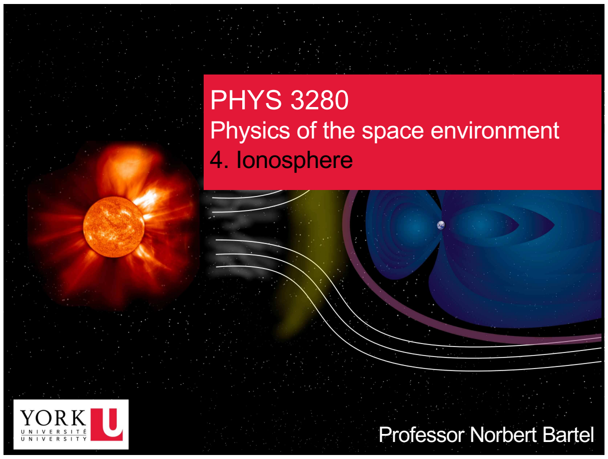
PHYS 3280
Physics of the space environment
4. Ionosphere



Professor Norbert Bartel

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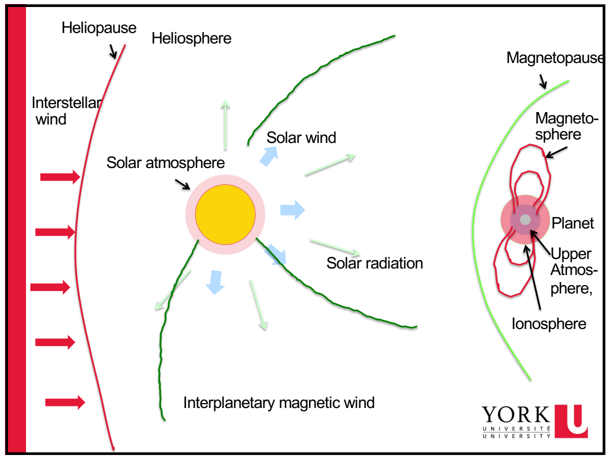

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Recall from
1) Introduction - History

Groundbased observations of the ionosphere

1901 Marconi – Transatlantic radio communication → speculation about conducting layer
1924 Breit, Tuve, Appleton (Nobel Prize), Barnett – Prove of existence of ionosphere
>1924 Chapman – Theory of ionosphere
1925 Espenschied – Ionospheric disturbance ←→ geomagnetic activity
1953 Storey – whistlers: low-frequency radio waves propagating in magnetized ionized gases, extension of ionosphere into magnetosphere

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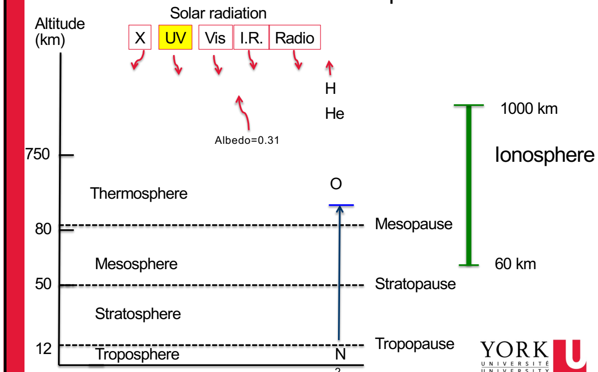
Absorption of solar radiation
UV radiation ionizes the thermosphere



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Atmospheric division (with more detail)

Height /km	Temperature	Composition	Vertical transport	Gravitational binding	Thermal plasma
100,000	Thermosphere	Hydrogensphere (Geocorona)	Effusosphere	Exosphere	
10,000					Plasmapause
1,000		Heterosphere		Exobase	Plasmasphere
			Diffusosphere		F-region
	Mesopause				E-region
100	Mesosphere	Homopause	Turbopause		D-region
	Stratopause			Barosphere	
	Stratosphere	Homosphere	Turbosphere		
10	Tropopause				
0	Troposphere				

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Thermal plasma - Ionosphere

- The ionosphere is the largest concentration of charges in the space environment of Earth
- Only 10^{-2} to 10^{-8} of particles are ionized
- The ionosphere extends from about 60 km to 1,000 km and includes the thermosphere and parts of the mesosphere and exosphere.
- It is ionized by solar radiation, plays an important part in atmospheric electricity and forms the inner edge of the magnetosphere.
- It has practical importance because, among other functions, it influences radio propagation to distant places on the Earth.

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Similarities to neutral gas

- The description of ionized gas is similar to the description of neutral gas (Section 2).
- Macroscopic state parameters – mass density, pressure, temperature
- Gas kinetic parameters – collision frequency, mean free path
- Defining equations remain the same

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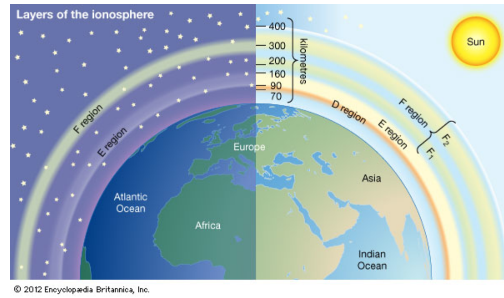
Layers of the ionoshere



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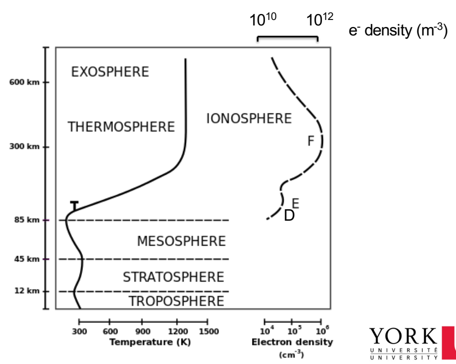
Ionosphere electron density profile



Wikipedia.org

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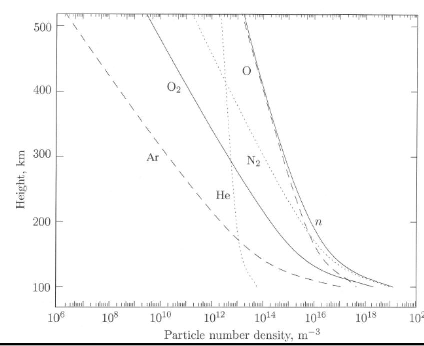
For comparison -- particle number density



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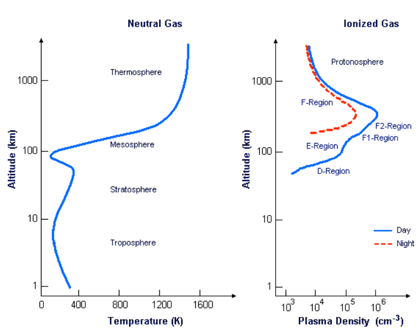
Ionosphere profile at day and night



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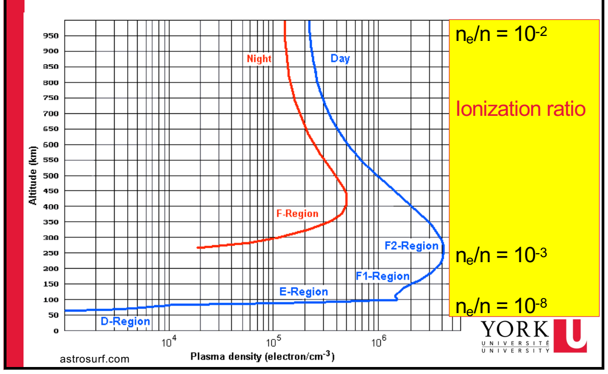
Ionosphere profile at day and night in detail



astrosurf.com

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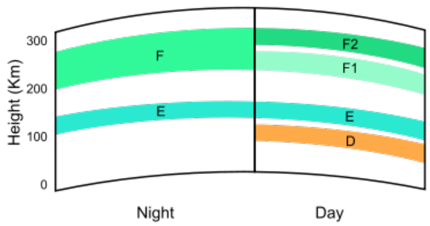
Ionospheric layers



- At night the F layer is the only layer of significant ionization present, while the ionization in the E and D layers is extremely low. During the day, the D and E layers become much more heavily ionized, as does the F layer, which develops an additional, weaker region of ionisation known as the F_1 layer. The F_2 layer persists by day and night and is the region mainly responsible for the refraction of radio waves.

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D, E, F layers

D layer

- The D layer is 60 to 90 km above the surface of the Earth. There are many more neutral air molecules than ions in this layer. Medium frequency (MF) and lower high frequency (HF) radio waves are significantly damped within the D layer, as the passing radio waves cause electrons to move, which then collide with the neutral molecules, giving up their energy. The lower frequencies move the electrons farther, with a greater chance of collisions. This is the main reason for absorption of HF radio waves, particularly at 10 MHz and below, with progressively smaller absorption as the frequency gets higher. This effect peaks around noon and is reduced at night due to a decrease in the D layer's thickness; only a small part remains due to cosmic rays. A common example of the D layer in action is the disappearance of distant AM broadcast band stations in the daytime.

E layer

- The E layer is 90 to 150 km above the surface of the Earth. Ionization is due to soft X-ray (1–10 nm) and far ultraviolet (UV) solar radiation ionization of molecular oxygen (O_2). Normally this layer can only reflect radio waves having frequencies lower than about 10 MHz and may contribute a bit to absorption on frequencies above. At night the E layer weakens because the primary source of ionization is no longer present. After sunset an increase in the height of the E layer maximum increases the range to which radio waves can travel by reflection from the layer. (560 mi) to 2,500 km (1,600 mi). Double-hop reception over 3,500 km (2,200 mi) is possible.

F layer

- The F layer extends from about 150 to more than 500 km above the surface of Earth. It is the densest point of the ionosphere, which implies signals penetrating this layer will escape into space. At higher altitudes, the number of oxygen ions decreases and lighter ions such as hydrogen and helium become dominant. The F layer consists of one layer at night, but during the day, a deformation often forms in the profile that is labeled F_1. The F_2 layer remains by day and night responsible for most sky wave propagation of radio waves, facilitating high frequency (HF, or shortwave) radio communications over long distances.

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Ionosphere composition

- Sum of positive ions = sum of electrons +sum of negative ions. Total charge is zero.

Sphere	Region	Height (km)	Main Constituents
Ionosphere	D	<90	H_3O^+ NO_3^-
	E	90-170	O_2^+ NO^+
	F	170-1000	O^+
Plasmasphere		>1000	H^+

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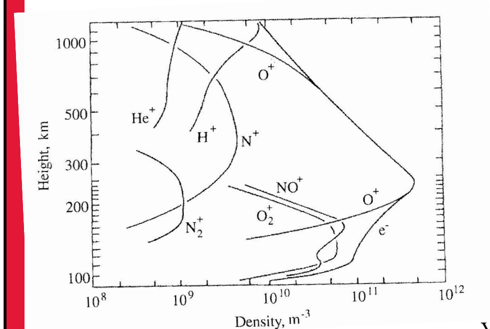
Ionosphere composition – midday, low solar activity



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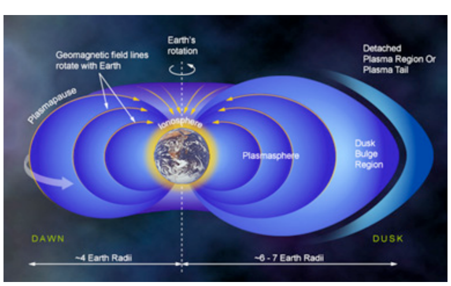

windows2universe

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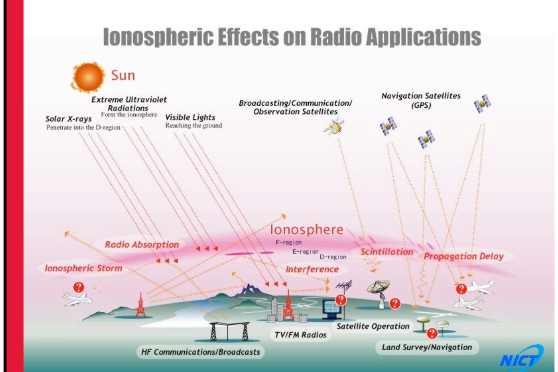

wdc.nict.go.jp

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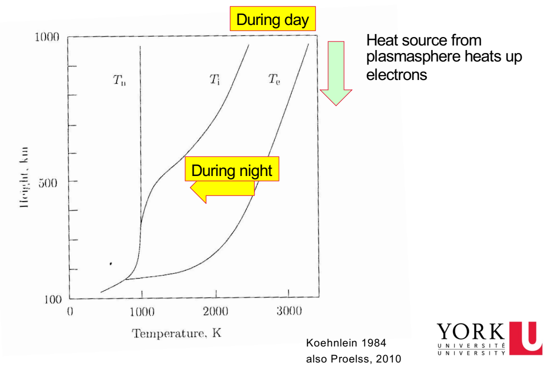
Temperature profiles for neutral gas, ions and e⁻



Koehnlein 1984
also Proelss, 2010

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Barometric density distribution for ion and electron gas

- Assuming pressure gradient and gravitational forces are in equilibrium for ion gas (s=i) and electron gas (s=e).

$$\frac{dp_s}{dz} = -\rho_s g$$

- If we assume that temperature, T_s, for the ion and electron gases are constant over a height range, then

$$n_s(h) = n_s(h_0)e^{-\frac{(h-h_0)}{H_s}}$$

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Recollection: Aerostatic equation

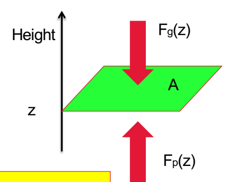
- We want to consider a massless membrane at height z and area A that experiences pressure from the gas below and the weight of the gas above.

Pressure force:

$$F_p(z) = Ap(z)$$

Weight of the gas:

$$F_g(z) = A\int_z^{\infty} \rho(z')g(z')dz'$$



- For static equilibrium, F_p=F_g

$$p(z) = \int_{z'=z}^{\infty} \rho(z')g(z')dz'$$

$$\frac{dp}{dz} = -\rho(z)g(z)$$ Aerostatic equation

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Recollection: Barometric law

- The aerostatic law gives relation between pressure as a function of height and mass density as a function of height. With the ideal gas law we can express the law in terms of T as a function of height.

$$p = nkT$$

$$\rho = \bar{m}\frac{p}{kT}$$

$$\frac{dp}{dz} = -\frac{\bar{m}(z)g(z)}{kT(z)}p(z) = -\frac{p}{H} \qquad \frac{dp}{dz} = -\frac{1}{H(z)}p(z)$$

$$\int_{p(h_0)}^{p(h)} \frac{dp}{p} = -\int_{h_0}^{h} \frac{\bar{m}(z)g(z)}{kT(z)}dz \quad \text{with} \quad H(z) = \frac{kT(z)}{\bar{m}(z)g(z)}$$ Pressure scale height

$$\ln\frac{p(h)}{p(h_0)} = -\int_{h_0}^{h} \frac{dz}{H(z)}$$

$$p(h) = p(h_0)\exp\left\{-\int_{h_0}^{h} \frac{dz}{H(z)}\right\}$$ Barometric law for the vertical pressure profile in an atmosphere

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Recollection: We can also derive a similar expression for the number density profile:

$$n(h)kT(h) = n(h_0)kT(h_0)\exp\left\{-\int_{h_0}^{h} \frac{dz}{H(z)}\right\}$$

$$n(h) = n(h_0)\frac{T(h_0)}{T(h)}\exp\left\{-\int_{h_0}^{h} \frac{dz}{H(z)}\right\}$$ Barometric law for the vertical density profile in an atmosphere

or

$$n(h) = n(h_0)\exp\left\{-\int_{h_0}^{h} \frac{dz}{H_n(z)}\right\} \quad \text{with} \quad \frac{1}{H_n} = \left(\frac{1}{H} + \frac{1}{T}\frac{dT}{dH}\right)$$

- For the pressure or the density, the scale height describes a vertical distance over which the pressure or the density changes significantly, namely approximately by 1/e or e.

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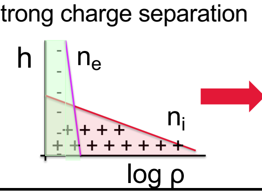
- However, the scale heights are very different for the ion gas and the electron gas.

$$H_i = \frac{kT_i}{m_i g} << H_e = \frac{kT_e}{m_e g}$$

- m_e / m_p=1/1836
- That would result in a density distribution with a strong charge separation



unrealistic

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- The separation of the charges, positive charge at lower height and negative charge at larger height, would produce a strong electrical polarization force.
- This would destroy the assumed equilibrium force between pressure from below and gravitational force per square meter from above.
- We have to introduce the electric force, since the electric cloud above would pull the ion cloud from below upwards and the ion cloud from below would pull the electric cloud downwards.

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- With, e, the elementary electric charge and, E_p, the electric polarization field strength, this leads to

$$\frac{dp_i}{dz} = -\rho_i g + n_i e \in_p$$

$$\frac{dp_e}{dz} = -\rho_e g - n_e e \in_p$$

Note-- Electric field:
A particle of charge, q, is subject to a force, F.

$$\vec{F} = q\vec{\in}$$

Units of electric field:
N/C or V/m

- Observations show that the field, E_p, is sufficiently strong to provide charge neutrality everywhere.

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- Summation, with $\rho_i >> \rho_e$ and $n_i = n_e = n$ leads to

$$\frac{d(p_i + p_e)}{dz} \approx -n m_i g$$

- If we again assume that $T_i = T_e$ then

$$\boxed{n(h) = n(h_0) e^{-\frac{(h-h_0)}{H_p}}}$$

with

$$\boxed{H_P = \frac{k(T_i + T_e)}{m_i g}}$$

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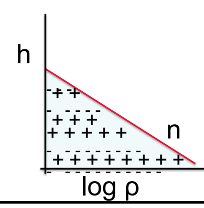
- The scale height of the ion gas is a bit more than doubled
- The scale height of the electron gas is reduced by a factor 10^4 (x m_e/ mass of particle, O_2 for instance)
- Light electron gas pulls the ion gas upward and the ion gas drags the electron gas downward



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Polarization field

The striving of a plasma toward charge neutrality

The electric polarization field, E_p, is obtained from previous equations and from $n(h) = n(h_0) e^{-\frac{h-h}{H_P}}$

$$\frac{dp_e}{dz} = -\rho_e g - n_e e \in_p = -n_e m_e g - n_e e \in_p = -n_{0,e} e^{-\frac{h-h_0}{H_P}} (m_e g + e \in_p)$$

$$\frac{dp_e}{dz} = -kT_e \frac{dn_e}{dz} = \frac{kT_e}{H_P} n_{0,e} e^{-\frac{h-h_0}{H_P}} = \frac{T_e m_i g}{T_i + T_e} n_{0,e} e^{-\frac{h-h_0}{H_P}}$$

$$-n_{0,e} e^{-\frac{h-h_0}{H_P}} (m_e g + e \in_p) = \frac{T_e m_i g}{T_i + T_e} n_{0,e} e^{-\frac{h-h_0}{H_P}}$$

$$(m_e g + e \in_p) = -\frac{T_e m_i g}{T_i + T_e}$$

For O ions:

$$\boxed{|\in_p| \approx \frac{m_i g}{e} \frac{T_e}{T_i + T_e}}$$

$$\in_p = \frac{16 \bullet 1.66 \bullet 10^{-27} \bullet 9.81}{1.60 \bullet 10^{-19}} \bullet \frac{2000}{1000 + 2000}$$

$$\in_p \approx 1 \mu V m^{-1}$$

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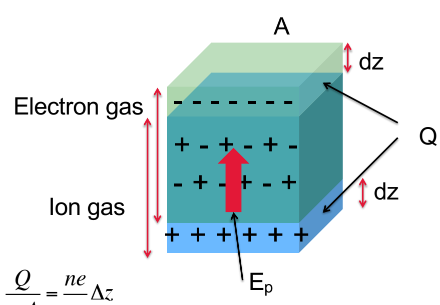
What does it take to produce such a small electrical field?



$$\in_p = \frac{Q}{\varepsilon_0 A} = \frac{ne}{\varepsilon_0} \Delta z$$

ε_0: permittivity or electric constant $= 8.854 \bullet 10^{-12}$ F m^{-1}

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$$\in_p = \frac{Q}{\varepsilon_0 A} = \frac{ne}{\varepsilon_0}\Delta z$$

$$\Delta z = \frac{\varepsilon_0 \in_p}{ne}$$

$$\Delta z = \frac{8.854 \bullet 10^{-12} \bullet 10^{-6}}{5 \bullet 10^{10} \bullet (1.60 \bullet 10^{-19})} \approx 1 \bullet 10^{-9} m$$

A displacement of only 10^{-9} m of the electron gas from the ion gas would produce such field! That is only a few diameters of an ion. That means that the ion gas is extremely well mixed with the electron gas and that indeed $n_i(h) = n_e(h)$ for all h.

A fundamental property of a plasma is charge neutrality as a whole.

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Systematic variations of ionisation density

- Temporal variations
  - Seasons
  - Solar activity
- Latitudinal variations

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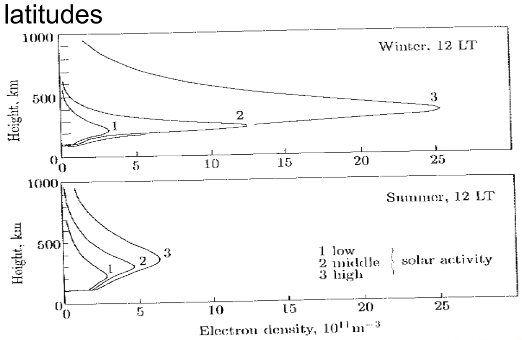
Electron density profile at noontime at mid-latitudes



Wright, 1967

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Ionization production

- Primary photoionization for an atom or molecule (X) in the upper atmosphere
  - X + photon (λ≤103 nm) ➔ X^+ + e
- Photoionization for the three predominant gases
  - O + photon ➔ O^+ + e
  - N_2+ photon ➔ N_2^+ + e
  - O_2+ photon ➔ O_2^+ + e

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Recall: Absorption processes

- Photodissociation (λ≤242 nm)
  - O_2 + photon ➔O + O
- Photoionization (λ≤103 nm)
  - O + photon ➔ O^+ + e
  - N_2+ photon ➔ N_2^+ + e
  - O_2+ photon ➔ O_2^+ + e
- Dissociative photoionization (λ≤72 nm)
  - N_2+ photon ➔ N^+ + N + e

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Recall: Extinction of photon flux in a gas volume

- Number of photons per unit area and time that get absorbed while passing through a distance ds of gas is..
  $\phi_{ph}(s)dw_{col}(ds)$
- Change in photon flux is…

$$\boxed{d\phi_{ph}(s) = -\phi_{ph}(s)dw_{col}(ds) = -\phi_{ph}(s)\sigma_A n(s)ds}$$

- Integration leads to..

$$\phi_{ph}(s) = \phi_{ph}(s_0)e^{-\tau(s)}$$

$$\tau = \int_{s_0}^{s} \sigma_A n(s')ds'$$

τ : Optical depth

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- The number of photons absorbed by the atmosphere per unit volume and time is

$$\frac{d\phi^{ph}}{ds} = \frac{dN_{ph}}{dVdt} = \sigma^{A} n \phi^{ph}$$

- When the photon is absorbed it can produce
  - Ionization
  - Dissociation
  - Excitation
- The fraction of absorbed photons that produce ionization is given by the ionization efficiency

$$\varepsilon_X^I$$

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- The number of ions produced from the main constituents , O, N_2 and O_2 by "primary photoionization (PI)" is

$$q_{X^+}^{PI} = \varepsilon_X^I \sigma_X^A n_X \phi^{ph}$$

- In the literature one can also find alternative expressions, for instance with the ionization cross section, σ_X^I

$$q_{X^+}^{PI} = \sigma_X^I n_X \phi^{ph}$$

- Or with the ionization frequency,

$$J_X = \varepsilon_X^I \sigma_X^A \phi_\infty^{ph}$$

$$q_{X^+}^{PI} = J_X n_X(h) e^{-\tau(h)}$$

X^+	Jx (10^{-7} s^{-1})
O^+	2 - 7
N_2^+	3 - 9
O_2^+	5 - 14
He^+	0.4 - 1
H^+	0.8 - 3

Proelss

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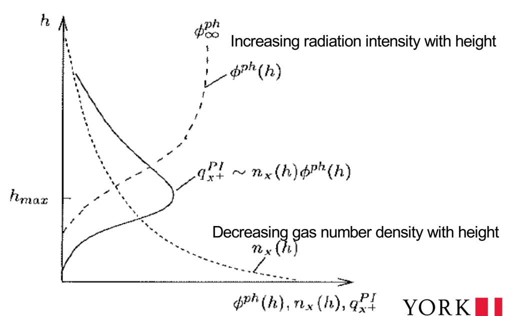
Formation of ionization production layers



Proelss

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- The interplay between $n_X(h)$, decreasing exponentially with height and $\phi^{ph}(h)$ increasing exponentially with height leads to a maximum for $q_{X^+}^{PI}$. That region around the maximum is called the production layer.
- This is a simplified derivation. A more realistic derivation has to take into account:
  - Not only one gas (X)
  - Not only photoionization but also dissociative ionization.

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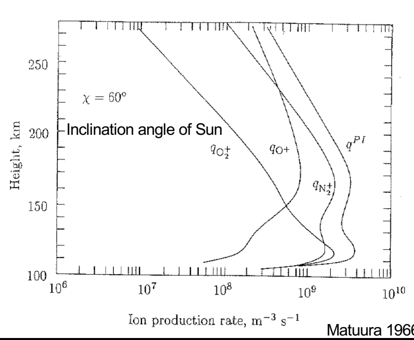
Ion production rate for most important constituents



Matuura 1966, Proelss

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Ionization losses

- If only ionization production were occurring, then ionization density would build up very quickly to an unrealistically high level.
  - Example: n(130 km)= 1×10^{11} m^{-3}
    q(130 km)= 3 x 10^9 m^{-3} s^{-1}
  - ➔ t = n/q= 30 s (time to build up the electron/ ion density)
- Ionization losses occur
  - dissociative recombination $XY^+ + e \rightarrow X^{(*)} + Y^{(*)}$
  - radiative recombination $X^+ + e \rightarrow X^{(*)}$ +photon
  - charge exchange $X^+ + Y \rightarrow X^{(*)} + Y^+$

(*) excited state

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Radio waves in the ionosphere

- Any plasma alters properties of an electromagnetic wave traversing it.
  - Propagation direction
  - Amplitude
  - Wave velocity
- Plasma characteristics can be determined by monitoring the changes
- Plasma can be used to change properties of the electromagnetic waves
- Key is to focus on the oscillations of the plasma electrons that are induced by the electric field of the radio wave.

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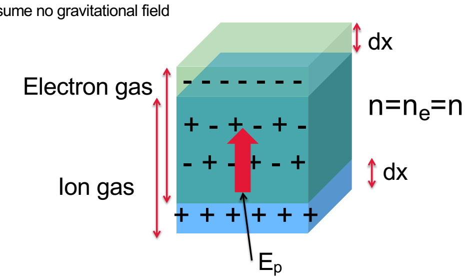
Plasma frequency

Assume no gravitational field



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Plasma frequency

- Applying an electric field will displace the electron gas from the ion gas and generate a polarization field, E_p.

$$\in_P = \frac{ne}{\varepsilon_0}\Delta x$$

- Turning E_p off, the electron gas will feel a force in the opposite direction that works against the displacement and accelerates the electrons in the direction of the ion gas. That force is

$$F_{\in_P} = -en\in_P = -\frac{e^2 n^2}{\varepsilon_0}\Delta x$$

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- The electrons overshoot the position of equilibrium causing the building-up of a polarization field again and the resulting force drives the electrons back into the other direction.
- The electron gas, and to a very small (ignorable) amount also the ion gas, oscillate about their equilibrium position.
- The electron – ion oscillation frequency is obtained from balancing the restoring force with the inertial force:

$$nm_e\frac{d^2(\Delta x)}{dt^2} = -\frac{e^2 n^2}{\varepsilon_0}\Delta x$$

- The solution is harmonic oscillation

$$\Delta x = (\Delta x)_0 \sin(\omega_p t)$$

Plasma frequency

Displacement amplitude

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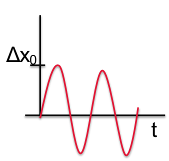
$$nm_e\frac{d^2[(\Delta x)_0 \sin(\omega_p t)]}{dt^2} = -\frac{e^2 n^2}{\varepsilon_0}(\Delta x)_0 \sin(\omega_p t)$$

$$nm_e\omega_p^2(\Delta x)_0 \sin(\omega_p t) = \frac{e^2 n^2}{\varepsilon_0}(\Delta x)_0 \sin(\omega_p t)$$

$$\boxed{\omega_p = \sqrt{\frac{e^2 n}{\varepsilon_0 m_e}} \quad \text{Plasma frequency}}$$



$f_P[Hz] \approx 9\sqrt{n[m^{-3}]}$ Rule of thumb

- However, damping processes have to be considered.
  - Friction $F_{fr}^* = -nm_e v_{e,s}^* u_e = -nm_e v_{e,s}^* \frac{d\Delta x}{dt}$ s: neutral and ion gas type, ➔ so two friction forces.
  - Radiative energy losses $F_{rd}^* = -nm_e v_{rd}^* u_e = -nm_e v_{rd}^* \frac{d\Delta x}{dt}$

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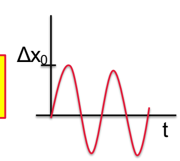
Example 4-1

$$\omega_p = \sqrt{\frac{(1.60\cdot 10^{-19})^2 \cdot n}{8.854\cdot 10^{-12}\cdot 9.11\cdot 10^{-31}}} =$$

$$nm_e\frac{d^2[(\Delta x)_0 \sin(\omega_p t)]}{dt^2} = -\frac{e^2 n^2}{\varepsilon_0}(\Delta x)_0 \sin(\omega_p t)$$

$$nm_e\omega_p^2(\Delta x)_0 \sin(\omega_p t) = \frac{e^2 n^2}{\varepsilon_0}(\Delta x)_0 \sin(\omega_p t)$$

$$\boxed{\omega_p = \sqrt{\frac{e^2 n}{\varepsilon_0 m_e}} \quad \text{Plasma frequency}}$$



$\boxed{f_P[Hz] \approx 9\sqrt{n[m^{-3}]}}$ Rule of thumb

- However, damping processes have to be considered.
  - Friction $F_{fr}^* = -nm_e v_{e,s}^* u_e = -nm_e v_{e,s}^* \frac{d\Delta x}{dt}$ s: neutral and ion gas type, ➔ so two friction forces.
  - Radiative energy losses $F_{rd}^* = -nm_e v_{rd}^* u_e = -nm_e v_{rd}^* \frac{d\Delta x}{dt}$

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Example 4-1

$\omega_p = \sqrt{\frac{(1.60\cdot 10^{-19})^2\cdot n}{8.854\cdot 10^{-12}\cdot 9.11\cdot 10^{-31}}}$ =n 56.3 rad/s

=n 8.97 Hz

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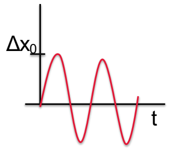

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Example 4-1

$\omega_p = \sqrt{\frac{(1.60\cdot 10^{-19})^2\cdot n}{8.854\cdot 10^{-12}\cdot 9.11\cdot 10^{-31}}}$ =56.3 $\sqrt{n}$ rad/s

f_p =8.97 $\sqrt{n}$ Hz

For n= 10^{12} m^{-3} ➡ $f_p = 9.0\ MHz$

$$nm_e\frac{d^2[(\Delta x)_0\sin(\omega_p t)]}{dt^2} = -\frac{e^2n^2}{\varepsilon_0}(\Delta x)_0\sin(\omega_p t)$$

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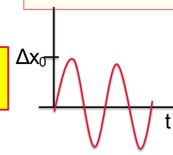

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    * : after collision
  - Radiative energy losses $F_{rd}^* = -nm_e v_{rd}^* u_e = -nm_e v_{rd}^* \frac{d\Delta x}{dt}$

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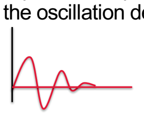
Damped oscillation

- The friction terms and radiative terms have to be added to the restoring force on the right side to yield:

$$nm_e\frac{d^2(\Delta x)}{dt^2} + nm_e v^*\frac{d\Delta x}{dt} + \frac{e^2n^2}{\varepsilon_0}\Delta x = 0$$

with $v^* = v_{e,n}^* + v_{e,i}^* + v_{rd}^*$

- The solution is an exponentially damped oscillation at the plasma frequency. That is, the plasma frequency does not change but the amplitude of the oscillation decreases exponentially with time.



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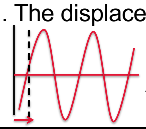
- Now we want to consider the case where the oscillation is forced by an external alternating electrical field with angular frequency ω and amplitude E_0. We then have

$$nm_e\frac{d^2(\Delta x)}{dt^2} + nm_e v^*\frac{d\Delta x}{dt} + \frac{e^2n^2}{\varepsilon_0}\Delta x = -en\in_0\sin(\omega t)$$

- After some time the oscillator, that is the electron gas oscillating around the ion gas, will assume the frequency of the forced oscillation with a phase shift, in general. The displacement then can be written as

$$\Delta x = (\Delta x)_0\sin(\omega t - \varphi)$$



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- With this equation inserted into the differential equation, we get an equation that defines amplitude and phase of the forced oscillation

$$-\omega^2(\Delta x)_0\sin(\omega t-\varphi) + \omega v^*(\Delta x)_0\cos(\omega t-\varphi) + \omega_p^2(\Delta x)_0\sin(\omega t-\varphi)$$

$$= -\left(\frac{e}{m_e}\right)\in_0\sin(\omega t)$$

- The solutions of this equation have two special forms
  - $\omega > \omega_P$:The ionosphere as a dielectric
  - $\omega < \omega_P$:The ionosphere as a conducting reflector

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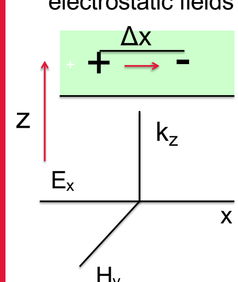
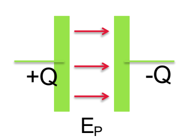
The ionosphere as a dielectric

- Dielectric: insulators but good supporter of electrostatic fields





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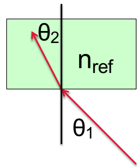
- Dielectric constant $\varepsilon = 1 - \frac{\omega_P^2}{\omega^2}$
- Index of refraction $n_{ref} = \sqrt{\varepsilon} = \sqrt{1 - \frac{\omega_P^2}{\omega^2}}$
- Shell's law $\sin\theta_2 = \frac{\sin\theta_1}{n_{ref}}$
- Phase velocity $\upsilon_{ph} = \frac{c_0}{n_{ref}} = \frac{c_0}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$
- Group velocity $\upsilon_{gr} = c_0 n_{ref} = c_0\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$



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(part) Derivation for pundits

- We want to consider a plane wave that travels along the z direction in a plasma with an angular frequency much larger than the plasma frequency. The E vector is oscillating along the x-axis and the vector $\hat{x}$ is the unit vector.

$$\vec{E} = E_0 e^{i(kz-\omega t)}\hat{x}$$

- The peak or any other part of the plane wave travels at the phase velocity

$$\upsilon_{ph} = \frac{\omega}{k} = \frac{c_0}{n_{ref}} = \frac{c_0}{\sqrt{\varepsilon}} = \frac{c_0}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$$

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- Equivalent to the last equation is the dispersion relation which can be shown to be

$$\omega^2 = k^2 c_0^2 + \omega_p^2$$

- The phase velocity in a plasma is greater than the speed of light in a vacuum!
- How can that be if no information can travel faster than the speed of light?
- ➔The phase velocity does not carry information!

$$\upsilon_{gr} = \frac{d\omega}{dk} = c_0\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

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- The group velocity carries the information. That follows from considering not a wave at infinitely small bandwidth but a wave with a bandwidth larger than 0.
- Group velocity

$$\upsilon_{gr} = \frac{d\omega}{dk} = c_0\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

- The group velocity is indeed smaller than the speed of light!

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The ionosphere as a conducting reflector

- When the frequency of the electromagnetic wave is smaller than the plasma frequency, then the index of refraction becomes imaginary.
- Phase and group velocities become imaginary.
- According to the dispersion relation, the associated wave number is

$$k = \frac{i\sqrt{\omega_P^2 - \omega^2}}{c_0} = i|k|$$

- The wave then becomes a decaying standing wave.

$$E_x = E_0 e^{i(i|k|z-\omega t)} = E_0 e^{-|k|z} e^{-i\omega t}$$

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- The wave will not propagate through the plasma
- The wave will be totally reflected
- This property of the plasma can be used to study the ionosphere
- A pulsed wave train at frequency ω can be transmitted upward through the ionosphere
- The reflection height, $h_{refl} = 0.5\ c_0 \Delta t$ can be measured from the travel time.
- The entire ionosation density profile can be determined.

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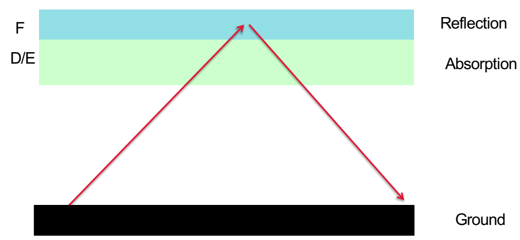
Subionospheric radio communication



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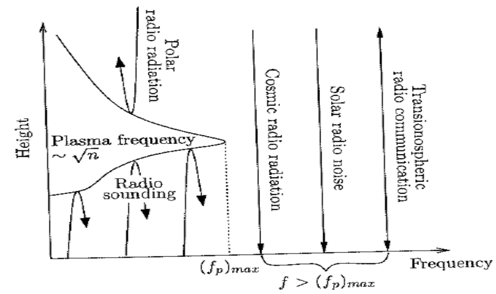
Reflection and transmission of radio waves



$f_{p,\,max}$: critical frequency

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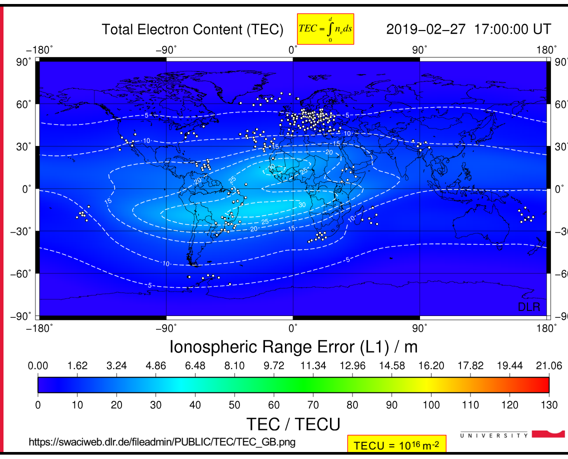

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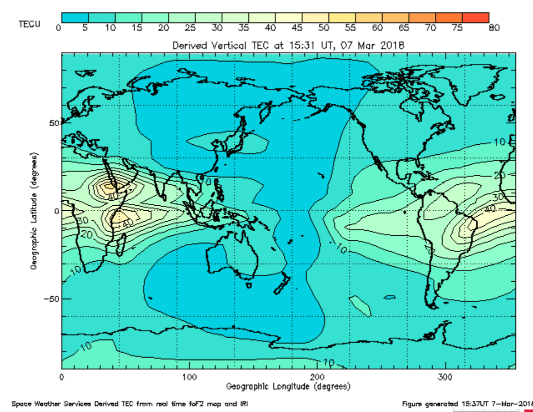

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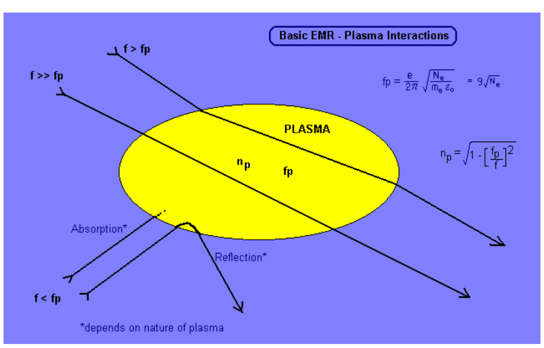

http://www.spaceacademy.net.au/env/spwx/raiono.htm

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