The Newtonian Synthesis

The Mathematical Principles of Natural Philosophy

The Falling Apple

- According to Newton, it was while he was in the orchard at Woolsthorpe during the plague years of 1665-1666 that he noticed an apple fall and realized that whatever made it fall also kept the Moon in its orbit around the Earth.

From Falling Apple to *Principia*

- The falling apple insight started Newton on the path that brought together the insights of Renaissance astronomy and physics into a comprehensive system.
- It took another 20 years before he was ready to put it all together in *Principia Mathematica – The Mathematical Principles of Natural Philosophy.*
Concepts considered by Newton

- *Kepler's Laws:*
  - 1. Elliptical orbits of planets.
  - 2. Planets sweep out equal areas in equal times.
  - 3. Harmonic law: \( D^3/T^2 = K \), providing a formula that relates the period of revolution of a planet, \( T \), to its distance from the Sun, \( D \).

Concepts considered by Newton, 2

- *Galileo's findings:*
  - 1. Times square law for falling bodies.
  - 2. Projectiles in parabolic path.

Concepts considered by Newton, 3

- *Descartes' Principles:*
  - 1. Motion is natural.
  - 2. Inertia: Bodies in motion tend to stay in motion in a straight line — unless forced from it.
  - 3. All motion due to impact.
    - Forces are occult — i.e., forbidden in a mechanical system.
Aristotle's philosophical approach to physics

1. Two separate realms:
   - The heavens and the earth.
2. Heavenly motions:
   - Eternal, changeless, and always circular.
3. Earthly motions:
   - Either natural or forced.
   - Natural motion either up (light things) or down (heavy things) – bodies seek their natural places.
   - Forced motions caused by pushes – Cannot occur "naturally."

Euclid’s Mathematical approach to certain knowledge

Axiomatic Structure:
- Definitions
- Axioms & Postulates
- Rules of reasoning
- Begin from reasonable assumptions and through logic and other strict rules of inference, build up a body of knowledge.

The Lucasian Professor of Mathematicks

- Newton returned to Cambridge after the plague.
- After a few years his former mathematics professor, Isaac Barrow, resigned his position, and recommended that Newton be his replacement.
- Newton became the 2nd Lucasian Professor of Mathematicks, a position he held for 27 years.
Newton’s sporadic output

- Over the next 15-20 years, Newton published a work on the calculus, the ideas of which he was accused of stealing from Leibniz, and some of his work on light, which Robert Hooke claimed he had conceived of first.
- Newton, disgusted, retreated into his own studies, publishing nothing.

Edmund Halley’s Visit

- One of Newton’s few friends was the astronomer, Edmund Halley.
- In 1684, Halley and architect Christopher Wren, speculated that the force that held the planets in their orbits must be inversely related to their distance from the sun.
- Halley thought Newton might be able to settle the matter.

Halley’s question

- Instead of asking Newton what kind of force would hold the planets in their orbits, Halley asked Newton what curve would be produced by a force of attraction that diminished with the square of the distance.
- Newton replied immediately, “An Ellipse.”
  - Halley asked for the proof, but Newton could not find it, and promised to send it to him.
Newton's first draft

- Newton sent Halley a nine page proof three months later.
- Halley urged Newton to publish it, but Newton refused, realizing that the consequences were far greater than the solution to that problem.
- For 18 months, Newton developed the theory farther.

The Principia

- Finally, three years after Halley's visit, Newton's results were published—at Halley's expense—in the single most important work in the history of science: *Philosophiæ Naturalis Principia Mathematica*, translated as *The Mathematical Principles of Natural Philosophy*, published in 1687.

The title tells all...

- Descartes' attempt at a new system of philosophy was *The Principles of Philosophy*.
- Newton adds two words:
  - Natural – referring to the physical world only, not to *res cogitans*.
  - Mathematical – perhaps not *all* of the principles of philosophy, just the mathematical ones.
The Axiomatic Structure of Newton’s *Principia*

- Definitions, axioms, rules of reasoning, just like Euclid.
- Examples:
  - Definition
    - 1. The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.
    - How Newton is going to use the term “quantity of matter.”

Rules of Reasoning

- 1. We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.
  - This is the well-known Principle of Parsimony, also known as Ockham’s Razor. In short, it means that the best explanation is the simplest one that does the job.

The Axioms

- 1. Every body continues in its state of rest of or uniform motion in right line unless it is compelled to change that state by forces impressed upon it.
  - This is Descartes’ principle of inertia. It declares that straight-line, constant speed, motion is the natural state. Force is necessary to change that motion.
  - Compare this to Aristotle’s need to explain motion.
The Axioms, 2

2. The change in motion is proportional to the motive force impressed and is made in the direction of the right line in which that force is impressed.

3. A force causes a change in motion, and does so in the direction in which the force is applied.

The Axioms, 3

3. To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts.

4. Push against any object; it pushes back at you. This is how any object is held up from falling, and how a jet engine works.

Known Empirical Laws Deduced

Just as Euclid showed that already known mathematical theorems follow logically from his axioms, Newton showed that the laws of motion discerned from observations by Galileo and Kepler followed from his axiomatic structure.
Galileo’s Laws

Galileo’s laws about physics on Earth:
- The law of free fall.
  - Galileo asserts that falling bodies pick up speed at a uniform rate.
  - Newton shows that a constant force acting in line with inertial motion would produce a constant acceleration. This is implied by his first 2 axioms.
- The parabolic path of a projectile.
  - Likewise, if a body is initially moving inertially (in any direction), but a constant force pushes it downwards, the resulting path will be a parabola.

Kepler’s Laws

- Newton’s very first proposition is Kepler’s 2nd law (planets sweep out equal areas in equal times).
- It follows from Newton’s first two axioms (inertial motion and change of motion in direction of force) and Euclid’s formula for the area of a triangle.

Kepler’s 2nd Law illustrated

- In the diagram, a planet is moving inertially from point A along the line AB.
- S is the Sun. Consider the triangle ABS as “swept out” by the planet.
- When the planet gets to B, Newton supposes a sudden force is applied to the planet in the direction of the sun.
- This will cause the planet’s inertial motion to shift in the direction of point C.
Kepler’s 2nd Law illustrated, 2

- Note that if instead of veering off to C, the planet continued in a straight line it would reach c (follow the dotted line) in the same time.
- Triangles ABS and BcS have equal area.
  - Equal base, same height.

Kepler’s 2nd Law illustrated, 3

- Newton showed that triangles BCS and BcS also have the same area.
  - Think of BS as the common base. C and c are at the same height from BS extended.
- Therefore ABS and BCS are equal areas.
  - Things equal to the same thing are equal to each other.

Kepler’s 2nd Law illustrated, 4

- Now, imagine the sudden force toward the sun happening in more frequent intervals.
  - The smaller triangles would also be equal in area.
- In the limiting case, the force acts continuously and any section taking an equal amount of time carves out an equal area.
The Same Laws of Motion in the Heavens and on Earth

- Newton's analysis showed that from the same assumptions about motion, he could account for the parabolic path of a projectile on Earth and for a planet (or the Moon) in orbit.

A Mechanical system

- Newton's axiomatic "principles" implied a mechanistic model of the universe.
  - This was all that made sense to Newton.
- The Clockwork Universe
  - God makes clock and winds it up.

Universal Gravitation

- A deduced effect
  - That which makes apples fall and the moon stay in orbit.
  - And the planets, and projectiles, etc.
- The gravitational force:
  \[ G = \frac{g(M_1 M_2)}{d^2} \]
  - The force varies inversely with square of distance.
  - It gets much weaker as the distance between objects is greater, but never disappears entirely.
Action at a Distance

- Gravity, and magnetism too, operate over apparently empty space.
- Is this an occult force?
- Newton postulates an "Aether" to transmit gravity, magnetism, etc.
- Makes empty space no longer empty.
  - Note the return to Parmenides' and Aristotle's denial of the existence of "nothing."

Hypotheses non fingi

- Unlike Aristotle (but like Galileo), Newton did not claim to have an explanation for everything.
- For example, he described how gravity works, on the basis of the effects seen. He does not say what gravity is.
- On this and other mystery subjects, Newton said that he "frames no hypotheses."

How the Newtonian world view transformed our view of Nature

Different ways of doing scientific work
The Whig Interpretation of History

- Seeing (British) history as a battle between
  - the progressive, forward-looking Whigs
  and
  - the reactionary, backward-looking Tories.

Herbert Butterfield

- *The Whig Interpretation of History*
- Butterfield showed that the "whig interpretation" was a fundamental problem in writing political history.

The Whig Interpretation of Science

- The same flaw occurred in writing the history of science.
- Even more acute in the history of science because of the inevitable conclusion that present day science is right and past science was wrong.
Hugh Kearney

- *Science and Change, 1500-1700.*
- An attempt to combat a whig interpretation of the scientific revolution.

Kearney’s Three Traditions in Science

- The Organic. The Magical. The Mechanist.

The Organic Viewpoint

- Common sense.
- Empirical.
- Coherent and logical.
- The goal was to explain the purpose (why) of something in nature.
- Focus on cycles.
  - Life cycles.
    - Generation and corruption.
  - Planetary cycles.
  - Ignored accident.
The Organic Tradition in Antiquity

- Aristotle
  - Biological interests
  - History of Animals
  - Purpose the ultimate cause
- Ptolemy
  - Cycles of the planets and the heavens
- Galen
  - Physiology

The Organic Tradition in the Middle Ages

- Domained the Middle Ages and the early Renaissance, especially in Europe.
- Scholasticism, 14th century.
  - William of Ockham, Ockham’s Razor.

Padua in the 15th and 16th century

- Andreas Vesalius
  - De Fabrica, 1543
- William Harvey
  - De Motu Cordis, 1628
The Organic Tradition in the Scientific Revolution

- Spokesman: Francis Bacon
  - Popularized experiment (i.e., observation).
  - Criticized acceptance of authority.
  - Science as induction from particulars.
  - Ignored mathematics.

Title page of Bacon's Great Instauration

The Magical Viewpoint

- The search for secrets.
- Solving the riddle of nature.
- Hidden structures, forces.
- Magical powers.
- The scientist as wizard, sorcerer, high priest, soothsayer.

The Magical Tradition in Antiquity

- Pythagoras
  - Number magic
  - Secretive cult
- Plato
  - Upper part of the Divided Line
  - Mathematics the key to higher understanding.

Intelligible World

Forms, Ideas

Mathematics, Logic
The Magical Tradition in Antiquity, 2

- Hermes Trismegistus
  - A mythical figure.
  - Held that the Sun was God, or a symbol of God.
  - Light, the source of life.
  - Mathematical harmony in the cosmos.

The Magical Tradition in the Middle Ages and Renaissance

- The Magical Tradition has never dominated, but has never been totally ignored.
- Neoplatonism.
- Hermeticism.
- Alchemy.
- Astrology.

The Magical Tradition viewed as out of touch

- A painting by Pieter Breughel, the Elder, showing alchemists as irresponsible and oblivious to the outside world.
The Magical Tradition in the Scientific Revolution

- Copernicus
  - The Sun the centre of the universe and the source of all life
  - Mathematics is for mathematicians.
  - Mathematical harmony trumps common sense.
- Giordano Bruno
  - The infinity of worlds.
  - The universe is magical.

The Magical Tradition in the Scientific Revolution, 2

- William Gilbert
  - *On the Magnet.*
  - Action at a distance.
- Paracelsus
  - The human body as a microcosm
  - Iatrochemistry.
- Johannes Kepler
  - Mathematical relationships are the ultimate secrets of the universe.

The Mechanist Viewpoint

- The world is (like) a machine.
- Understand the world through analogies to machines.
- Everything to be explained by combinations of pushes and pulls.
- No hidden forces or mysterious influences.
- Emphasis on “how” – not “why.”

![A mechanical hand](image-url)
The Mechanist Tradition in Antiquity and the Middle Ages

Archimedes
- Levers, pulleys, floating bodies, ingenious machines.
- Archimedes asked how does it work?

In the Middle Ages
-Craftsmen, builders of windmills, waterwheels, devices of all sorts.
-What later became “engineers.”

The Mechanist Tradition in the Scientific Revolution

Niccolo Tartaglia
- Cannonball trajectory.
-Translated Archimedes and Euclid.
The Mechanist Tradition in the Scientific Revolution, 2

- Galileo
  - Simplify problem, make model, find mechanism.
  - Describe mathematically
  - Avoid system building

- René Descartes
  - Forces are occult

The Mechanist Tradition in the Scientific Revolution, 3

- Evangelista Toricelli and Blaise Pascal
  - Atmospheric pressure and the barometer
  - The Puy de Dôme experiment, carrying a barometer up the mountain and noting the fall in atmospheric pressure—the sea of air.

- Robert Boyle
  - Measurement in chemistry
  - Boyle’s law, PV=K
The Mechanist Tradition in the Scientific Revolution, 5

- The Royal Society of London for Improving Natural Knowledge
  - Founded in 1662
  - Patron, Charles II
  - Founded on Baconian precepts (build knowledge from observation), members became later committed to the mechanist viewpoint.

The Genius of Isaac Newton

- Newton combined the mechanist and the magical viewpoints.
- The mathematical formulation was the true description, according to Newton.

The clockwork universe—that needed winding up and resetting

- The world operates as a vast machine—the "clockwork universe."
  - God (a supernatural and definitely not mechanical force) made things work when the mechanism failed.
  - The Universe is a riddle.
  - Gravity is action at a distance.
At the end of the Scientific Revolution

- Mechanism triumphs.
- The Organic viewpoint is in disrepute.
- The incompatibility of the Magical and the Mechanist views are forgotten or ignored.
- Mathematics is accepted as the language of science.
- The mechanical model is accepted as the ultimate explanation.

The Newtonian Model for true knowledge

- Axiomatic presentation.
- Mathematical precision and tight logic.
- With this Euclidean style, Newton showed that he could (in principle) account for all observed phenomena in the physical world, both in the heavens and on Earth.
- Implication: All science should have this format.
- This became the model for science.

Newtonianism

- The application of the Newtonian model beyond physics, e.g. in philosophy, psychology, sociology, economics.
- John Locke, *Essay on Human Understanding*
- Benedict Spinoza, *Tractatus Theologia*
- Adam Smith, *Wealth of Nations*