}{c}$$

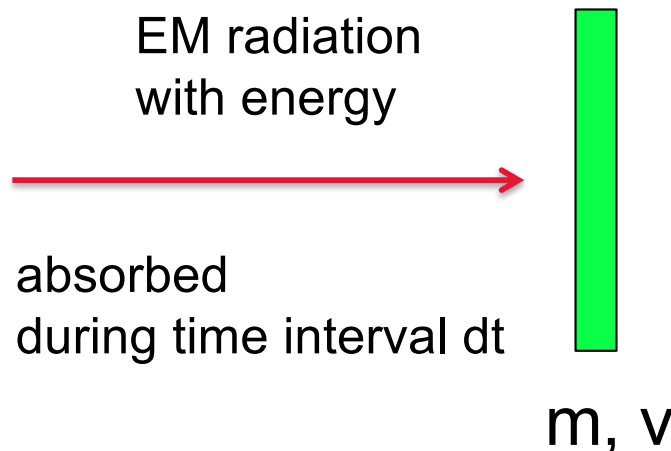
Change of momentum due to
EM absorbed by surface

$$mdv = \text{energy} / c$$

$$F = d/dt (mdv)$$

$$= d(\text{energy}/c)/dt$$

$$= \text{power}/c$$



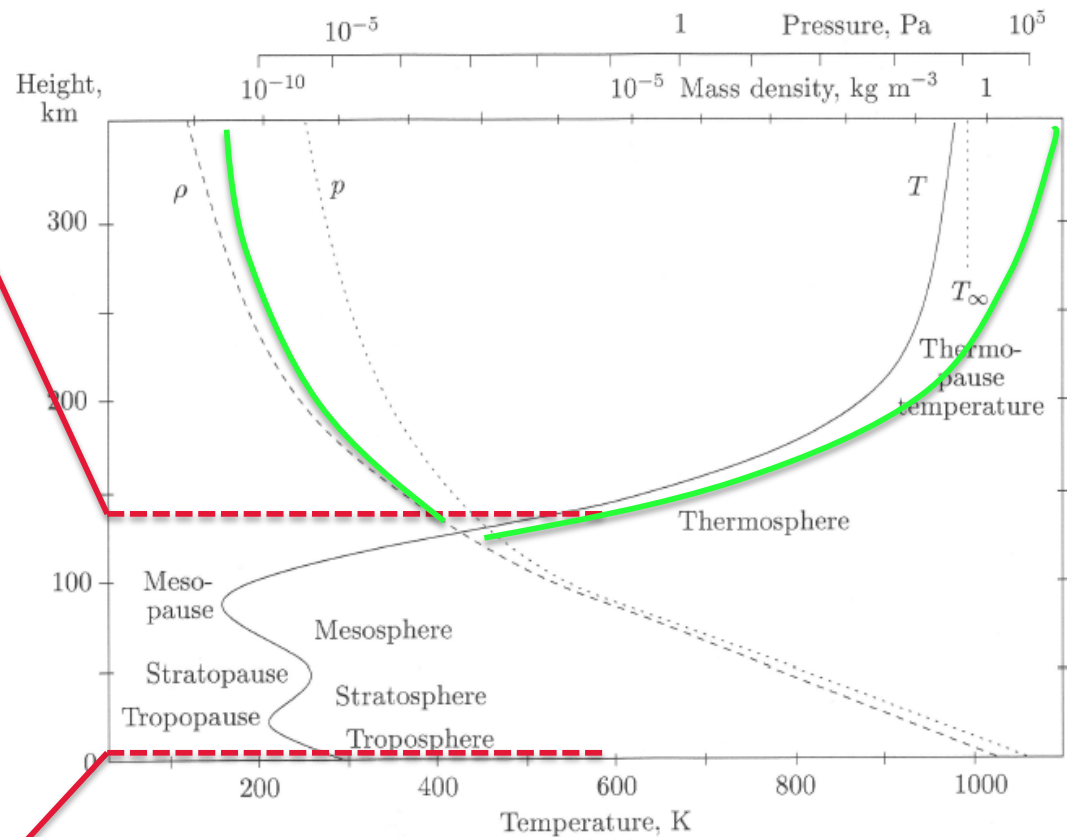
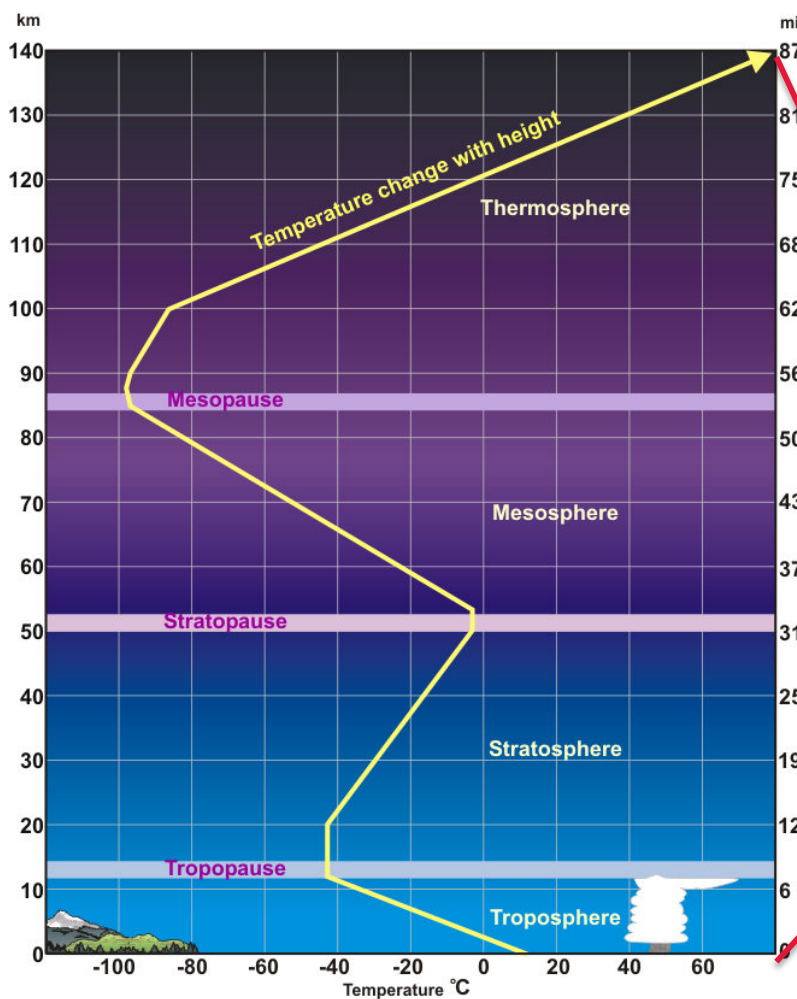
$$P_{\text{rad, abs}} = F/A$$
$$= (\text{power}/\text{m}^2)/c$$

For spacecraft near Earth and $\alpha=0$:

$$P_{\text{rad, abs}} = 1370/c$$
$$= 4.6 \times 10^{-6} \text{ Pa}$$

Temperature, density variations

Green indicates possible change from nominal



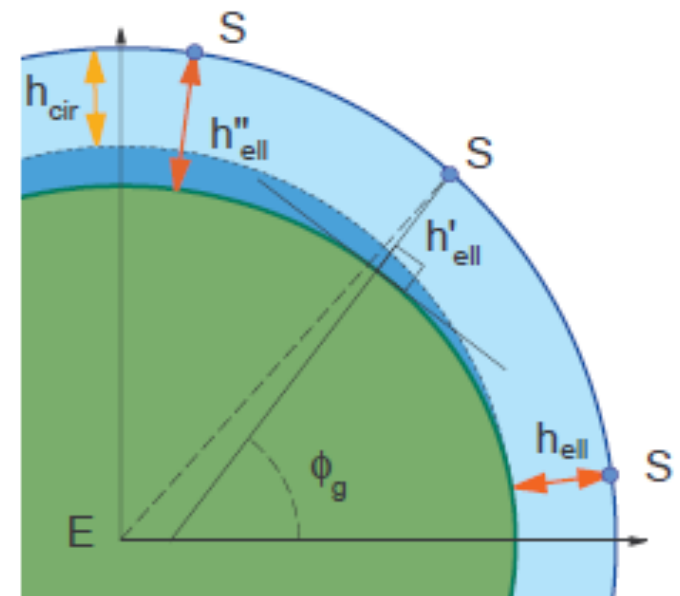
Molecular interactions

- The molecular interactions are characterized by the Knudsen number $K_n = \frac{L}{d}$, where
 - L is the mean free path and
 - d is the satellite characteristic length.
- We distinguish between:
 - $K_n \gg 1$; free molecular flow, particles interact one-by-one with body, no disturbance of the incident flow.
 - $K_n \sim 1$: reentry-like flow.
 - $K_n \ll 1$; classical aerodynamics, e.g., shockwave for supersonic speeds.

Static models

- Simple model with reasonable results.
- Errors are $< 40\%$ during mean solar activities and $< 60\%$ during max. solar activities.
- Values change only with location:
 - Height: hydrostatic equilibrium + perfect gas $\rho = \rho_0 e^{\frac{h_0 - h}{H}}$
 - Latitude: change of height because flattening of Earth toward poles.
 - Height over ellipsoid changes with longitude

$$\Delta h_{ell} = 0 - 21 \text{ km} \Leftrightarrow \Delta \rho$$

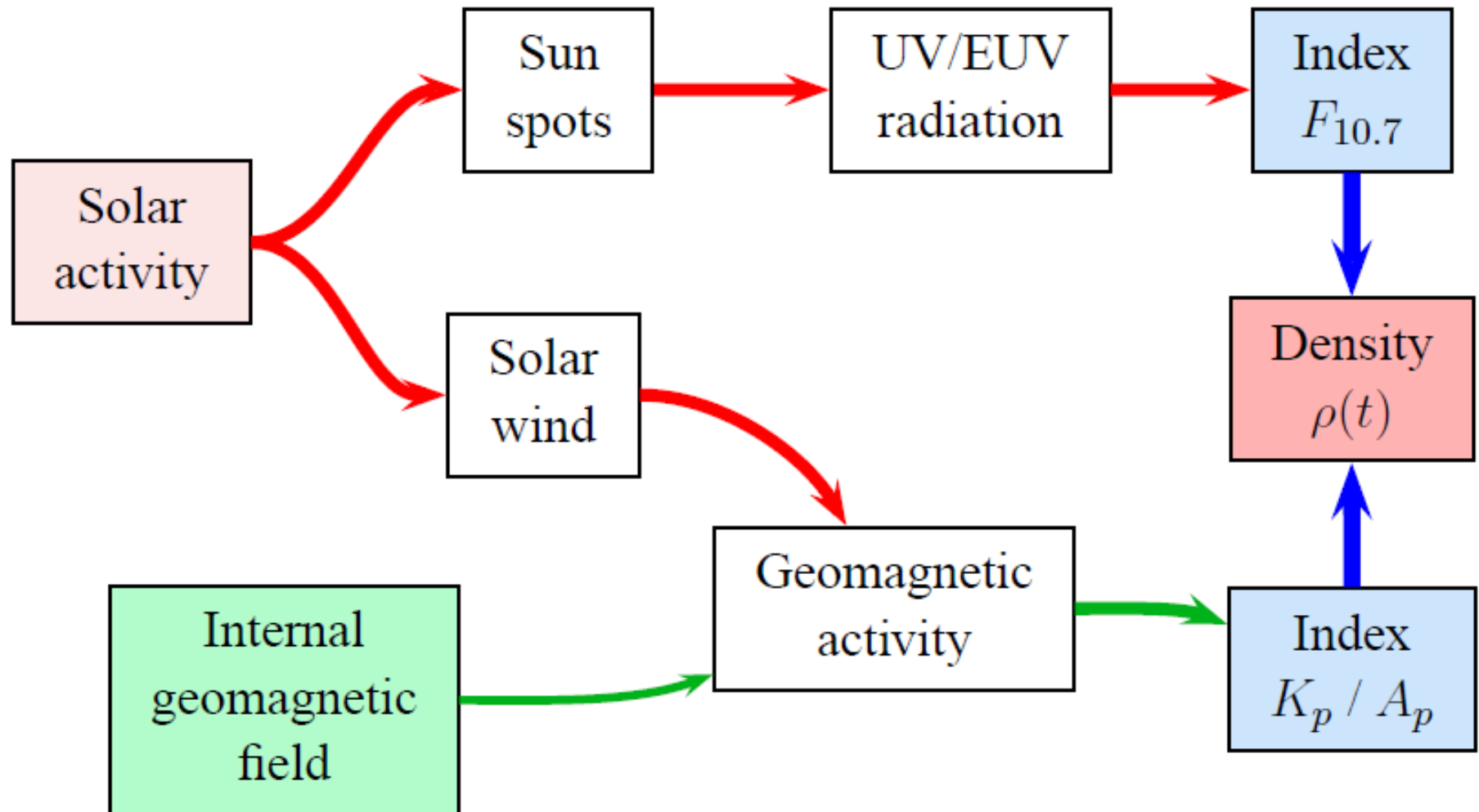


- Longitude:
 - Diurnal change, subsolar hump
 - Small space variation (seas, mountains), mainly at low heights
- Spherical symmetry, co-rotating with Earth, that is the atmosphere rotates with the Earth as a rigid body.
- Reference density, ρ_0 , and reference height, h_0 .
- Scale height, H .

Different types of static models

- 1976 standard model
- CIRA 65-90
 - Semi-static model with some free variables
 - Data mainly from satellite drag and ground based measurements
 - CIRA-72 and CIRA-86 incorporate dynamic models for altitudes over 100 km.
- Harris-Priester

Time-varying models



Diurnal variations

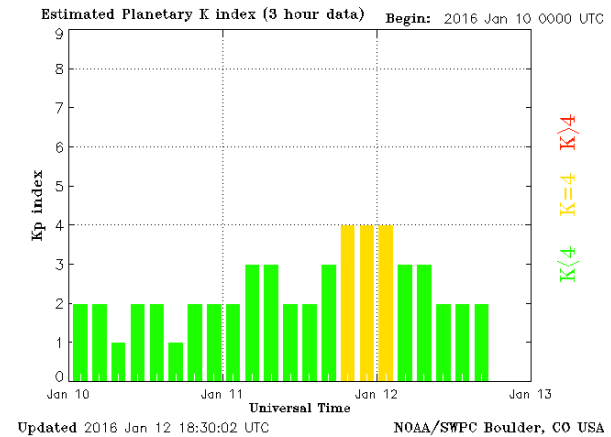
- Solar UV radiations heats up the atmosphere: density ρ increases.
- Maximum solar hump at 2:00 to 2:30 pm (delayed).
- Density ρ depends on:
 - Apparent local solar time of satellite
 - Solar declination
 - Geodetic latitude of satellite

Magnetic storms

- Earth's field fluctuations.
- Solar storms – they are of short duration but have large effects.
- Influence on ρ through the geomagnetic indices K_p or A_p .

K_p , A_p

K_p -index



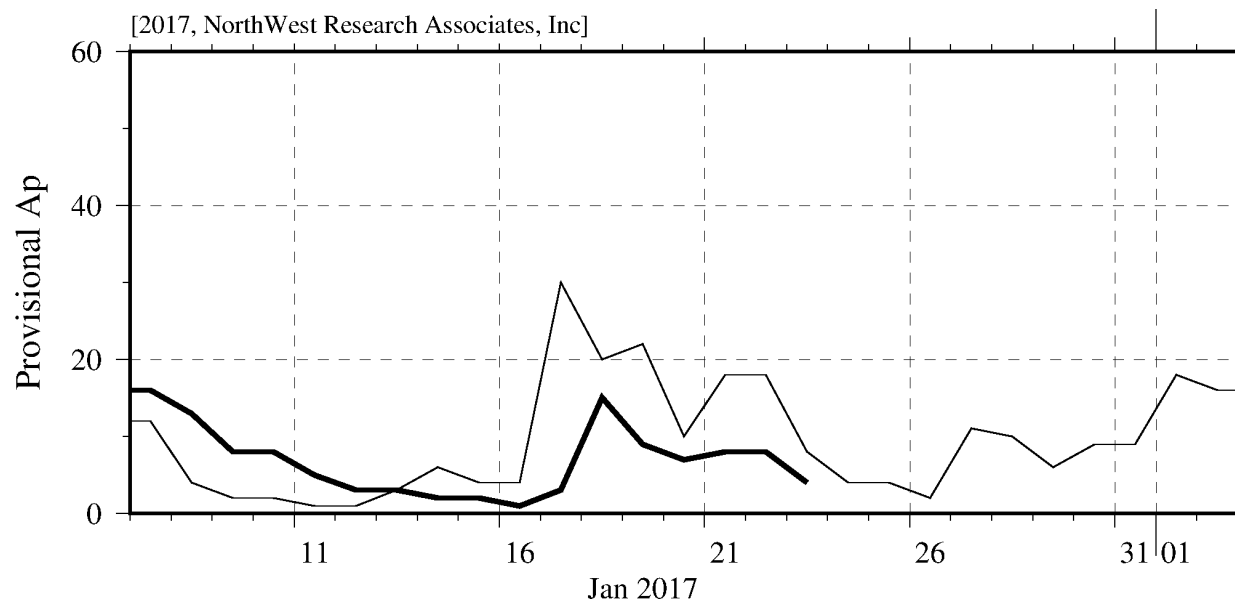
- The K-index characterizes the magnitude of geomagnetic storms. It is a code that is related to the maximum fluctuations of horizontal components observed on a magnetometer relative to a quiet day, during a three-hour interval.
- The A-index is calculated for individual magnetometer stations. The value is calculated as the average of eight, three-hour station indices observed during a UT day and provides a single, average value to indicate the activity for that day.
- The planetary K_p -index is a globally averaged indicator of the worldwide level of geomagnetic activity. The official index is derived by calculating a weighted average of K-indices from a predetermined network of geomagnetic observatories, the official K_p network.
- The network of contributing stations include the U.S. Geological Survey, Natural Resources Canada, the British Geological Survey, the German Research Center for Geosciences, Geoscience Australia, and contributions from the Institute de Physique du Globe de Paris, and the Korean Space Weather Center.
- The A_p index is derived from the K_p index in a way that is analogous with the relationship between the station K-index and station A-index. Each individual three-hour K_p index is converted to an equivalent amplitude index, and the average of eight of eight three-hour indices produces the A_p index.



NorthWest Research Associates, Inc.
Space Weather Services

A_p index

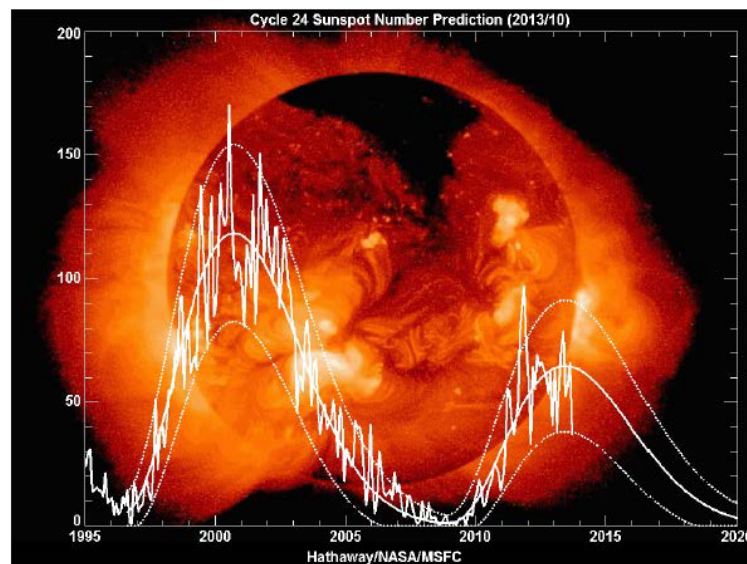
Geomagnetic Disturbance Index



Note: Heavy (dark) line is for the current 27-day Bartel rotation; light line is for the last rotation.

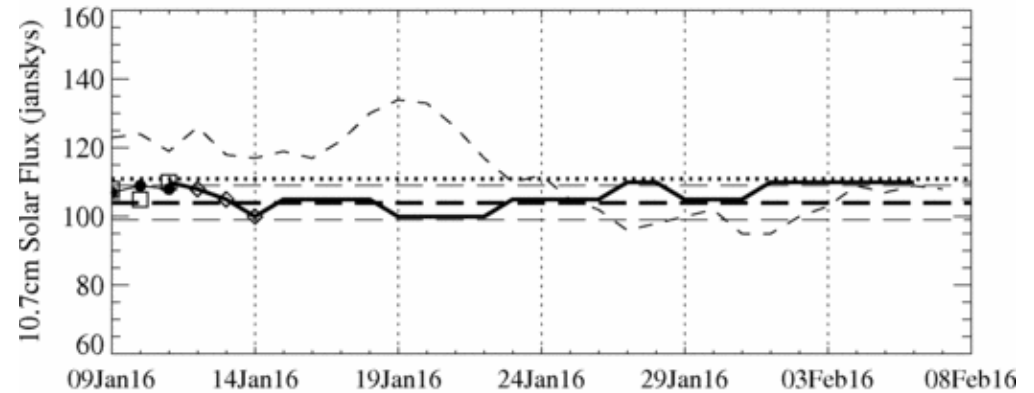
Solar affects

- Solar rotation period: 27 days
 - Visible sun spots change
 - Extreme UV (EUV) radiation changes
 - Affects p through F10.7 and its 81-day average
- Semi-annual variation: Sun distance changes
- Solar cycle variations: 11-year cycles are not regular but change in intensity



F10.7

DRAO

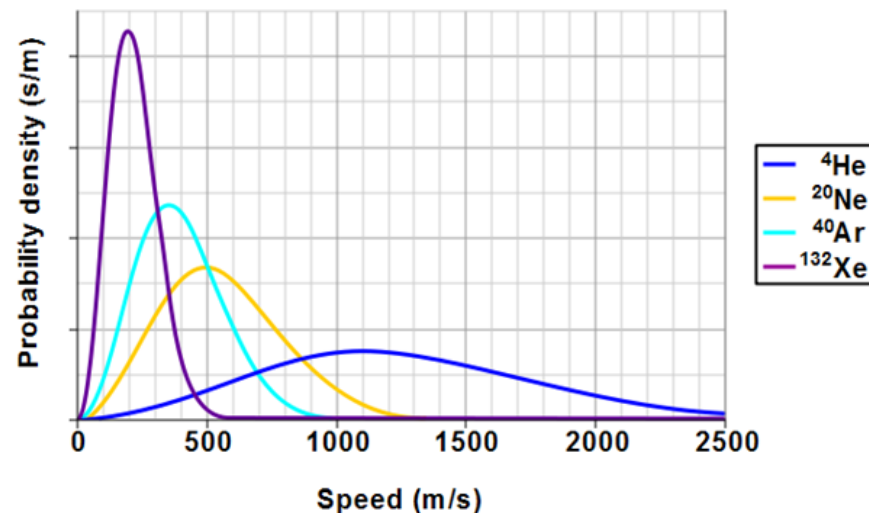


- The radio emission from the Sun at a wavelength $\lambda=10.7$ cm (the 10 cm flux density) correlates well with the sunspot number.
- Sunspot number is defined from counts of the number of individual sunspots as well as the number of sunspot groups and must be reduced to a standard scale taking into account the differences in equipment and techniques between observatories.
- On the other hand, the radio flux density at 10.7 cm can be measured relatively easily and quickly and has replaced the sunspot number as an index of solar activity for many purposes.
- The global daily value is measured at local noon at the Dominion Radio Astronomy Observatory (DRAO) in Pendicton, BC, Canada.
- This index is used as a surrogate for the solar output in wavelengths that produce photoionization in the Earth's atmosphere in the UV.

Exosphere and escaping particles

- In the exosphere most of the particles are H and He.
- What is their escape velocity?
- What is the portion, b_{esc} , of these particles having sufficient energy to overcome the gravitational attraction?
- This portion can be computed from the Maxwell-Boltzmann speed distribution.

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases



Remember.....

Escape velocity

- In order to escape Earth, a particle with mass, m , must be moving with sufficiently high velocity to overcome earth's gravity. The velocity that is just sufficiently large to do that is the escape velocity, v_{esc} .
- The escape velocity can be computed from the kinetic energy of the particle being equal to the work required to overcome the gravitational field along the trajectory

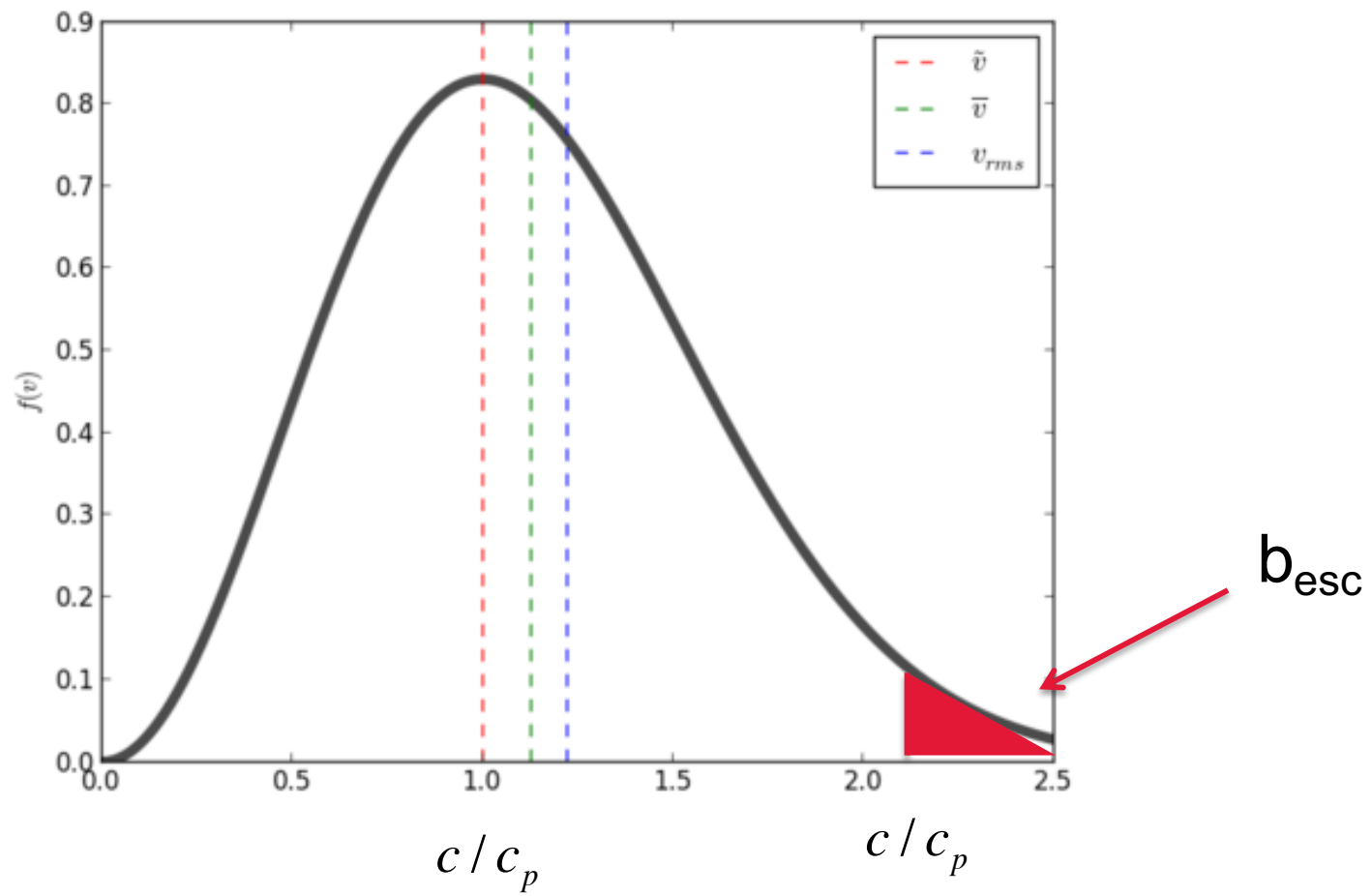
$$\frac{1}{2}mv_{esc}^2 = \int_r^{\infty} mg(r') dr' = \int_r^{\infty} mGM_E r'^{-2} dr' = -mGM_E r'^{-1} \Big|_r^{\infty} = mGM_E r^{-1} = mg(r)r$$

where $r = R_E + h$, is the distance from Earth's centre, $R_E = 6371$ km, is the mean radius of Earth and h is the height above mean sea level, and $g(r) \approx GM_E/r^2$, the gravitational acceleration.

$$v_{esc}(h=0) = 11.18 \text{ km s}^{-1}$$

$$v_{esc} = \sqrt{2g(r)r} = \frac{v_{esc}(h=0)}{\sqrt{1 + \frac{h}{R_E}}}$$

Escape velocity



The integration of $f(c)$ over all speeds is ?

The integration of $f(c)$ from c_{esc} to infinity is b_{esc} , the portion of particles that have speeds larger than c_{esc} .

$$b_{esc} = \int_{c_{esc}}^{\infty} f(c) dc$$

$$c_p = \sqrt{\frac{2kT}{m}}$$

$$b_{esc} = \int_{c_{esc}}^{\infty} \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} 4\pi c^2 e^{-\left(\frac{mc^2}{2kT}\right)} dc$$

$$b_{esc} = 4\pi \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} \left(\frac{2kT}{m} \right)^{\frac{3}{2}} \int_{\frac{c_{esc}}{c_p}}^{\infty} \left(\frac{c}{c_p} \right)^2 e^{-\left(\frac{c^2}{c_p^2}\right)} d\left(\frac{c}{c_p} \right)$$

$$\frac{c}{c_p} = x \quad \frac{c_{esc}}{c_p} = y$$

$$b_{esc} = \frac{4}{\sqrt{\pi}} \int_y^{\infty} x^2 e^{-x^2} dx$$

$$\int_a^b uv' dx = uv \Big|_a^b - \int_a^b vu' dx$$

$$b_{esc} = \frac{4}{\sqrt{\pi}} \left(-\frac{x}{2} e^{-x^2} \Big|_y^{\infty} - \int_y^{\infty} e^{-x^2} \left(-\frac{1}{2} \right) dx \right)$$

$$u = -\frac{x}{2} \quad v' = -2xe^{-x^2} \quad v = e^{-x^2}$$

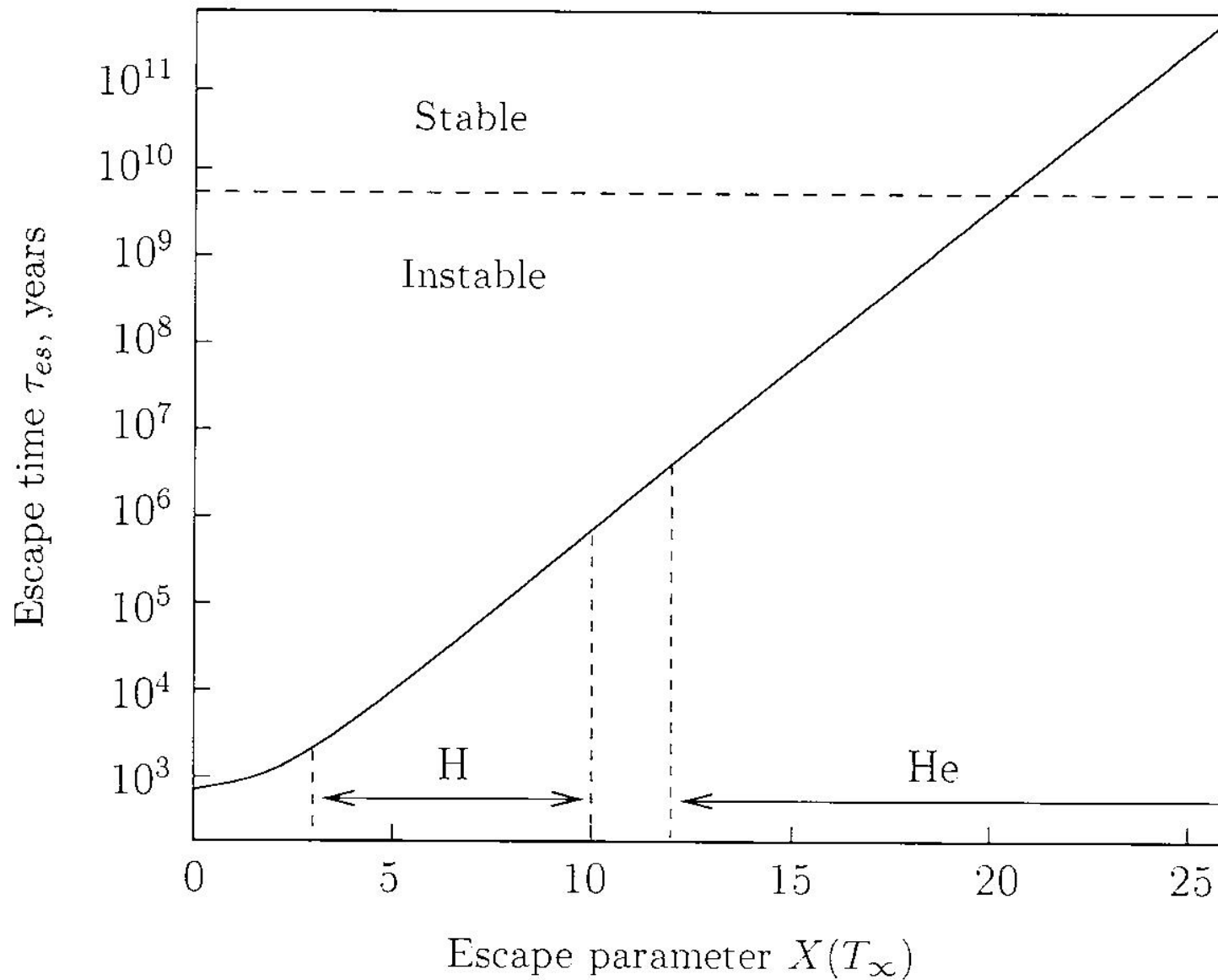
$$b_{esc} = \frac{4}{\sqrt{\pi}} \frac{1}{2} ye^{-y^2} + \left(\int_0^{\infty} e^{-x^2} dx - \int_0^y e^{-x^2} dx \right)$$

$$b_{esc} = \frac{2}{\sqrt{\pi}} ye^{-y^2} + 1 - \text{erf}(y)$$

$$\text{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2} dx \approx 1 - \frac{e^{-y^2}}{y\sqrt{\pi}} \left[1 - \frac{1}{2y^2} + \frac{1 \cdot 3}{2^2 y^4} - \frac{1 \cdot 3 \cdot 5}{2^3 y^6} + \dots \right]$$

Escape time

- How long does it take to completely remove the gas constituents from the atmosphere?
- b_{esc} may be a small portion of 1, but over time some of the light gas constituents, like H, may be lost completely.
- This “escape time” can be computed by considering an escape flux through an area parallel to the exopause.
- Particles moving through this area outward from all possible directions have to be considered.
- An integration over the angles φ , θ over a hemisphere needs to be done.
- As a result we get the escape parameter, X , an important parameter for the atmospheres of planets and moons.
- The larger X , the smaller the escape flux. For Jupiter, $X \sim 1400$. For Mercury, $X \sim 2$.



$$X(T_{\infty}) = \left(\frac{c_{esc}}{c_p} \right)_{EB}^2 = \frac{g_{EB} r_{EB} m}{k T_{\infty}}$$

EB: Exobase