PHYS 1420 (F19)
Physics with Applications to Life Sciences

2019.11.15

Relevant reading:
Kesten & Tauck ch. N/A

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Ref. (re images):
Wolfson (2007), Knight (2017),
Kesten & Tauck (2012)
Here are four different views of the same cube.

What would the cube look like unfolded?
Announcements & Key Concepts (re Today)

→ Written HW #2: Posted and due TODAY (Friday 11/15) in class

→ Online HW #8 (re fluids): Posted and due next Friday (11/22)

→ Final exam: Saturday, Dec. 14 (start preparing!)

Some relevant underlying concepts of the day...

- Molecular motors
- Swimming when you are small....
- Fluid basics
- Reynold's #
Recall: Biophysical notion of *Passive vs Active*

- **Passive**: movement is subject to the medium you are in moving you around
- **Active**: you move yourself around (e.g., swim)

→ What is the mechanism(s) underlying "active"?

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**Motile Behavior of Bacteria**

January 2000  Physics Today

*E. coli*, a self-replicating object only a thousandth of a millimeter in size, can swim 35 diameters a second, taste simple chemicals in its environment, and decide whether life is getting better or worse.

Howard C. Berg
Bacterial motility: Motor is required

- Some sort of “flagellum” and energy-consuming “motor” is required

**Figure 2. Bacterial Motor and Drive Train.** (a) Rotationally averaged reconstruction of electron micrographs of purified hook-basal bodies. The rings seen in the image and labeled in the schematic diagram (b) are the L ring, P ring, MS ring, and C ring. (Digital print courtesy of David DeRosier, Brandeis University.)
Bacterial motility: Motor powers the propeller

**Newton’s first law of motion:** A body in uniform motion remains in uniform motion, and a body at rest remains at rest, unless acted on by a nonzero net force.

**Newton’s second law of motion:** The rate at which a body’s momentum changes is equal to the net force acting on the body:

\[ \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \]  

(Newton’s 2nd law) (4.2)

**Newton’s third law of motion:** If object A exerts a force on object B, then object B exerts an oppositely directed force of equal magnitude on A.

→ Newton's Laws still apply!

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**Fig. 6.3.** Analysis of viscous drag on two segments of a flagellar filament moving slowly to the right and turning rapidly counterclockwise. The velocity of each segment, \( v \), is decomposed into velocities normal and parallel to the segment, \( v_n \) and \( v_p \), respectively. The segment shown on the left is moving upward in front of the plane of the paper; the one shown on the right (denoted by primes) is moving downward behind the plane of the paper. The frictional drags normal and parallel to each segment, \( F_n \) and \( F_p \), act in directions opposite to \( v_n \) and \( v_p \), respectively. Note that their magnitudes are in the ratios \( F_n/F_p = 2v_n/v_p \). \( F_n \) and \( F_p \) are decomposed into components normal and parallel to the helical axis, \( F_n' \) and \( F_p' \), respectively. \( F_n' \) and \( F_p' \) act in opposite directions and form a couple that contributes to the torque. \( F_v' \) and \( F_v' \) act in the same direction and contribute to the thrust.

Berg (1993)
Fluids

> Implicitly or explicitly, much of our recent efforts have revolved around the notion of _fluids_...
Moving along....

**Question:**
What differences are there for micro- vs. macro-scopic motors?

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**Life at low Reynolds number**

E. M. Purcell  
*Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138*  
(Received 12 June 1976)

American Journal of Physics, Vol. 45, No. 1, January 1977

But I want to take you into the world of very low Reynolds number—a world which is inhabited by the overwhelming majority of the organisms in this room. This world is quite different from the one that we have developed our intuitions in.

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**Note:** Purcell (1912-1997) won the 1952 Nobel Prize for his work on NMR
Reynolds Number

Reynolds number ($R$) is a dimension-less number that indicates the ratio of inertial to viscous forces.

\[ \frac{\text{inertial forces}}{\text{viscous forces}} \approx \frac{avp}{\eta} \]

\[ R = \frac{avp}{\eta} = \frac{av}{\nu} \]

\[ = 10^2 \text{ cm}^2/\text{sec} \quad \text{for water} \]
Aside: Density

We define density $\rho$ in terms of mass $M$ and volume $V$:

$$\rho = \frac{M}{V}$$

$\rho_{\text{water}} = 1000 \text{ kg/m}^3 = 1 \text{ g/cm}^3$.

<table>
<thead>
<tr>
<th>Substance (at 0°C unless otherwise noted)</th>
<th>Density ($\text{kg/m}^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>air at 20°C</td>
<td>1.217</td>
</tr>
<tr>
<td>alcohol (ethanol)</td>
<td>$0.806 \times 10^3$</td>
</tr>
<tr>
<td>aluminum</td>
<td>$2.70 \times 10^3$</td>
</tr>
<tr>
<td>bone</td>
<td>$1.7-2.0 \times 10^3$</td>
</tr>
<tr>
<td>copper</td>
<td>$8.93 \times 10^3$</td>
</tr>
<tr>
<td>earth (average)</td>
<td>$5.52 \times 10^3$</td>
</tr>
<tr>
<td>glass (common)</td>
<td>$2.4-2.8 \times 10^3$</td>
</tr>
<tr>
<td>gold</td>
<td>$19.3 \times 10^3$</td>
</tr>
<tr>
<td>helium</td>
<td>0.1786</td>
</tr>
<tr>
<td>ice</td>
<td>$0.917 \times 10^3$</td>
</tr>
<tr>
<td>iron</td>
<td>$7.8 \times 10^3$</td>
</tr>
<tr>
<td>lead</td>
<td>$11.3 \times 10^3$</td>
</tr>
<tr>
<td>mercury</td>
<td>$13.6 \times 10^3$</td>
</tr>
<tr>
<td>seawater at 4°C</td>
<td>$1.025 \times 10^3$</td>
</tr>
<tr>
<td>steam at 100°C</td>
<td>0.6</td>
</tr>
<tr>
<td>water at 4°C</td>
<td>$1.000 \times 10^3$</td>
</tr>
<tr>
<td>water at 20°C</td>
<td>$0.998 \times 10^3$</td>
</tr>
<tr>
<td>wood (maple)</td>
<td>$0.75 \times 10^3$</td>
</tr>
<tr>
<td>wood (pine)</td>
<td>$0.50 \times 10^3$</td>
</tr>
</tbody>
</table>
Aside: Viscosity

- Viscosity ($\eta$) deals how a liquid “deforms” due to stress (i.e., forces) applied to it.

- Tied to how individual fluid molecules interact and friction arising from such.

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Table 5.1: Density, viscosity and viscous critical force for some common fluids at 25°C.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$\rho_m$ (kg m$^{-3}$)</th>
<th>$\eta$ (Pa · s)</th>
<th>$f_{crit}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
<td>$2 \cdot 10^{-5}$</td>
<td>$4 \cdot 10^{-10}$</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>0.0009</td>
<td>$8 \cdot 10^{-10}$</td>
</tr>
<tr>
<td>Olive oil</td>
<td>900</td>
<td>0.080</td>
<td>$4 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>Glycerine</td>
<td>1300</td>
<td>1</td>
<td>0.0008</td>
</tr>
<tr>
<td>Corn syrup</td>
<td>1000</td>
<td>5</td>
<td>0.03</td>
</tr>
</tbody>
</table>
For “swimmers”, $R$ varies quite a lot depending upon size.....
Low Reynolds Number

At "low Reynolds number): If you stop swimming, you *stop* (i.e., no coasting)

\[ \eta = 1 \text{ centipoise} \quad \nu = 10^{-2} \text{ cm}^2/\text{sec} \]

\[ R = 3 \times 10^{-5} \]

\[
\begin{align*}
\text{coasting distance} &= 0.1 \text{ Å} \\
\text{coasting time} &= 0.3 \text{ microsec.}
\end{align*}
\]

Figure 4.
Low Reynolds Number

- When $R$ is small, “weird” things can happen.....

Figure 5.1: (Photographs.) An experiment showing the peculiar character of low Reynolds-number flow. (a) A small blob of colored glycerine is injected into clear glycerine in the space between two concentric cylinders. (b) The inner cylinder is turned through four full revolutions, apparently mixing the blob into a thin smear. (c) Upon turning the inner cylinder back exactly four revolutions, the blob reassembles, only slightly blurred by diffusion. The finger belongs to Sir Geoffrey Taylor. [Copyrighted figure; permission pending.][From (Shapiro, 1972).]
Low Reynolds Number

Wait, you can simply “unmix” by “reversing” time?

Mixing is hard!

Tied to that: swimming is different!

Figure 5.2: (Schematics.) Shearing motion of a fluid in laminar flow, in two geometries. (a) Cylindrical (ice-cream maker) geometry, viewed from above. The central cylinder rotates while the outer one is held fixed. (b) Planar (sliding plates) geometry. The top plate is pushed to the right while the bottom one is held fixed.
Low Reynolds Number

If $R \ll 1$:
Time doesn't matter. The pattern of motion is the same, whether slow or fast, whether forward or backward in time.

- “Reciprocal motion”

The Scallop Theorem

- A scallop wouldn’t be able to swim for low $R$

- Need more than one degree of freedom
Low Reynolds Number

A scallop wouldn’t be able to swim for low $R$

Figure 7.6  (a) A scallop with a single hinge is unable to move forward at low Reynold’s number due to the time symmetry of the Navier–Stokes equation. (b) Purcell’s hypothetical two hinge organism is able to propel forward using a series of conformational changes.
Possible motor designs....

Figure 5.5: (Schematic.) Three swimmers. (a) The flapper makes reciprocal motion. (b) The twirler cranks a stiff helical rod. (c) The spinner swings a stiff, straight rod.

Figure 5.9: (Schematic; photomicrograph.) (a) The bacterial flagellar motor, showing elements analogous to those of a macroscopic rotary motor. The inner part of the motor assembly develops a torque relative to the outer part, which is anchored to the polymer network (the “peptidoglycan layer”), turning the flagellum. The peptidoglycan layer provides the rigid framework for the cell wall; it is located in the periplasmic space between the cell’s two membranes. (b) Composite electron micrograph of the actual structure of the motor assembly. Top to bottom, about 75 nm. [Digital image kindly supplied by D. Derosier; see (Derosier, 1998).] [Copyrighted figure; permission pending.]
Low Reynolds Number

Figure 7.11 Range of different strategies used by micro-organisms for motility. (a) Amoeba use cytoplasmic streaming to crawl across a surface, (b) euglena have a single flagellum and the cell body acts as a propeller, (c) sperm cells use flagellae to swim, (d) spirochetes swim using a cork screw motion, (e) chrysophytes (golden algae) have a hairy flagellum for propulsion, (f) paramecium use the coordinated beating of cilia in a metachronal wave to propel themselves, and (g) chlamydomonas (green algae) swim with multiple flagellae.
7.10 Range of velocities used for transport in cells and micro-organisms. The mechanism of motility can be classified as crawling/swimming, extension/contraction and internal transport. [Copyright 2000 from Cell: From Molecules to Motility by Bray. Reproduced by permission of Garland Science/Taylor & Francis LLC.]
Brings us back to that more general question: What are the basic mechanisms by which "stuff" moves around?
Fluids

→ Implicitly or explicitly, much of our recent efforts have revolved around the notion of *fluids*...
Fluids: Basic Considerations

- Density
- Pressure
- Pascal's Principle
- Buoyancy & Archimedes
- Fluid statics vs dynamics
- Bernoulli → Convection REVISITED
Recall (re pressure)

Pressure (a scalar, i.e., not a vector) is directly related to force

But that must mean “area” is a vector too(!!?!)

Changes in pressure is how sound energy propagates

\[ pV = NkT \] (ideal-gas law)

\[ p = \frac{F}{A} \] (pressure)

The fluid exerts pressure internally as well as on the container. The internal pressure is the same in all directions.

\[ \vec{F} \] is the force on the area \( A \), so the pressure is \( p = \frac{F}{A} \).

**FIGURE 15.1** Pressure, the force per unit area, is exerted equally in all directions.
Recall (re a barometer)

A vacuum has zero pressure, so \( p_o = 0 \) at the mercury’s surface in the tube.

Basic concepts stemming from Newtonian mechanics apply to fluids and gases too.

\[ \text{Atmospheric pressure presses on surface . . .} \]

\[ \text{760 mm} \]

\[ p_{\text{atmosphere}} \]

\[ \text{Mercury} \]

\[ \text{. . . and pushes mercury up the tube until the mercury’s weight balances the pressure force.} \]

\[ \text{FIGURE 15.4 A mercury barometer.} \]

\[ \text{Fluid element} \]

\[ \text{Pressure force on the bottom must be greater in order to balance gravity.} \]

\[ \text{FIGURE 15.3 Forces on a fluid element in hydrostatic equilibrium.} \]
Fluids: Pressure

\[ \vec{F} = P \vec{A} \]

1 Pa = \( \frac{N}{m^2} \)

So is it okay to divide two vectors then?

The fluid exerts pressure internally as well as on the container. The internal pressure is the same in all directions.

\( \vec{F} \) is the force on the area \( A \), so the pressure is \( p = \frac{F}{A} \).

**FIGURE 15.1** Pressure, the force per unit area, is exerted equally in all directions.
Fluids: Pressure & Depth (& Atmospheric pressure)

The upward force $\vec{F}_u$ and the pressure below the volume are related.

$$\vec{F}_u + \vec{F}_d + \vec{W} = 0$$

The imaginary volume is stationary because the net upward and downward forces cancel.

$$P = P_0 + \rho gd$$

The downward force $\vec{F}_d$ and the pressure above the volume are related.

$$P_{atm} = 1 \text{ atmosphere} = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$$
Fluids: Pressure difference

Changes in pressure is how sound energy propagates
Sound...

- squawk
- brr
- toot!
- wind sound
- clip
- swoosh
- clop
- swoosh
- quack!
- rustle
- chirp chirp
What is sound?

Note the periodic nature present....
Can think of sound as introducing a bit of a bias!