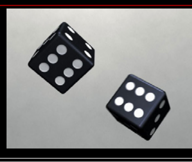
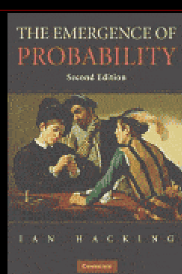
The History of Probability



Text for this lecture:

- *The Emergence of Probability*, 2nd Ed., by Ian Hacking
- Hacking was professor of the philosophy of science at the University of Toronto.



Hacking's thesis

- Probability emerged as a coherent concept in Western culture around 1650.
- Before then, there were many aspects of chance phenomena noted, but not dealt with systematically.

Gaming

- Gaming apparently existed in the earliest civilizations.
  - E.g., the *talus* – a knucklebone or heel bone that can land in any of 4 different ways. – Used for amusement.

Randomizing

- The talus is a randomizer. Other randomizers:
  - Dice.
  - Choosing lots.
  - Reading entrails or tea leaves.
- Purpose:
  - Making “fair” decisions.
  - Consulting the gods.

Emergence of probability

- All the things that happened in the middle of the 17th century, when probability “emerged”:
  - Annuities sold to raise public funds.
  - Statistics of births, deaths, etc., attended to.
  - Mathematics of gaming proposed.
  - Models for assessing evidence and testimony.
  - “Measurements” of the likelihood/possibility of miracles.
  - “Proofs” of the existence of God.

The Pascal – Fermat correspondence of 1654

- Often cited in histories of mathematics as the origin of probability theory.

The Problem of Points

- Question posed by a gambler, Chevalier De Mere and then discussed by Pascal and Fermat.
  - There are many versions of this problem, appearing in print as early as 1494 and discussed earlier by Cardano and Tartaglia, among others.
- Two players of equal skill play a game with an ultimate monetary prize. The first to win a fixed number of rounds wins everything.
- How should the stakes be divided if the game is interrupted after several rounds, but before either player has won the required number?

Example of the game

- Two players, A and B.
  - The game is interrupted when A needs *a* more points to win and B needs *b* more points.
  - Hence the game can go at most $a + b - 1$ further rounds.
  - E.g. if 6 is the total number of points needed to win and the game is interrupted when A needs 1 more point while B needs 5 more points, then the maximum number of rounds remaining is 1+5-1=5.

The Resolution

- Pascal and Fermat together came to a resolution amounting to the following:
- A list of all possible future outcomes has size 2^{a+b-1}
- The fair division of the stake will be the proportion of these outcomes that lead to a win by A versus the proportion that lead to a win by B.

The Resolution, 2

- Previous solutions had suggested that the stakes should be divided in the ratio of points already scored, or a formula that deviates from a 50:50 split by the proportion of points won by each player.
- These are all reasonable, but arbitrary, compared with Pascal & Fermat's solution.
- Note: It is assumed that all possible outcomes are equally likely.

The historian's question: why 1650s?

- Gambling had been practiced for millennia, also deciding by lot. Why was there no mathematical analysis of them?
- The Problem of Points appeared in print in 1494, but was only solved in 1654.
  - What prevented earlier solutions?

The "Great Man" answer:

- Pascal and Fermat were great mathematical minds. Others simply not up to the task.
- Yet, all of a sudden around 1650, many problems of probability became commonplace and were understood widely.

The Determinism answer:

- Science and the laws of Nature were deterministic. What sense could be made of chance if everything that happened was fated? Why try to understand probability if underneath was a certainty?

"Chance" is divine intervention

- Therefore it could be viewed as impious to try to understand or to calculate the mind of God.
  - If choosing by lot was a way of leaving a decision to the gods, trying to calculate the odds was an impious intervention.

The equiprobable set

- Probability theory is built upon a fundamental set of equally probable outcomes.
- If the existence of equiprobable outcomes was not generally recognized, the theory of them would not be built.
  - Viz: the ways a talus could land were not equally probable. But Hacking remarks on efforts to make dice fair in ancient Egypt.

The Economic necessity answer:

- Science develops to meet economic needs. There was no perceived need for probability theory, so the explanation goes.
  - Error theory developed to account for discrepancies in astronomical observations.
  - Thermodynamics spurred statistical mechanics.
  - Biometrics developed to analyze biological data for evolutionary theory.

Economic theory rebuffed:

- Hacking argues that there was plenty of economic need, but it did not spur development:
  - Gamblers had plenty of incentive.
  - Countries sold annuities to raise money, but did so without an adequate theory.
    - Even Isaac Newton endorsed a totally faulty method of calculating annuity premiums.

A mathematical answer:

- Western mathematics was not developed enough to foster probability theory.
- Arithmetic: Probability calculations require considerable arithmetical calculation. Greek mathematics, for example, lacked a simple numerical notation system.
  - Perhaps no accident that the first probabilists in Europe were Italians, says Hacking, who first worked with Arabic numerals and Arabic mathematical concepts.
  - Also, a “science of dicing” may have existed in India as early as year 400. Indian culture had many aspects that European culture lacked until much later.

Duality

- The dual nature of the understanding of probability that emerged in Europe in the middle of the 17th century:
  - Statistical: concerned with stochastic laws of chance processes.
  - Epistemological: assessing reasonable degrees of belief.

The Statistical view

- Represented by the Pascal-Fermat analysis of the problem of points.
- Calculation of the relative frequencies of outcomes of interest within the universe of all possible outcomes.
  - Games of chance provide the characteristic models.

The Degree of Belief view

- Represented by efforts to quantify the weighing of evidence and/or the reliability of witnesses in legal cases.
- Investigated by Gottfried Leibniz and others.

The controversy

- Vast and unending controversy over which is the “correct” view of probability:
  - The frequency of a particular outcome among all possible outcomes, either in an actual finite set of trials or in the limiting case of infinite trials.
- Or
  - The rational expectation that one might hold that a particular outcome will be a certain result.

Independent concepts, or two sides of the same issue?

- Hacking opines that the distinction will not go away, neither will the controversy.
- Compares it to distinct notions of, say, weight and mass.

The "probable"

- Earlier uses of the term probable sound strange to us:
  - Probable meant approved by some authority, worthy of approprobation.
    - Examples from Gibbon's *Decline and Fall of the Roman Empire:* one version of Hannibal's route across the Alps having more probability, while another had more truth. Or: "Such a fact is probable but undoubtedly false."

Probability versus truth

- Pascal's contribution to the usage of the word probability was to separate it from authority.
- Hacking calls it the demolition of probabilism, decision based upon authority instead of upon demonstrated fact.

Opinion versus truth

- Renaissance scientists had little use for probability because they sought incontrovertible demonstration of truth, not approbation or endorsement.
- Opinion was not important, certainty was.
  - Copernicus's theory was "improbable" but true.

Look to the lesser sciences

- Physics and astronomy sought certainties with definitive natural laws. No room for probabilities.
- Medicine, alchemy, etc., without solid theories, made do with evidence, indications, signs. The probable was all they had.
- This is the breeding ground for probability.

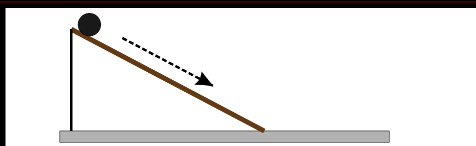
Evidence

- Modern philosophical claim:
  - Probability is a relation between an hypothesis and the evidence for it.
  - Hence, says Hacking, we have an explanation for the late emergence of probability:
    - Until the 17th century, there was no concept of evidence (in any modern sense).

Evidence and Induction

- The concept of evidence emerges as a necessary element in a theory of induction.
- Induction is the basic step in the formation of an empirical scientific theory.
  - None of this was clarified until the Scientific Revolution of the 16th and 17th centuries.

The classic example of evidence supporting an induction:



- Galileo's inclined plane experiments.
  - Galileo rolled a ball down an inclined plane hundreds of times, at different angles, for different distances, obtaining data (evidence) that supported his theory that objects fell (approached the Earth) at a constantly accelerating rate.

Kinds of evidence

- Evidence of things – i.e., data, what we would accept as proper evidence today.
  - Called "internal" in the *Port Royal Logic*.

Versus

- Evidence of testimony – what was acceptable prior to the scientific revolution.
  - Called "external" in the *Port Royal Logic.*
    - *The Port Royal Logic*, published in 1662. To be discussed later.

Signs: the origin of evidence

- The tools of the "low" sciences: alchemy, astrology, mining, and medicine.
- Signs point to conclusions, deductions.
  - Example of Paracelsus, appealing to evidence rather than authority (yet his evidence includes astrological signs as well as physiological symptoms)
  - The "book" of nature, where the signs are to be read from.

Transition to a new authority

- The book written by the "Author of the Universe" appealed to by those who want to cite evidence of the senses, e.g. Galileo.
- High science still seeking demonstration. Had no use for probability, the tool of the low sciences.

Calculations

- The incomplete game problem.
  - This is the same problem that concerned Pascal and Fermat.
  - Unsuccessful attempts at solving it by Cardano, Tartaglia, and G. F. Peverone.
  - Success came with the realization that every possible permutation needs to be enumerated.
- Dice problems
  - Confusion between combinations and permutations
- Basic difficulty of establishing the Fundamental Set of equiprobable events.

What about the Pascal-Fermat correspondence?

- Hacking says it set the standard for excellence for probability calculations.
- It was reported by many notables:
  - Poisson: "A problem about games of chance proposed to an austere Jansenist [Pascal] by a man of the world [Méré] was the origin of the calculus of probabilities."
  - Leibniz: "Chevalier de Méré, whose *Agréments* and other works have been published—a man of penetrating mind who was both a gambler and philosopher—gave the mathematicians a timely opening by putting some questions about betting in order to find out how much a stake in a game would be worth, if the game were interrupted at a given stage in the proceedings. He got his friend Pascal to look into these things. The problem became well known and led Huygens to write his monograph *De Aleae*. Other learned men took up the subject. Some axioms became fixed. Pensioner de Witt used them in a little book on annuities printed in Dutch."

The Roannez Circle

- Artus Gouffier, Duke of Roannez, 1627-1696
- His salon in Paris was the meeting place for mathematicians and other intellectuals, including Leibniz, Pascal, Huygens, and Méré.
- Méré posed several questions to Pascal about gambling problems.
  - Solving the problem led Pascal to further exploration of the coefficients of the binomial expansion, known to us as Pascal's triangle.

Pascal and Decision Theory

- Hacking attributes great significance to Pascal's "wager" about belief in God, seeing the reasoning in it as the foundation for decision theory. ("How aleatory arithmetic could be part of a general 'art of conjecturing'.")
- Infini—rien (infinity—nothing)
  - Written on two sheets of paper, covered on both sides with writing in all directions.

Decision theory

- The theory of deciding what to do when it is uncertain what will happen.
- The rational, optimal decision, is that which has the highest *expected value*.
  - Expected value is the product of the value (payoff) of an outcome multiplied by its probability of occurrence.
    - E.g. expected value of buying a lottery ticket = sum of product of each prize times probability of winning it.

Decision theory, 2

- Three forms of decision theory argument:
  - Dominance: one course of action is better than any other under all circumstances.
  - Expectation: one course of action, A_i, has the highest expected value:
    - Let p_i = probability of each possible state, S_i
    - Let U_{ij} = utility of action A_j in state Si
    - Expectation of $A_j = \sum p_i U_{ij}$ over I
  - Dominating expectation: where the probabilities of each state is not known or not trusted, but partial agreement on probabilities assigns one action a higher probability than any other, then that action has dominating expectation.

Pascal's Wager as decision theory

- Two possible states: God exists or He does not.
- Two possible actions: Believe and live a righteous life or don't believe and lead a life of sin.
- Four outcomes:
  - God exists X righteous life ➔ salvation
  - God exists X sinful life ➔ eternal damnation
  - God does not exist X righteous life ➔ no harm done
  - God does not exist X sinful life ➔ finite life span of riotous living
- Dominance case: Believing simply dominates over non-believing if the situation is equivalent in the case that God does not exist.
- Expectation case: But if believing (and living righteously) foregoes the pleasures of sin, then believing does not simply dominate. However if the consequences in the case of God's existence are greatly in excess of those in the event of non-existence (salvation vs damnation as opposed to indifference vs. fun), then believing has the highest expected value.
- Dominating expectation: Since the probability of God existing is not known, Pascal appeals to dominance of one expectation over another: *Infinite salvation or damnation* versus something finite.

Pascal's Wager

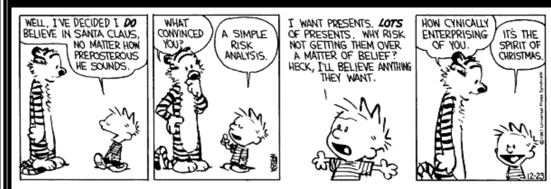
		Existence	
		No	Yes
Conduct	Righteous	Oh Well	Whew
	Sinful	Fun (short-term)	Damn (eternal)

Pascal's wager much quoted, often misrepresented

- It was transformed and re-stated by many theologians and used as an argument for the existence of God or for righteous living.
- It was criticized as faulty by many who saw it as manipulative and impious.
  - E.g., William James suggestion that those who became believers for the reasons given by Pascal were not going to get the payoff anticipated.

Cartoon versions:

- Calvin & Hobbes:



Cartoon versions:

Epistemic probability

- Chance, understood as “odds” of something happening is a quantitative notion.
- Not so with evidence, in the sense of legal evidence for a charge.
- The concept of epistemic probability did not emerge until people though of measuring it, says Hacking.

The word "probability" itself

- First used to denote something measurable in 1662 in the Port Royal Logic.
  - *La logique, ou l'art de penser* was the most successful logic book of the time.
    - 5 editions of the book from 1662 to 1683.
    - Translations into all European languages.
    - Still used as a text in 19th century Oxford & Edinburgh.

Port Royal Logic

- Written by Pascal's associates at Port Royal, esp. Pierre Nicole and antoine Arnauld.
- Arnaud seems to have written all of Book IV, the section on probability.
- Arnauld also wrote the Port Royal *Grammar*, his chief contribution to philosophy

Probability measured in the Port Royal Logic

- Example given of a game where each of 10 players risks one coin for an even chance to win 10.
- Loss is 9 times more probable "*neuf fois plus probable"* than gain. And later, there are "nine degrees of probability of losing a coin for only one of gaining nine."
- These are the first occasions in print where probability is measured.

Frequency used to measure chance of natural events

- Author of Port Royal advocates that people's fear of thunder should be proportional to the frequency of related deaths (lightning, etc.).
- Frequency of similar past events used here as a measure of the probability of the future event.
- Note that the frequency measure does not work if the payoff is not finite. Hence Pascal's wager: slight chance of eternal salvation trumps all other options.

Difficulties of quantifying evidence

- Measuring the reliability of witnesses.
  - How? Past reliability? Reputation? How to make judgements comparable?
  - Very difficult is the evidence is of totally different kinds.
  - Example of verifying miracles.
  - Internal vs. external evidence

Language, the key to understanding nature

- Big subject of interest in mid 17th century was language. Thinking was that if language was properly understood then Nature would become understandable.
  - The notion that their was an inherent "Ur-language" that underlies every conventional language.
  - Underlying assumption, that there is a plan to nature. Understanding it's "true" language will lead to understanding nature itself.

Probability as a tool of jurisprudence

- As a young man of 19, Leibniz published a paper proposing a numerical measure of proof for legal cases: “degrees of probability.”
- His goal was to render jurisprudence into an axiomatic-deductive system akin to Euclid.

Natural jurisprudence

- Evidence (a legal notion), to be measured by some system that will make calculation of justice possible.
  - Leibniz more sanguine that this can be done than Locke, who viewed it as “impossible to reduce to precise rules the various degrees wherin men give their assent.”
  - Leibniz believed that a logical analysis of conditional implication will yield such rules.

The dual approach to probability revealed

- Hacking’s thesis is that our concept of probability in the West emerged as a dual notion:
  1. *Frequency* of a particular outcome compared to all possible results
  2. *Degree of belief* of the truth of a particular proposition.
- This duality can be seen in the 17th century thinkers 1st publications.

Port Royal Logic and frequency

- The Port Royal Logic text and the Pascal-Fermat correspondence concern random phenomena.
  - The actual cases come from gaming, where there are physical symmetries that lead to easy assignment of the equipossible event and hence of simple mathematical calculation in terms of combinations and permutations.
- Or, applications are made to such statistics as mortality, with an assumption of a random distribution.

Leibniz and the epistemic approach

- Leibniz began from a legal standpoint, where the uncertainty is the determination of a question of right (e.g., to property) or guilt.
- Leibniz believed that mathematical calculations were possible, but did not have the model of combinations and permutations in mind.

Expectation and the Average

- Hacking remarks that mathematical expectation should have been an easier concept to grasp than probability.
  - In a random situation, such as gaming or coin tosses, the mathematical expectation is simply the average payoff in a long run of similar events.
  - But the problem is that the notion of "average" was not one people were familiar with in the mid-17th century.

Expectation in Huygens' text

- Christiaan Huygens, *Calculating in Games of Chance*, 1657 (De rationcinis in aleae ludo), the first printed textbook of probability.
- Huygens had made a trip to Paris and learned of the Pascal-Fermat correspondence. He became a member of the Roannez Circle and met Méré.

Expectation as the "fair price" to play

- Huygens' text is about gambling problems. His concept of mathematical expectation, the possible winnings multiplied by the frequency of successes divided by all possible outcomes, was given as the "fair price" to play.
- In the long run (the limit of successive plays) paying more than the expectation will lose money, paying less will make money. The expected value expresses the point of indifference.

But that is in the limit, implying potentially infinite rounds of playing

- A major difficulty arises when the assertion arises that the mathematical expectation is the price of indifference for a single play.
- Hacking cites the example of the Coke machine that charges 5 cents for a bottle of Coke that retails at 6 cents, but one in every six slots in the machine is empty.
  - 5 cents is the expected value, but a given customer will either get a 6-cent Coke or nothing.

Expectation in real life

- A major practical application of probability calculations is to calculate the fair price for an annuity.
- Here the question of expectation is that of expected duration of life.
  - A major complication here is confusion as to the meaning of averages, e.g., the *mean* age at death of a newly conceived child was 18.2 years as calculated from mortality tables by Huygens, but the *median* age, at which half of those newly conceived would die was 11 years old.
  - This illustrates the problem of using a theory built upon simple games of chance in real life, where the relevant factors are much more complex.

Political Arithmetic, a.k.a. statistics

- John Graunt's *Natural and Political Observations*, 1662, was the first treatise that analyzed publicly available statistics, such as birth and death records, to draw conclusions about public issues.
  - Population trends
  - Epidemics
  - Recommendations about social welfare.

Social welfare: the guaranteed annual wage

- Graunt recommended that Britain establish a guaranteed annual wage (welfare) to solve the problem of beggars. His reasoning:
  1. London is teeming with beggars.
  2. Very few actually die of starvation.
  3. Therefore there is clearly enough wealth in the country to feed them, though now they have to beg to get money to eat.
  4. It's no use putting them to work, because their output will be substandard and will give British products a bad reputation, driving up imports and losing business to Holland (where there already was a system of welfare payments).
  5. Therefore, the country should feed the beggars and get them off the streets where they are a nuisance.

What actually happened…

- Britain passed the Settlement and Removal law, establishing workhouses for the poor.
- Result: Just what Graunt predicted, shoddy goods were produced and Britain lost its reputation as makers of high quality products.

Graunt's innovation

- What Graunt advocated was not new with him. Several other British leaders had suggested similar actions and other European countries had actually established welfare systems.
- But what was new was supporting his arguments with statistics.

Other uses

- Graunt used birth and death data, an estimate of the fertility rate of women, and some other guessed parameters to estimate the size of the population of the country and of the cities. His estimating technique included taking some sample counts in representative parishes and extrapolating from that.
- With such tools, Graunt came up with informed estimates of the population much more reliable than anything else available. He could also use the same techniques to calculate an estimate for years past.
- He was able to show that the tremendous growth of the population of London was largely due to immigration rather than procreation.

Graunt's mortality table

- The statistics of mortality being kept did not include the age of people at death.
- Graunt had to infer this from other data. He did so and created a table of mortality, indicating the survival rates at various ages of a theoretical starting population of 100 newborns.

Age	Survivors
0	100
6	64
16	40
26	25
36	16
46	10
56	6
66	3
76	1

What went into the table

- Since Graunt had no statistics on age at death, all of these are calculations. The figure of 36% deaths before the age of 6 results from known data on causes of death, assuming that all those who died of traditional children's diseases were under 6 and half of those who died from measles and smallpox were under 6. That gave him the data point of 64 survivors at age 6. He also concluded that practically no one (i.e., only 1 in 100) lived past 75.
- That gave him the two data points for ages 6 and 75.
- The other figures come from solving $64(1-p)^7=1$, and rounding off to the nearest integer. Solving gives $p \approx 3/8$.

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66	3
76	1

The power of numbers

- Graunt's table was widely accepted as authoritative. It was based upon real data and involved real mathematical calculations. It must be correct.
- Note the assumption that the death rate between 6 and 76 is uniform.
  - Hacking remarks that actually it was not far from the truth, though Graunt could hardly have known this.

Annuities

- Annuities distinguished from loans with interest:
  - Loan: *A* transfers an amount to *B. B* pays *A* a series of regular installments which may be all interest, in which case the loan is perpetual, or combined interest and principal, which eventually pays back the original amount.
  - Annuity: Very much the same except that principal and interest were not distinguished. AND it was not conceived as a loan and therefore not subject to the charge of usury.

Kinds of Annuities

- Perpetual: paid the same amount out at the stated interval forever. Identical to an interest only loan.
- Terminal: paid a fixed amount at each regular interval for a designated *n* number of intervals and then ceased.
- Life: paid a fixed amount out at the stated intervals for the life of the owner and then ceased.
- Joint: paid a fixed amount out at the stated intervals until the death of the last surviving owner.

Fair price to purchase an annuity

- First establish the proper interest rate, *r*, i.e. the time value of money.
- Perpetual annuities: Fair price, *F*, is the amount that generates the regular payment, *p*, at the interest rate, *r*. Hence $F=p/r$.
- Terminal: Calculate the same as for a mortgage, except the payment *p*, and the number of periods *n* are known and the fair price *F* is the unknown.
- Life: Calculate as a terminal annuity, where *n* is the life expectancy of the owner.
- Joint: A more difficult calculation where it is necessary to determine the life expectancy of all the owners.

What actually was done

- Ullpian, Roman jurist of the 3rd century left a table of annuities in which a 20-year old had to pay £30 to get £1 per year for life and a 60-year old had to pay £7 to get £1 per year.
  - Hacking comments that neither price is a bargain. The amounts were not calculated by actuarial knowledge.
- On the other hand in England in 1540 a government annuity was deemed to cost "7 years' purchase," meaning that it was set equivalent to a terminal annuity with a period *n* of 7 years.
  - The contract made no stipulation about the age of the buyer. Not until 1789 did Britain tie the price of an annuity to the age of the buyer.

Problems to be worked out

- No mortality tables could be relied upon on which to base fair values.
- Even with some data, the tendency was to try to force the data into a smooth function, e.g. a uniform death rate.
- Not even certain what people's ages were, so unless the annuity was purchased at birth, one could not be certain of the age of the owner.

Equipossibility

- A fundamental problem in all probability calculations is the Fundamental Probability Set of outcomes that have equal probability of happening.
- Once probabilities are to be applied outside of artificial situations such as gaming, it is considerably more difficult to establish what events are equally probable.

The principle of indifference

- A concept named in the 20th century by John Maynard Keynes, but articulated in the 17th century by Leibniz and then stated as a fundamental principle by Laplace in the 18th century.
- Two events are viewed as equally probable when there is no reason to favour one over the other.

Inductive logic

- Leibniz, anxious to use the mathematical apparatus of probability to decide questions of jurisprudence, proposed a "new kind of logic" that would calculate the probability of statements of fact in order to determine whether they were true. The statements with the highest probability score would be judged to be true.

The Art of Conjecturing

- Jacques Bernoulli's *Ars conjectandi* appeared in 1713
  - Probability emerges fully with this book.
  - Contains the first limit theorem of probability.
  - Establishes the addition law of probability for disjoint events.
  - The meaning of the title: The "Port Royal Logic" was titled *Ars cogitandi*, the Art of Thinking.
    - The art of conjecturing takes over where thinking leaves off.

Degree of certainty

- Bernoulli states that “Probability is degree of certainty and differs from absolute certainty as the part differs from the whole.”
- Etymological distinction: “certain” used to mean decided by the gods.
- Therefore events that were uncertain were those where the gods could not make up their minds.

Question: Does uncertain mean undetermined?

- Does the existence of uncertainty imply a principle of indeterminism?
- These are questions still debated by philosophers.

The first limit theorem

- Bernoulli’s theorem, in plain language, is that for repeatable and or in all ways comparable events (e.g. coin tosses), the probability of a particular outcome is the limit of the ratio of that outcome to all outcomes as the number of trials increases to infinity.

More formally expressed…

- If p = the true probability of a result
- s_n = the number of such results after n trials
- e = the error, or deviation of the results from the true probability, i.e. $| p - s_n |$
- Bernoulli shows how to calculate a number of trials n necessary to guarantee a moral certainty that the error e is less than some specified number. (A confidence interval.)

Hacking on Bernoulli

- Ian Hacking discusses the manifold meanings that can be given to Bernoulli's calculation and its implication for questions about the nature of chance, the temptation to view unlike events as comparable, so as to apply the rule, and so on.

Design

- Probability laws applied:
- In the 18th century, scientific laws were absolute: Newton's laws, for example, describe an absolutely deterministic universe. Our only uncertainty is our knowledge.
- Meanwhile, the world around is full of variations and uncertainties.

Design, 2

- The living world, in particular exhibited immense variations, yet there was an underlying stability.
- The discovery of stable probability laws and stable frequencies of natural events (e.g., the proportions of males to females) suggested a guiding hand.
- Hence the Design Argument.

The Design Argument and Probability

- Stability in probabilities suggested that there was a divine plan. The frequencies exhibited in nature came not from an inherent randomness, but from a divine intervention that caused the proportion of males to females, set the average age of death, the amount of rainfall, and so on.

Induction

- Finally, Hacking takes up the matter of induction in science: the stating of universal principles of nature on the basis of incomplete knowledge of the particulars.
- Hacking holds that the entire philosophical discussion of induction was not even possible until such time as probability emerged:
  - Until the high sciences of mathematics, physics, and astronomy found a way to co-exist with the low sciences of signs: medicine, alchemy, astrology. This ground was found through probability.